Slow Light and Superluminal Propagation

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Interest in Slow and Fast light

- Fundamental aspect in optical physics
- How can we stop (reversibly) the light ?
- Can light travels faster than c?

- Applications: optical storage, optical information, telecom (buffering)

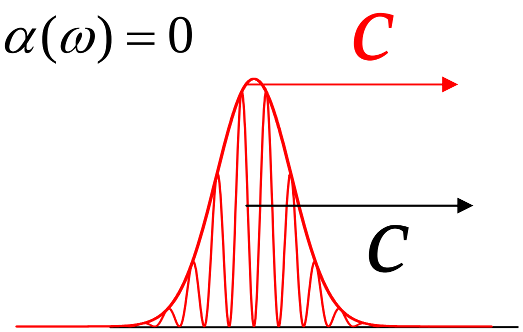
Outlook

- Group velocity, case of a resonant system
- (Ultra) Slow light: - **E**lectromagnetic **I**nduced **T**ransparency
  - **C**oherent **P**opulation **O**scillations
  - **Z**eeman **C**oherence **O**scillations. Coherent control of the optical response
- Superluminal propagation: - Relativity implications
  - Different velocities for a light pulse
  - Backward propagation
- Conclusion

Group Velocity

$n(\omega) = 1;$

$\alpha(\omega) = 0$



$$E(t,z) = E_0(t - z/c)\, e^{i(k_0 z - \omega_0 t)}$$

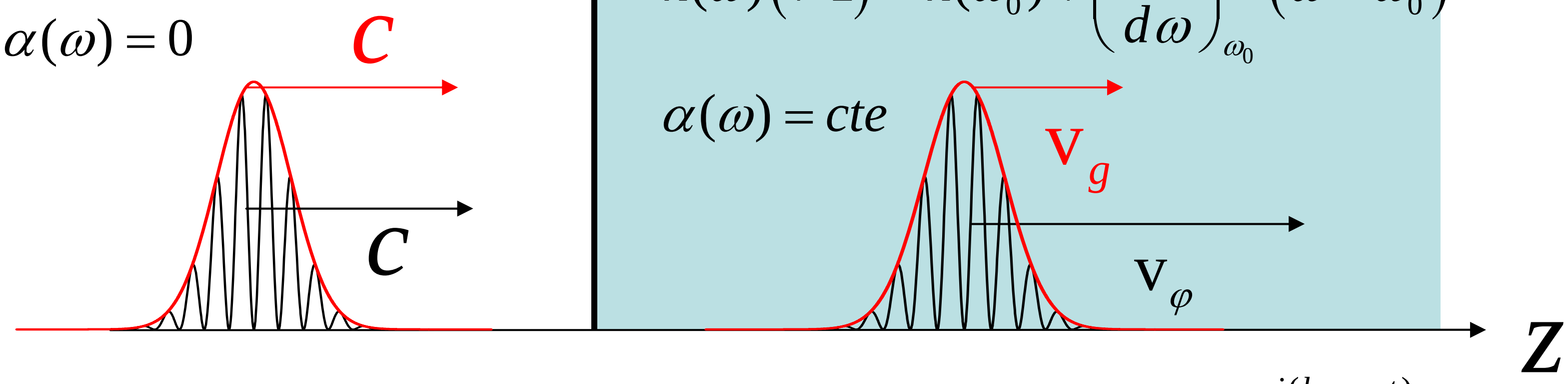
$$k_0 = \frac{\omega_0}{c}$$

Group Velocity

$$n(\omega) = 1;$$
$$\alpha(\omega) = 0$$

$$n(\omega)(\neq 1) \simeq n(\omega_0) + \left(\frac{dn}{d\omega}\right)_{\omega_0} (\omega - \omega_0)$$

$$\alpha(\omega) = cte$$



$$E(t,z) = E_0(t - z/c)\, \mathrm{e}^{i(k_0 z - \omega_0 t)}$$

$$E(t,z) = G\, E_0(t - z/\mathrm{v}_g)\, \mathrm{e}^{i(kz - \omega_0 t)}$$

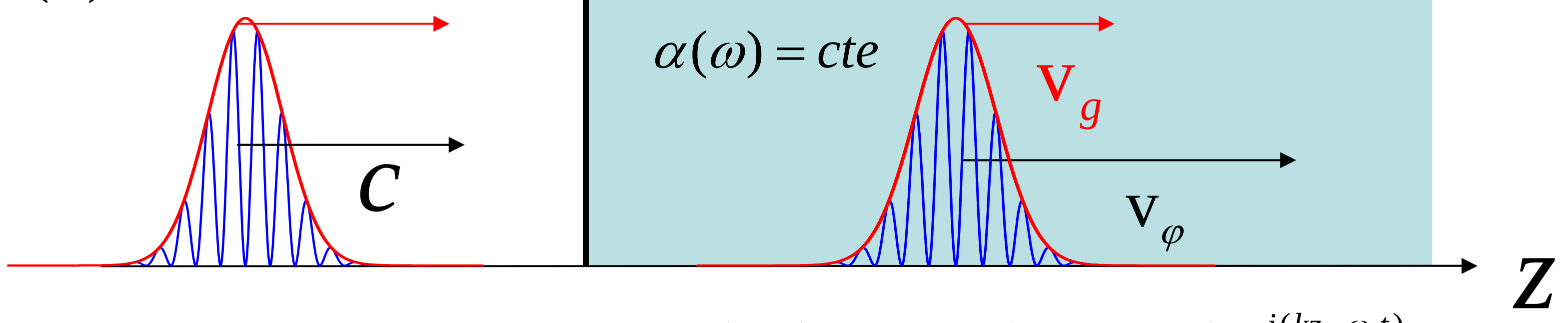
$$k_0 = \frac{\omega_0}{c}$$

Group Velocity

$$n(\omega)=1;\quad \alpha(\omega)=0$$

$$n(\omega)(\neq 1)\simeq n(\omega_0)+\left(\frac{dn}{d\omega}\right)_{\omega_0}(\omega-\omega_0)$$

$$\alpha(\omega)=cte$$



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$$E(t,z)=G\,E_0(t-z/v_g)\,e^{i(kz-\omega_0 t)}$$

$$k_0=\frac{\omega_0}{c}$$

$$k=\frac{\omega_0}{v_\varphi}$$

$v_\varphi = c/n(\omega_0)$: Phase velocity

Group Velocity

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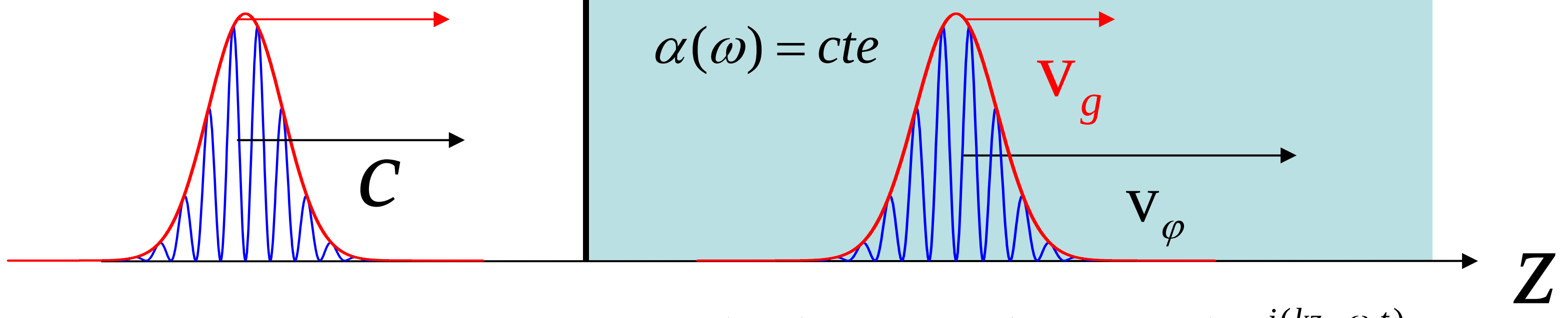

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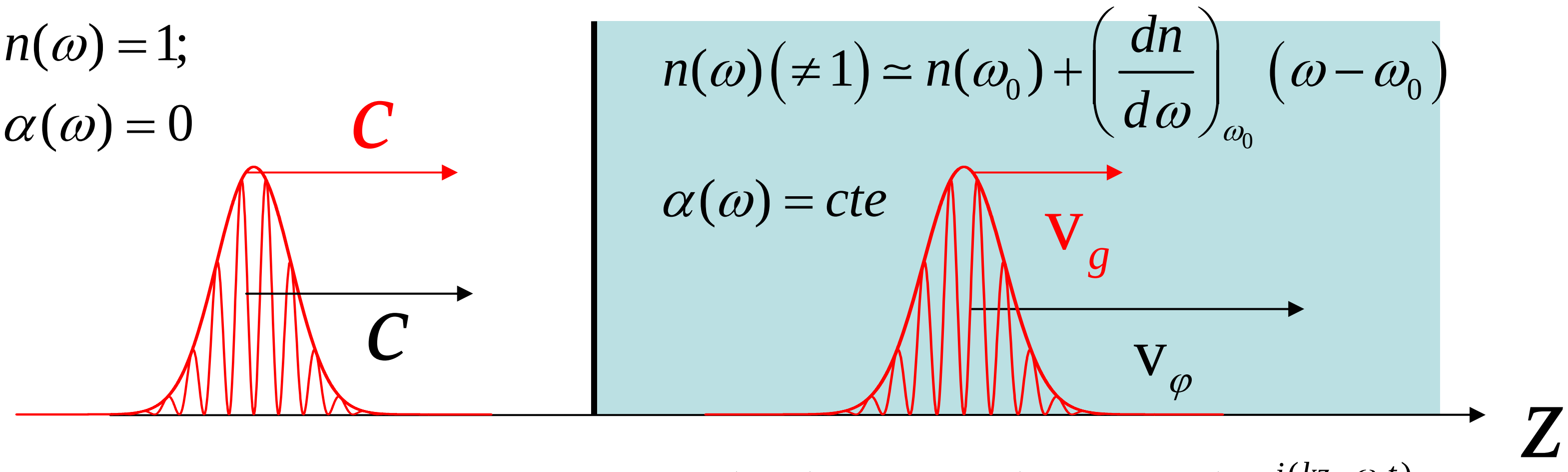
$$k = \frac{\omega_0}{\mathrm{v}_\varphi}$$

$$\mathrm{v}_\varphi = c / n(\omega_0):$$ Phase velocity

$$\mathrm{v}_g = \frac{c}{n(\omega_0) + \omega_0 \left(\frac{dn}{d\omega}\right)_{\omega_0}}:$$ Group velocity

Group Velocity



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$\mathrm{v}_g=\dfrac{c}{n(\omega_0)+\omega_0\left(\dfrac{dn}{d\omega}\right)_{\omega_0}}:$ Group velocity

linear-dispersive medium with constant spectral gain/abs: PROPAGATION WITHOUT DISTORTION

High orders contributions: RESHAPING EFFECTS $e^{i(\omega/c)nL}$, $e^{\alpha L}$

$$V_g = \frac{c}{n_g}; \quad n_g = n + \omega \frac{dn}{d\omega}: \quad \textit{Group index}$$

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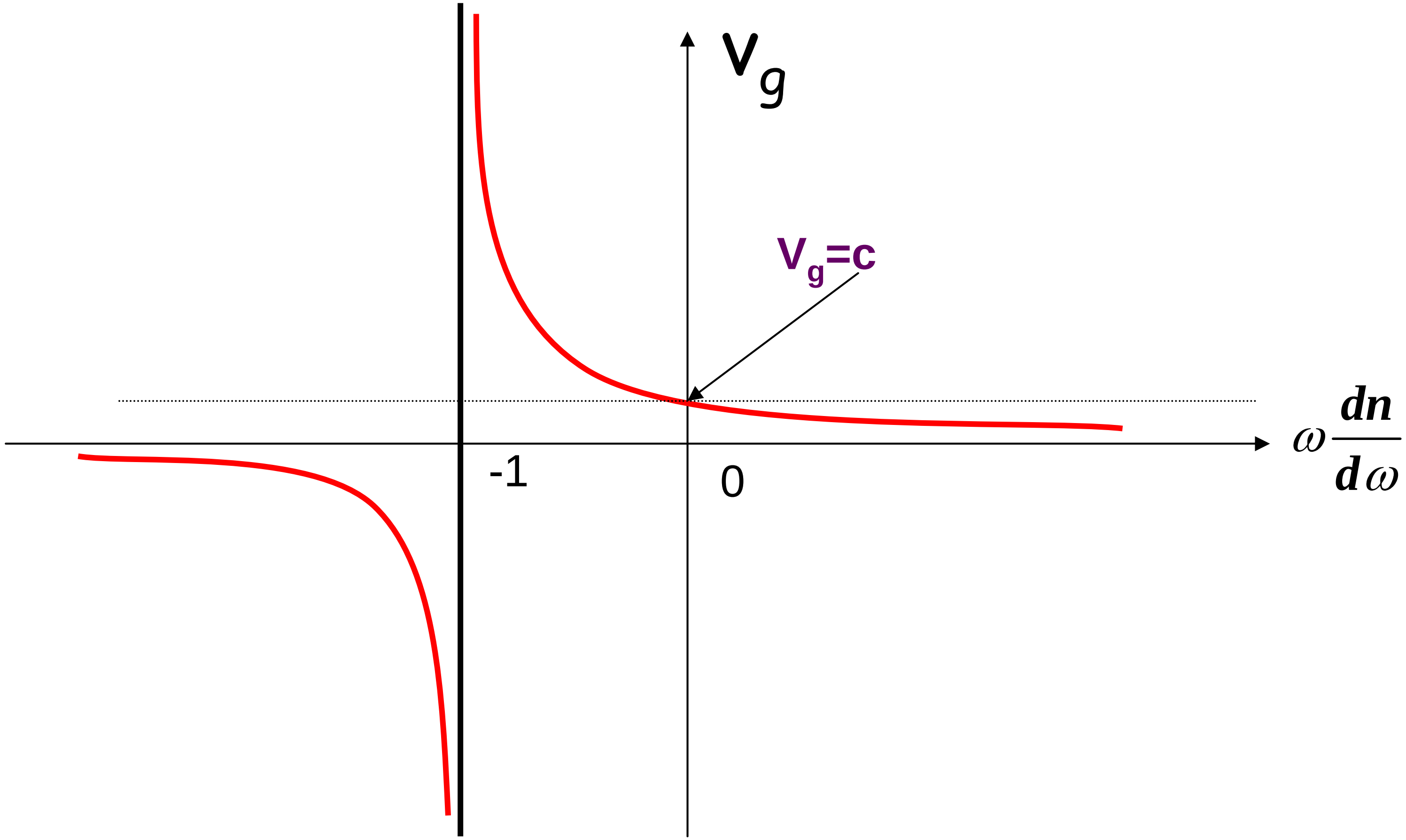

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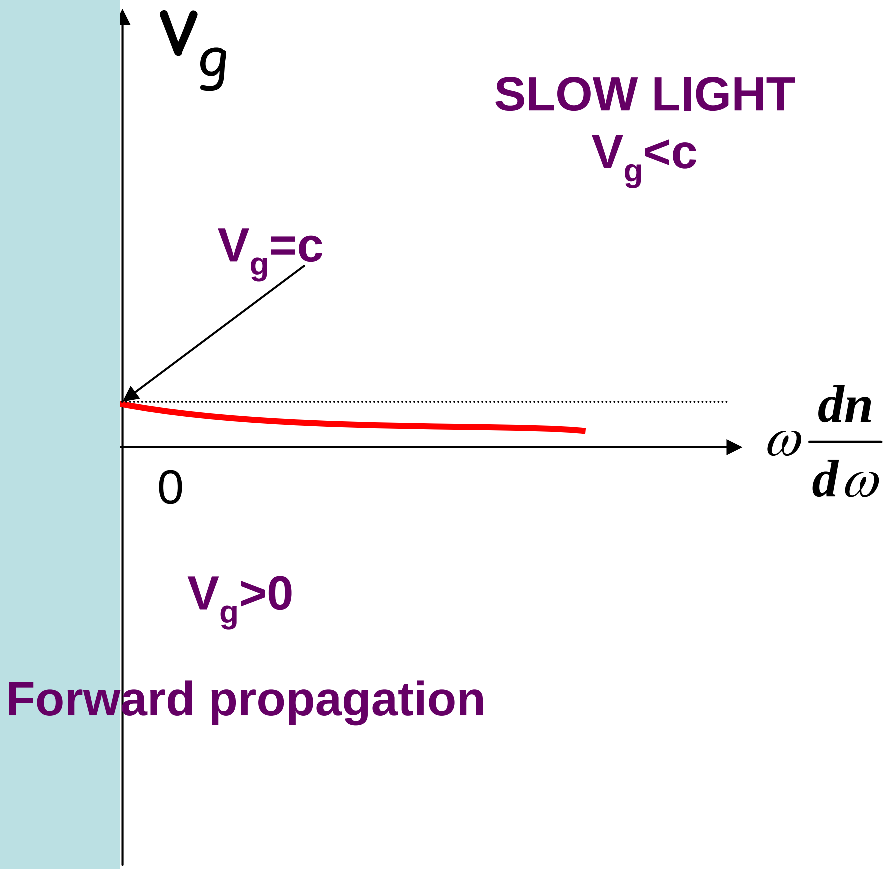

NORMAL DISPERSION

$$V_g = \frac{c}{n_g}; \quad n_g = n + \omega \frac{dn}{d\omega}: \quad \textit{Group index}$$

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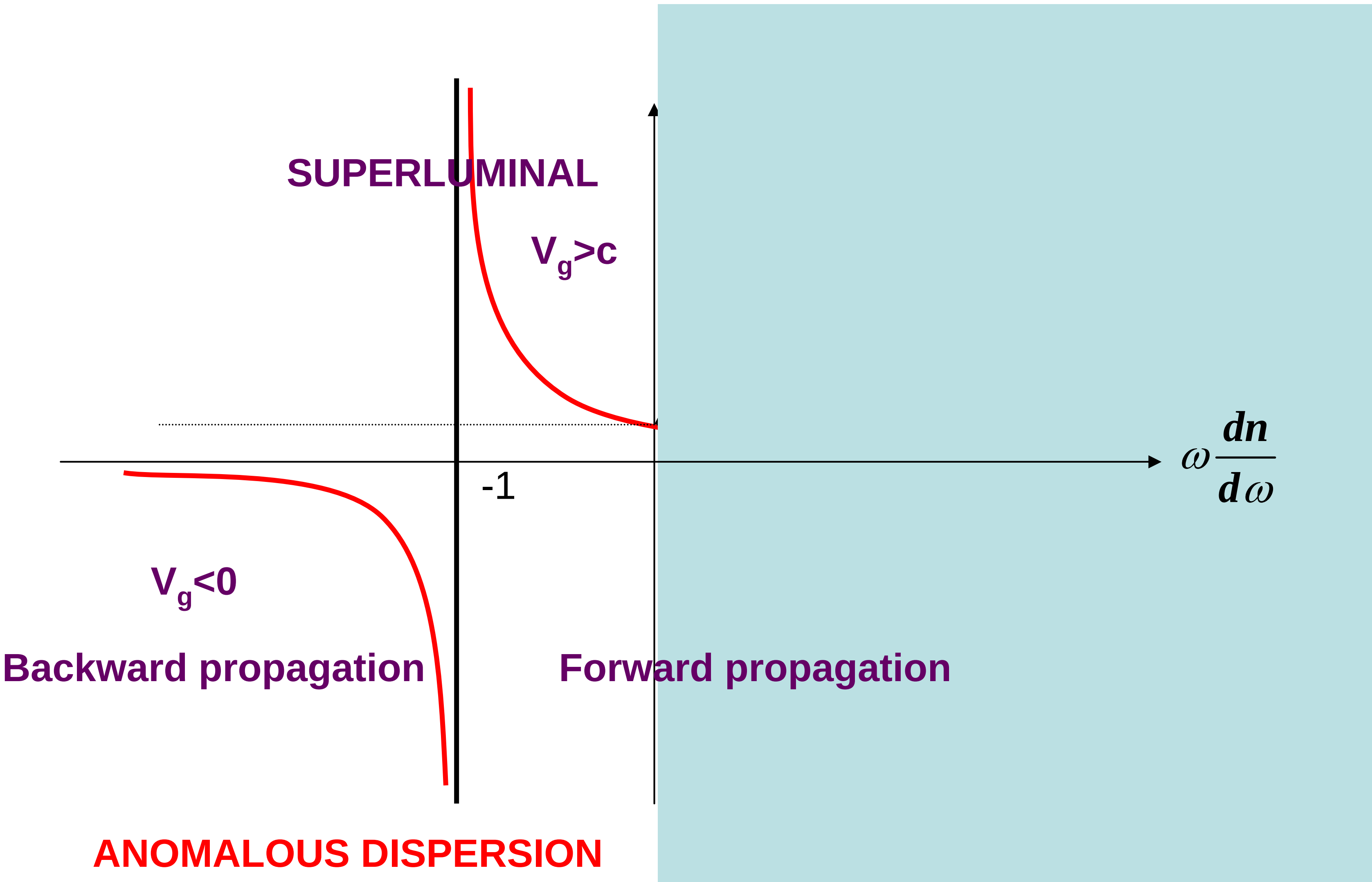


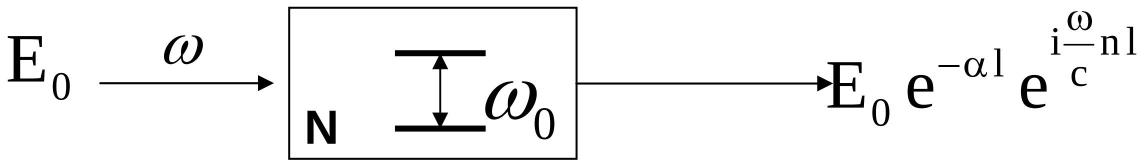
- Ultra slow light needs w dn/dw >>1

How to achieve ??

- $v_g > c$: What about relativity?

- $v_g < 0$: Propagation of energy?

What happens near a resonant transition?



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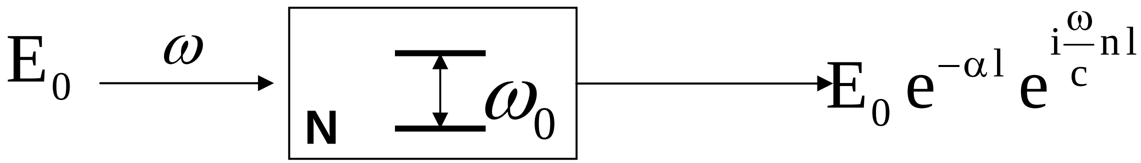
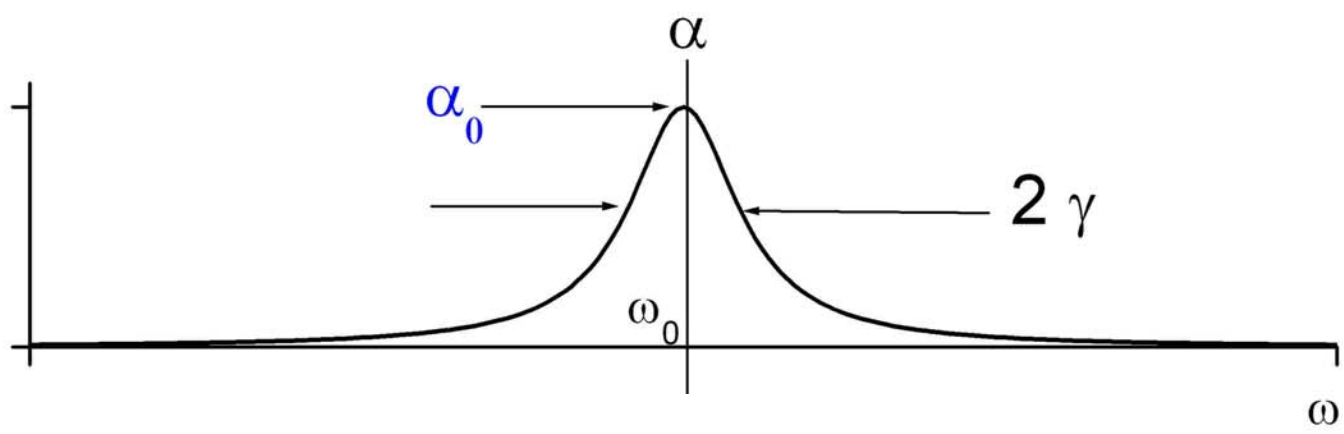


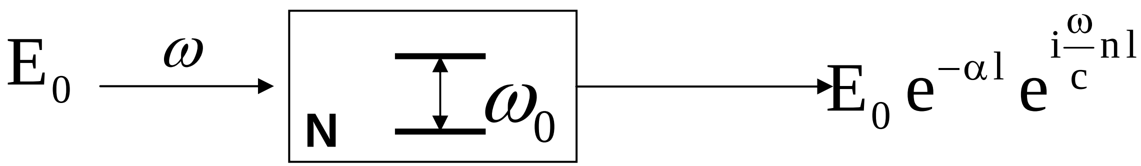
$$\alpha = \alpha_0 \frac{\gamma^2}{(\omega_0 - \omega)^2 + \gamma^2} : \textit{absorption coefficient}$$

$$\alpha_0 = \frac{N \mu^2 \omega_0}{c \varepsilon_0 \hbar \gamma} : \textit{absorption at resonance frequency}, \quad \gamma : \textit{relaxation rate}$$



α
α0
2 γ
ω0
ω

What happens near a resonant transition?



$$n = 1 + \frac{2c\alpha_0\gamma}{\omega_0}\frac{\omega_0-\omega}{(\omega_0-\omega)^2+\gamma^2} : \textit{refraction index}$$

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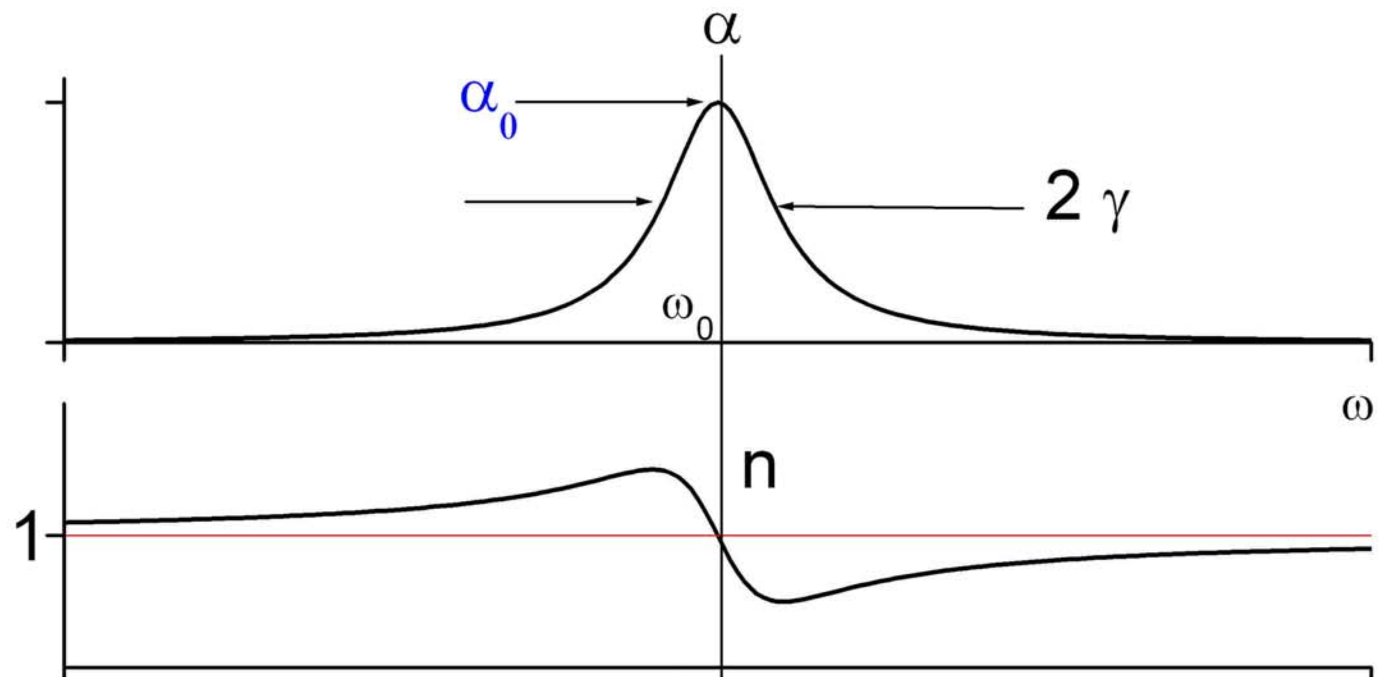
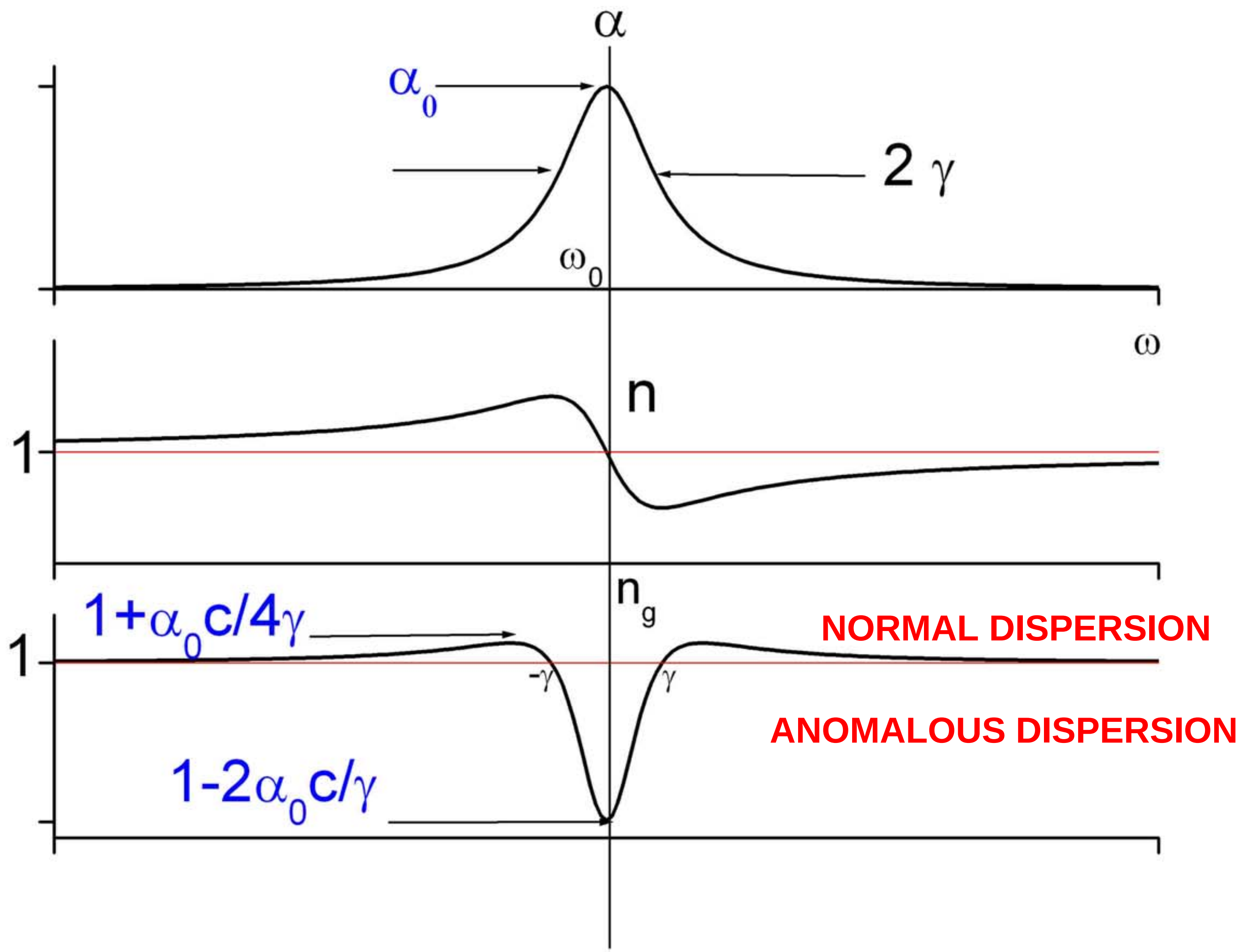


α
α0
2 γ
ω0
ω
n
1



Increasing $|n_g|$ is possible but absorption also increases!
Difficult to observe experimentally

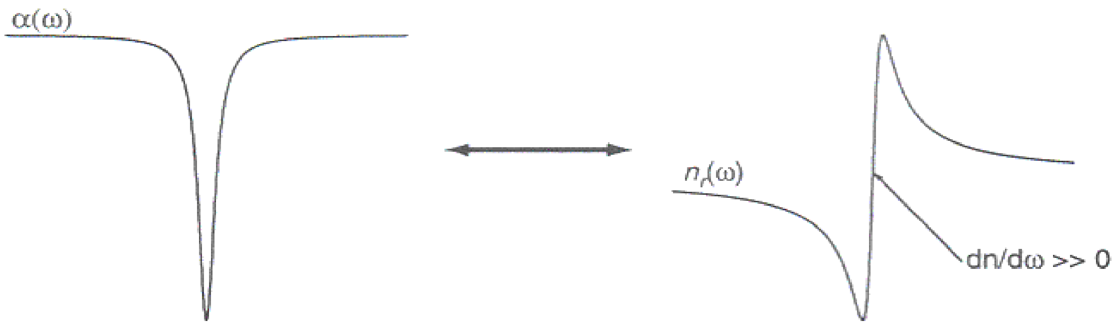
I- How to produce slow-fast light?

$$V_g = \frac{c}{n + \omega \frac{dn}{d\omega}}:$$

Kramers-Kronigs relations

$$n(\omega) - 1 = \frac{c}{\pi} \wp \int_0^\infty \frac{\alpha(\omega')}{\omega'^2 - \omega^2} d\omega'$$

$$\alpha(\omega) = -\frac{4\omega^2}{\pi c} \wp \int_0^\infty \frac{n(\omega') - 1}{\omega'^2 - \omega^2} d\omega'$$



A narrow spectral hole produce a strong normal dispersion

Slow light

- Electromagnetic Induced Transparency

- Coherent Population Oscillations

- Zeeman Coherence Oscillations

II-a Electromagnetic Induced Transparency

VOLUME 66, NUMBER 20 PHYSICAL REVIEW LETTERS 20 MAY 1991

Observation of Electromagnetically Induced Transparency

K.-J. Boller, A. Imamoğlu, and S. E. Harris
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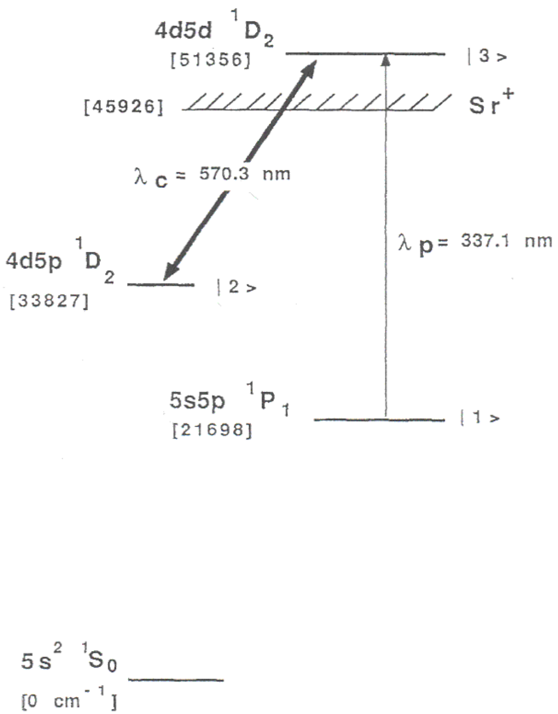

FIG. 1. Energy-level diagram of neutral Sr. Inset: Dressed-state picture.

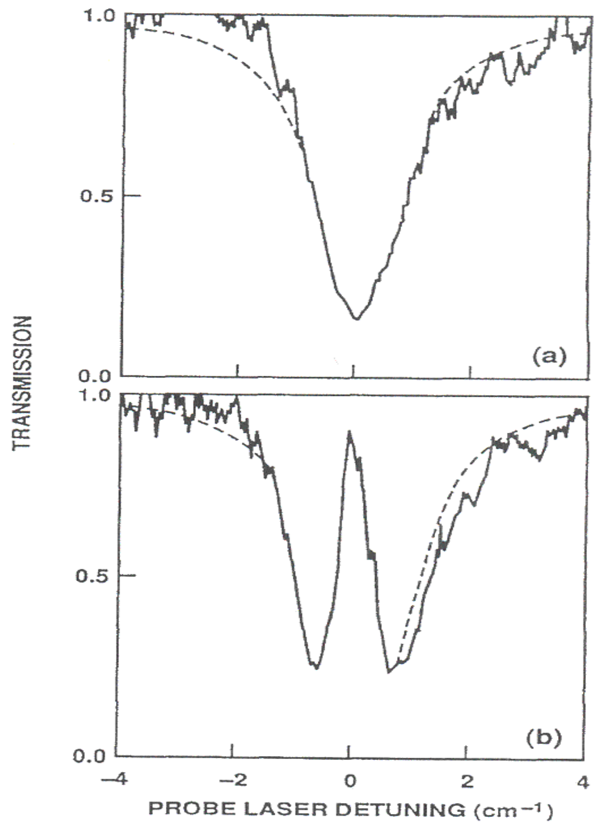

FIG. 2. Transmission vs probe laser detuning for (a) $\Omega_{23}=0$ and (b) $\Omega_{23}=1.3$ cm^{-1}, $\Delta\omega_c = -0.2$ cm^{-1}. Minimum transmission is exp(−1.7).

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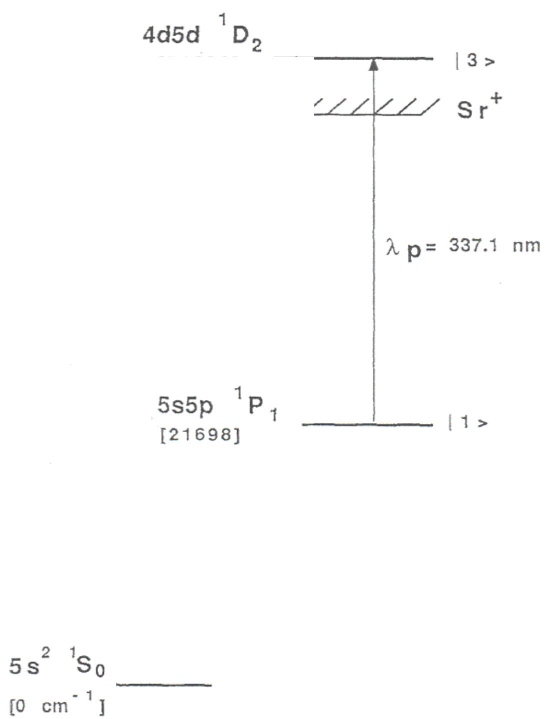

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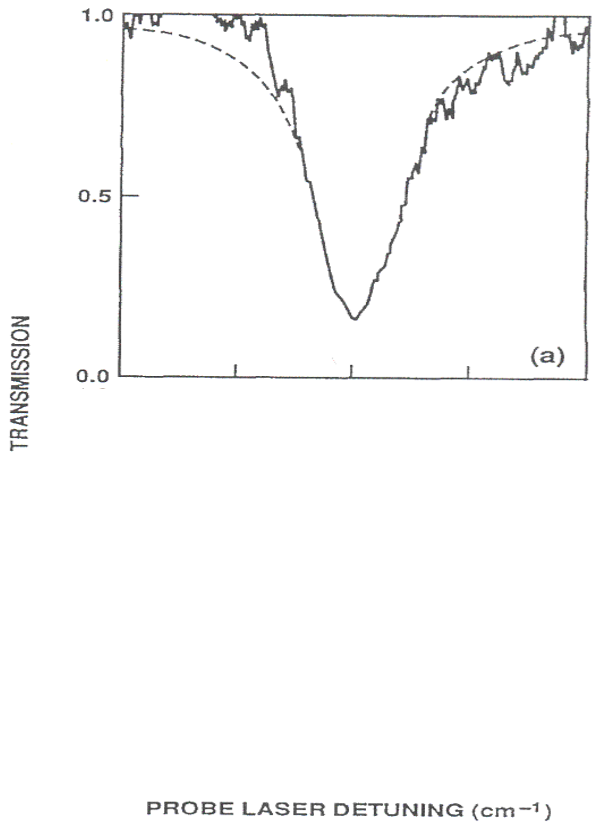

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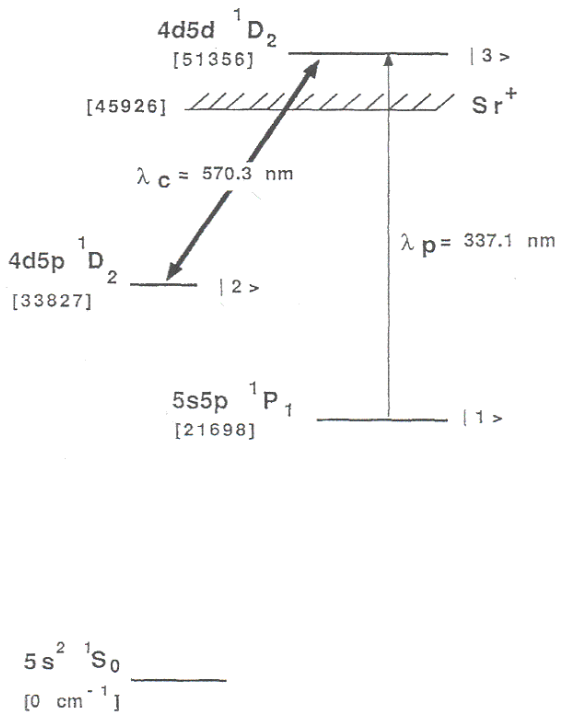

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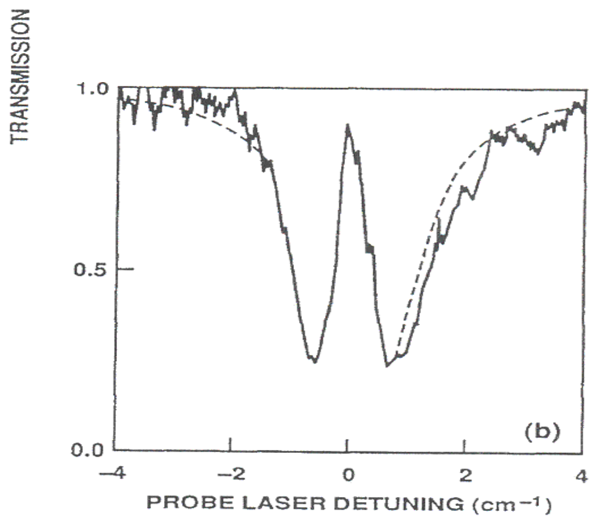

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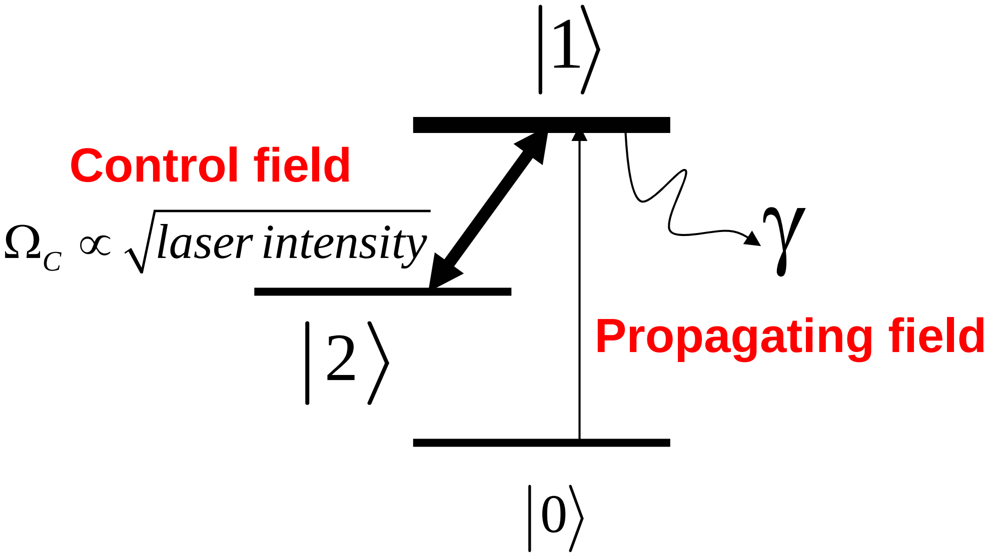
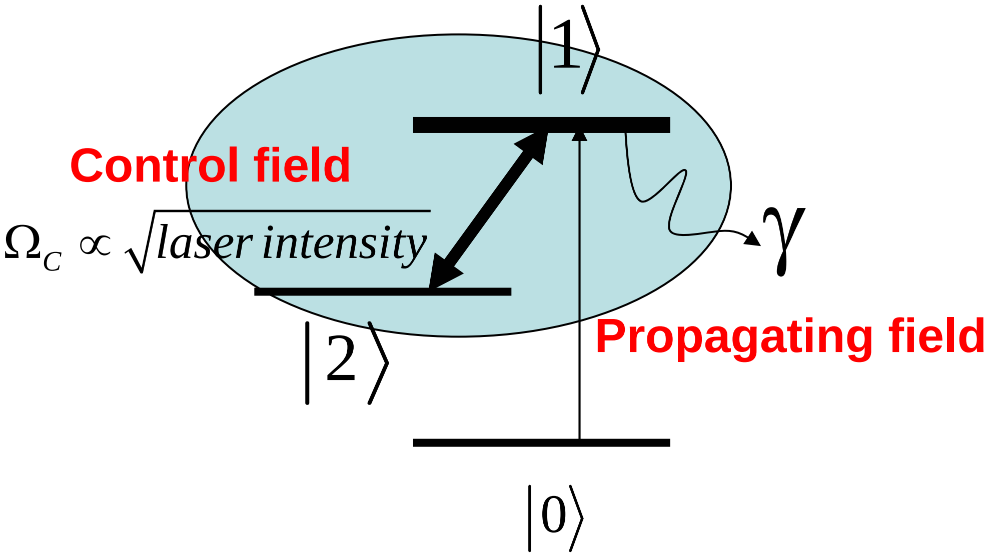
Interpretation



Interpretation

$$|\pm\rangle = \frac{|1\rangle \pm e^{-i\omega t}|2\rangle}{\sqrt{2}} : adiabatic\ states$$



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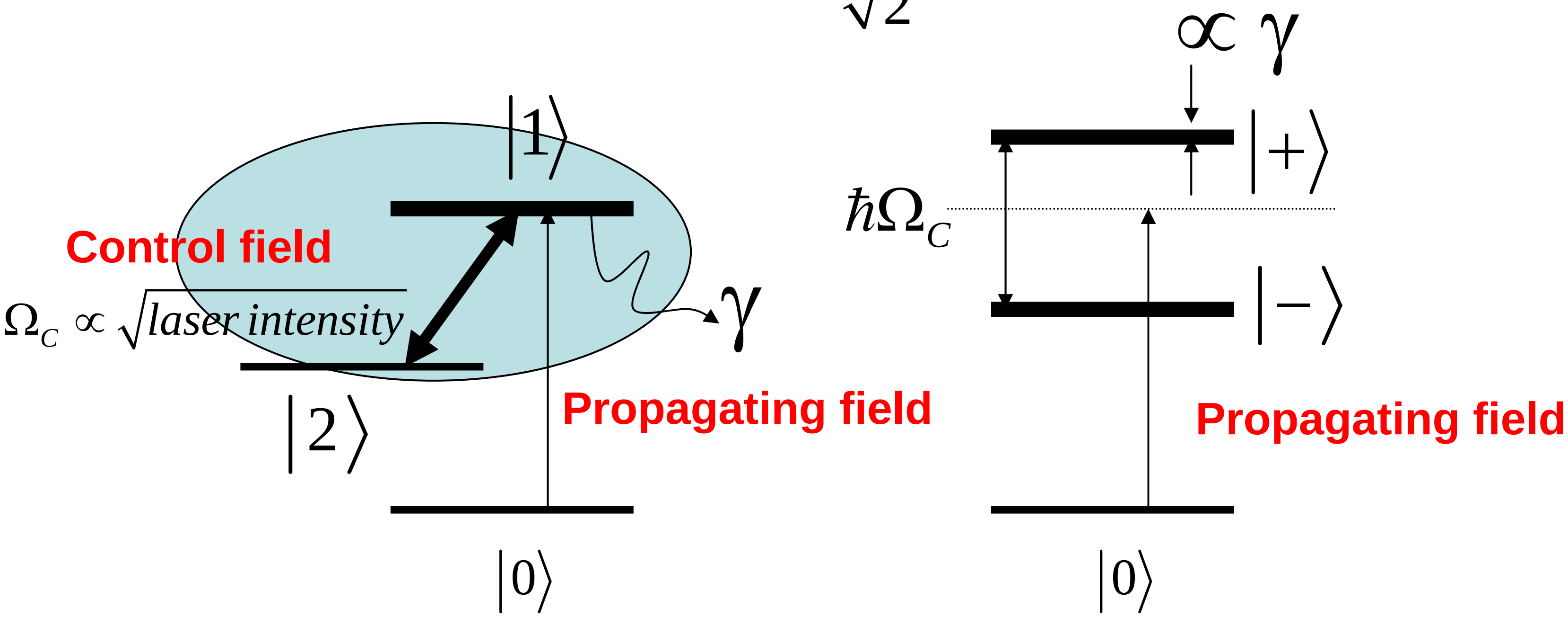

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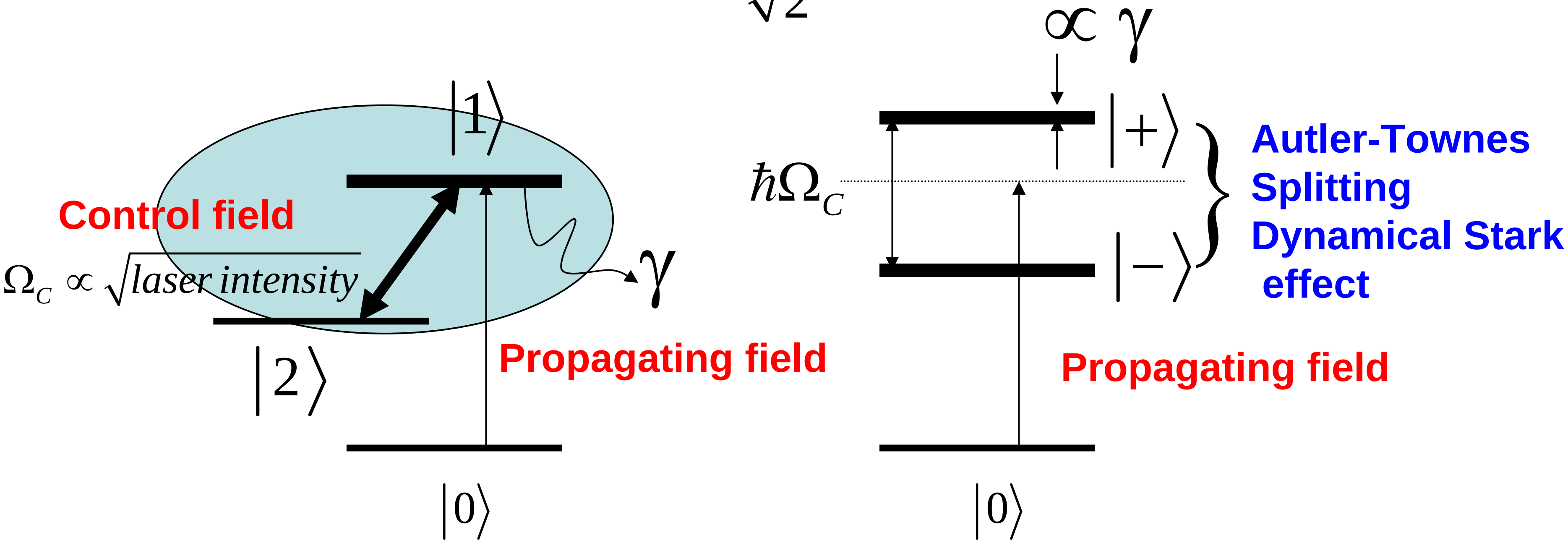

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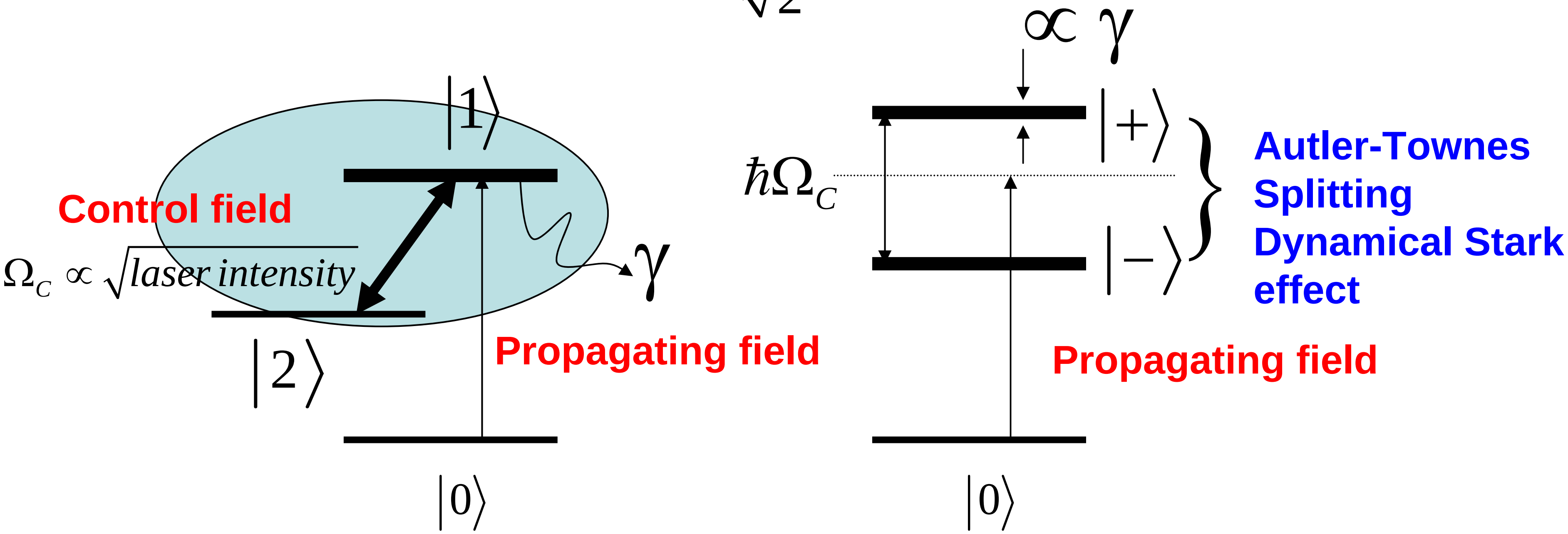
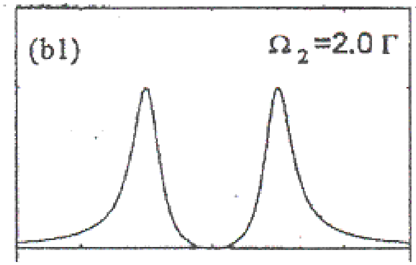


$\hbar\Omega_C >> \gamma$: **Propagating field is non-resonant**



ABSORPTION COEFFICIENT

$\omega_{propag.} - \omega_{10}$

Interpretation

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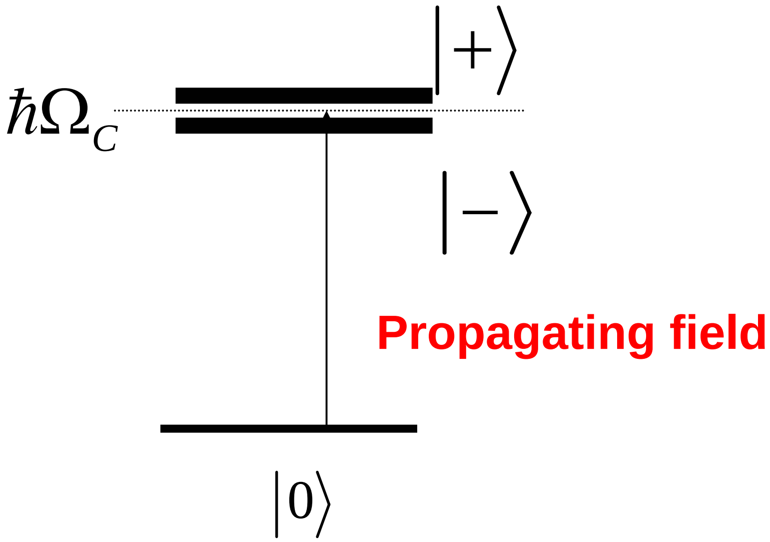
$$\hbar\Omega_C \leq \gamma$$



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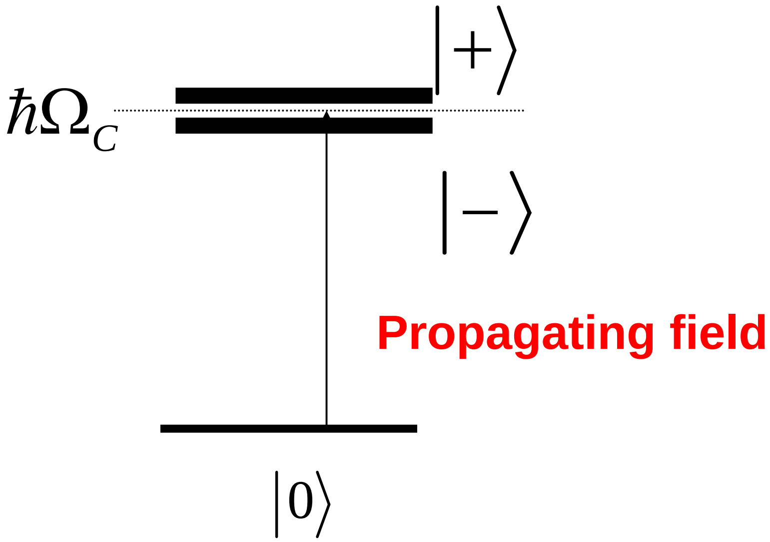
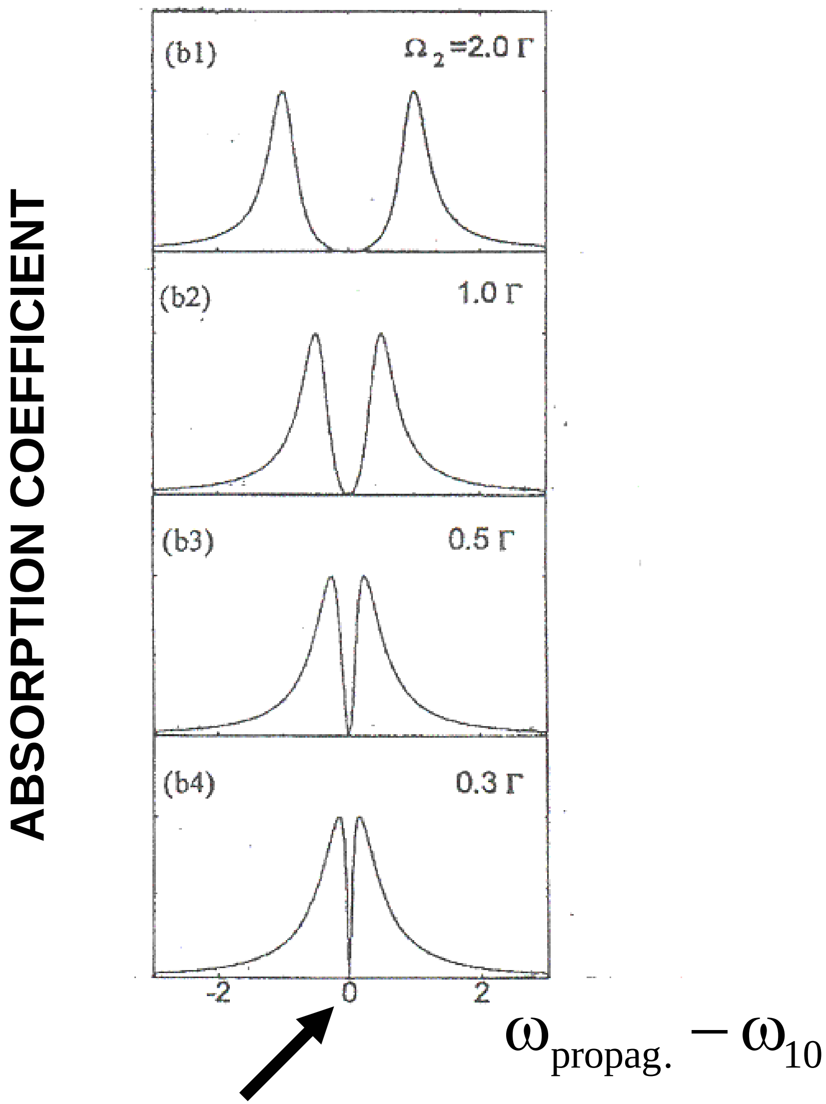
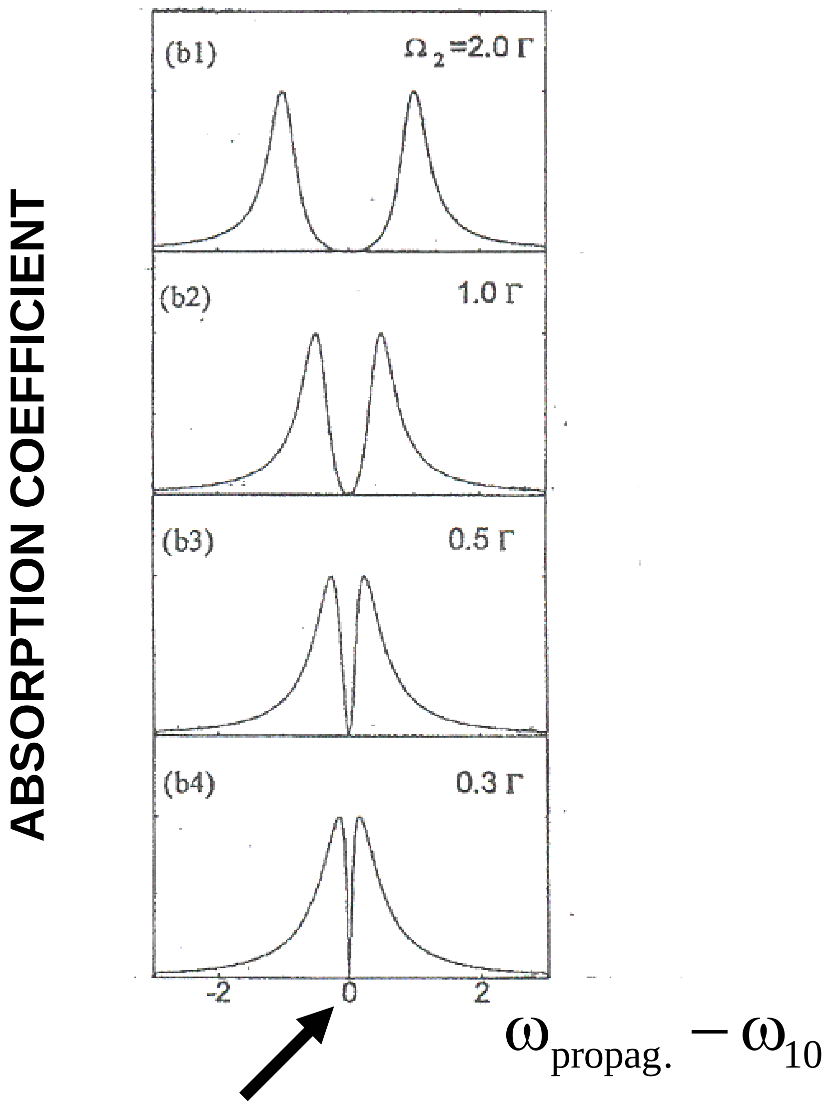


Interference between quantum paths $|0\rangle \rightarrow |+\rangle$ and $|0\rangle \rightarrow |-\rangle$

RESULT ?

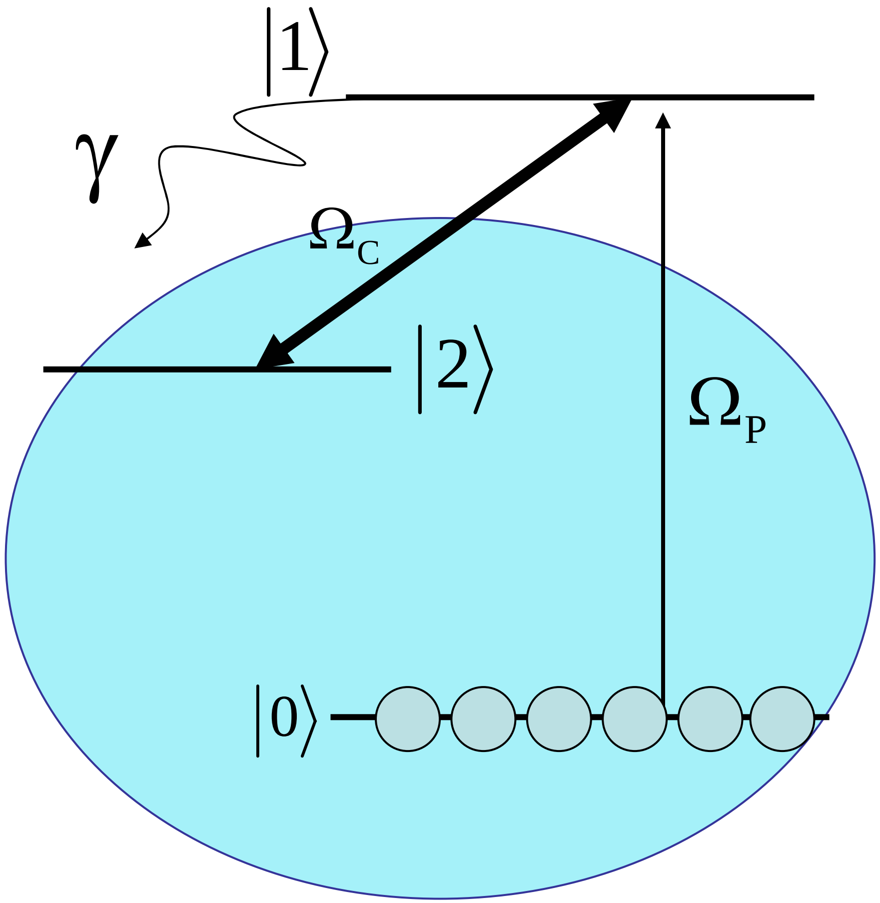


ABSORPTION COEFFICIENT
(b1)
Ω2=2.0 Γ
(b2)
1.0 Γ
(b3)
0.5 Γ
(b4)
0.3 Γ
-2
0
2
ω propag. − ω 10



TOTAL DESTRUCTIF INTERFERENCE

Coherent Population Trapping



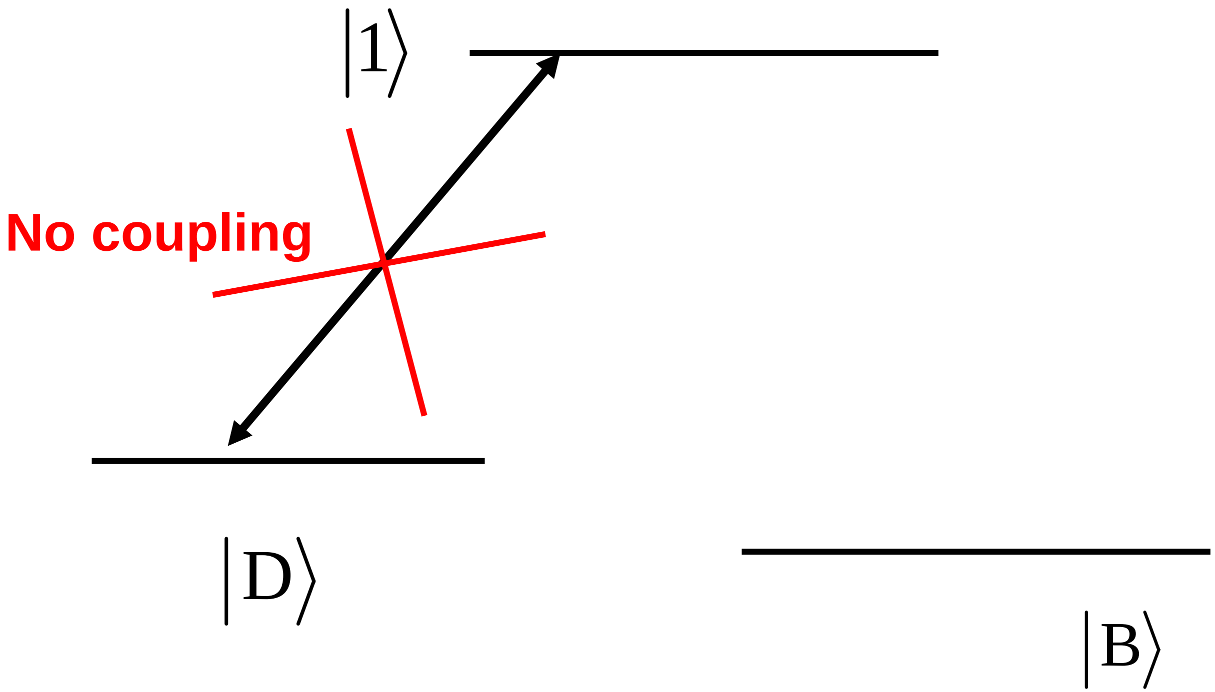
$$|B\rangle = \sin\varphi|0\rangle + \cos\varphi|2\rangle : \text{Bright state}$$

$$|D\rangle = -\sin\varphi|2\rangle + \cos\varphi|0\rangle : \text{Dark state}$$

$$\tan\varphi = \frac{\Omega_p}{\Omega_c}$$

Coherent Population Trapping

$$\omega_p = \omega_{10}$$



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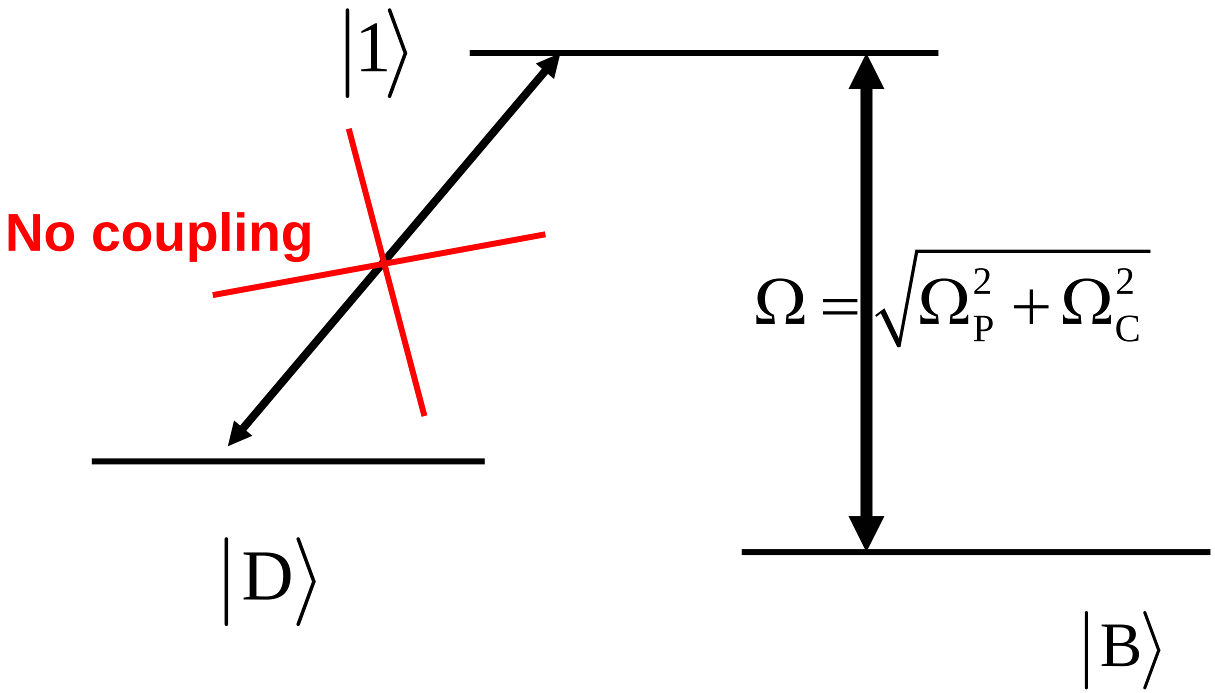

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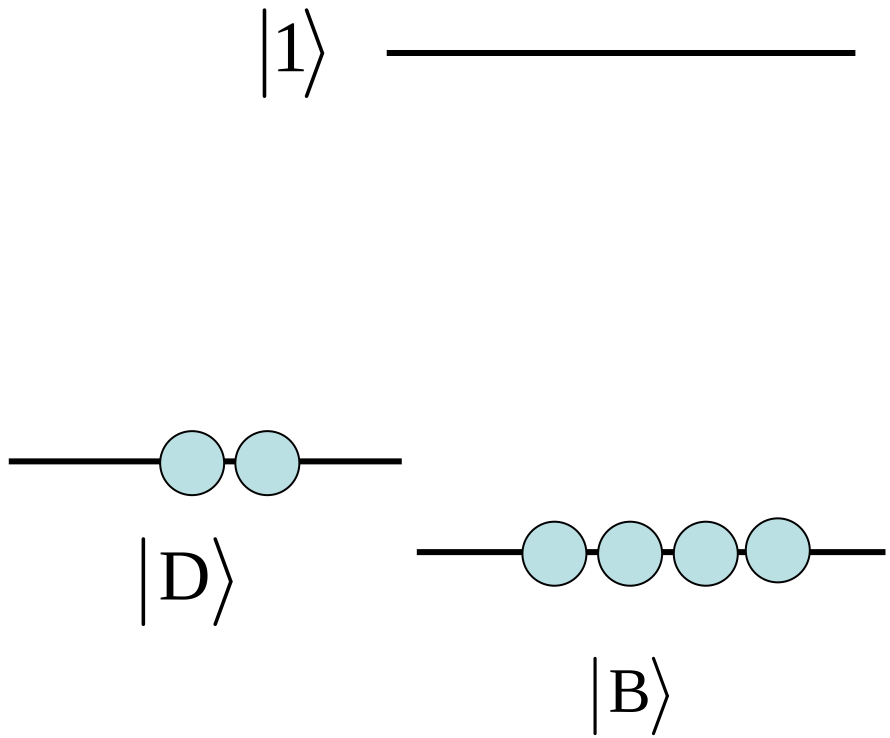

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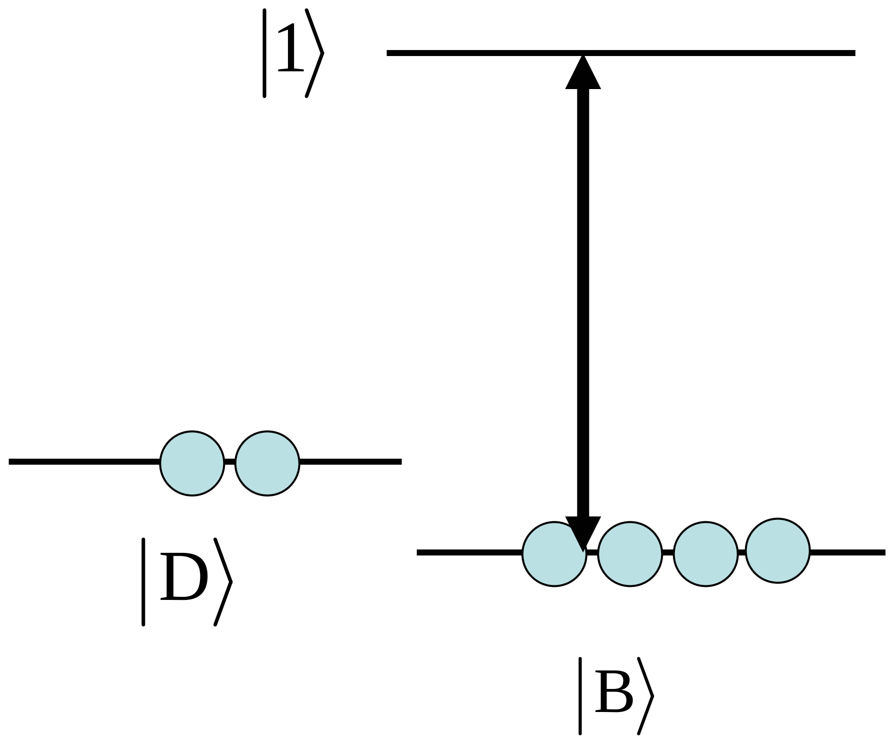

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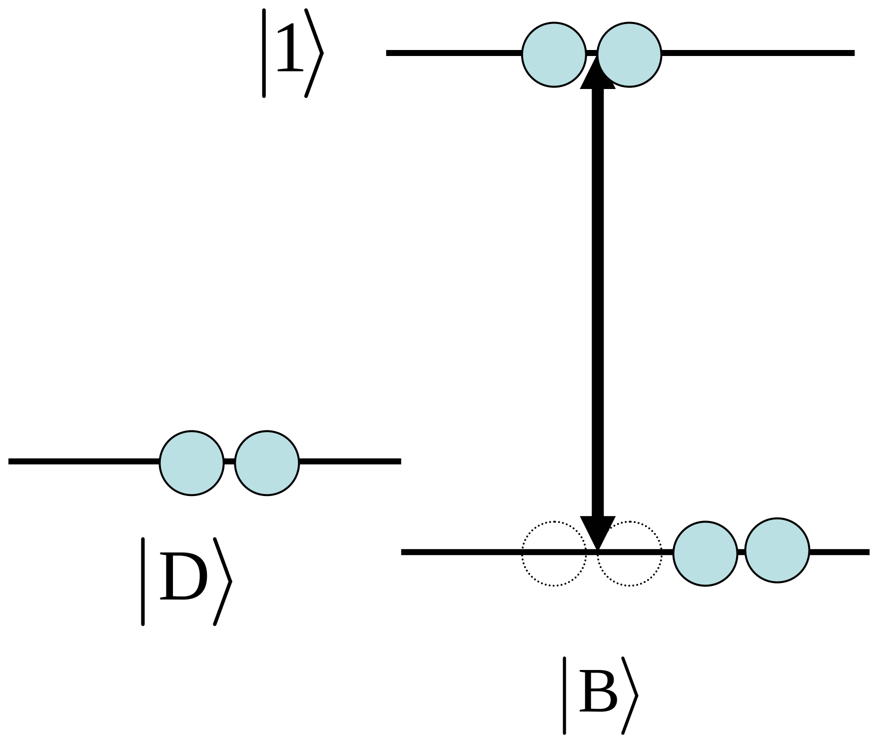

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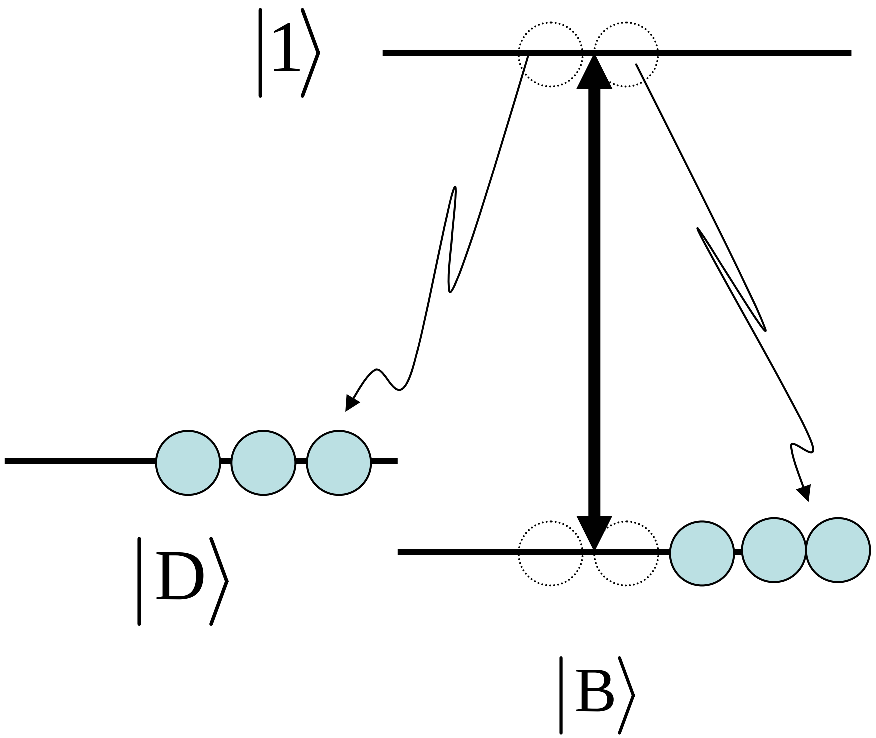

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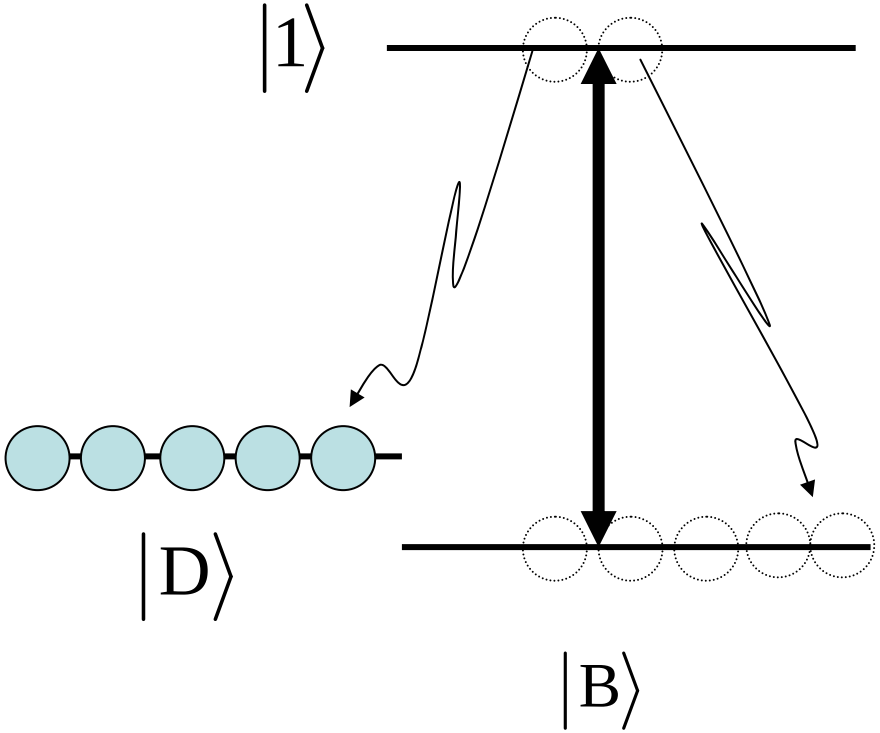
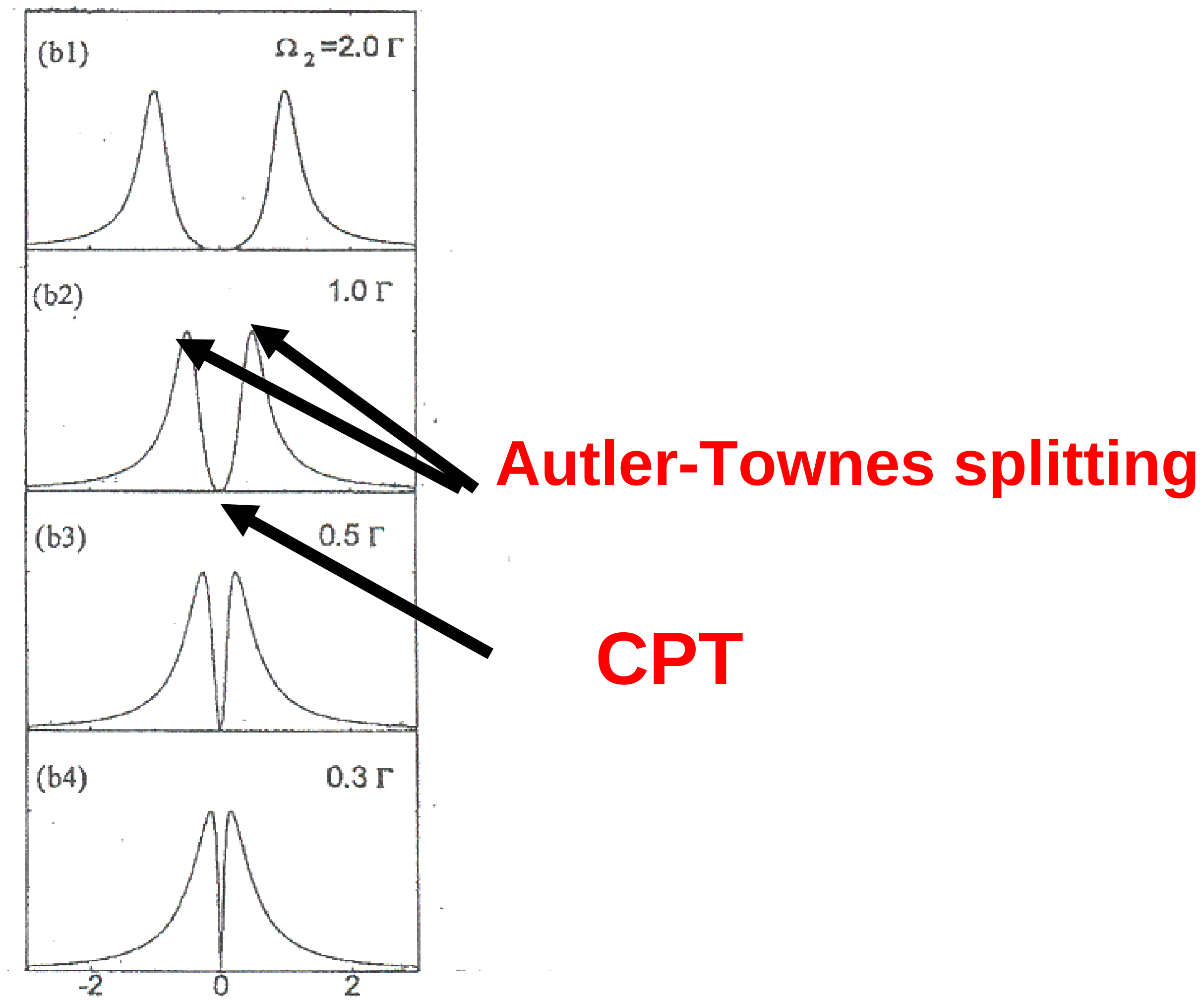


ALL THE ATOMS ARE IMMUNE TO INTERACTION:

perfect transparency


(b1)
Ω_2=2.0 Γ
(b2)
1.0 Γ
(b3)
0.5 Γ
(b4)
0.3 Γ
-2
0
2
Autler-Townes splitting
CPT

Light speed reduction to 17 metres per second in an ultracold atomic gas

Lene Vestergaard Hau*†, S. E. Harris‡, Zachary Dutton*† & Cyrus H. Behroozi*§

** Rowland Institute for Science, 100 Edwin H. Land Boulevard, Cambridge, Massachusetts 02142, USA*

† Department of Physics, § Division of Engineering and Applied Sciences, Harvard University, Cambridge, Massachusetts 02138, USA

‡ Edward L. Ginzton Laboratory, Stanford University, Stanford, California 9 USA

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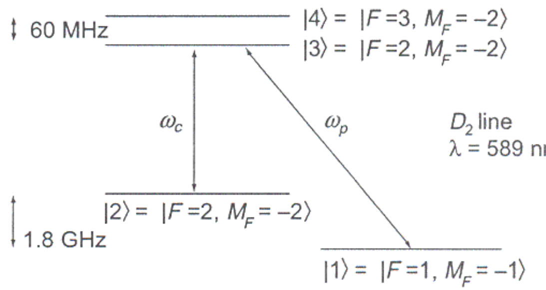
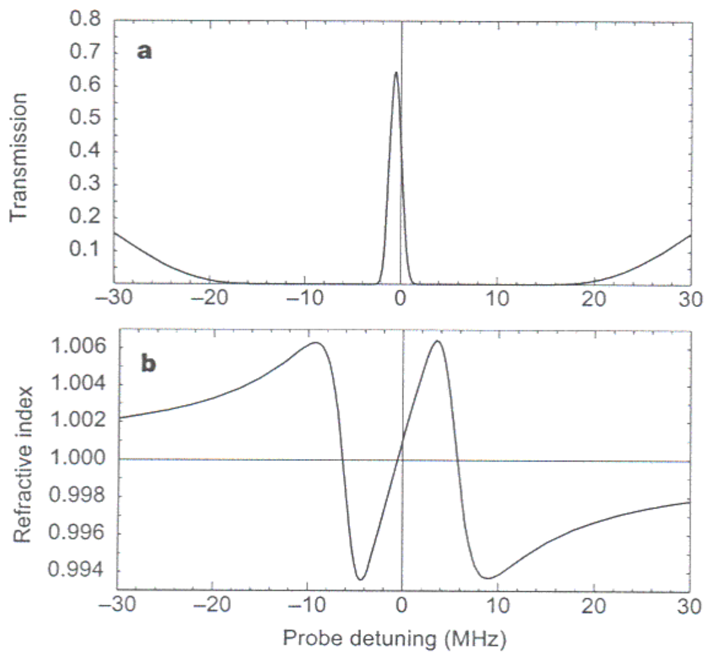
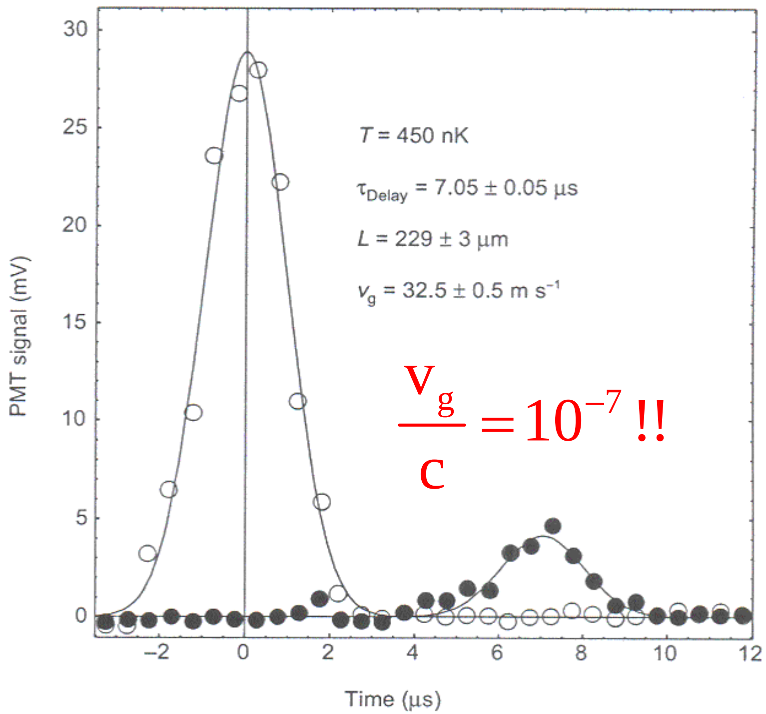


$$v_g = \frac{c}{n(\omega_p) + \omega_p \frac{dn}{d\omega_p}} \approx \frac{\hbar c \epsilon_0}{2\omega_p} \frac{|\Omega_c|^2}{|\mu_{13}|^2 N}$$





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V_g decreases with Ω_C

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V_g decreases with Ω_C

Is it possible to stop the light ?

$$v_g = \frac{c}{n(\omega_p) + \omega_p \frac{dn}{d\omega_p}} \approx \frac{\hbar c \epsilon_0}{2\omega_p} \frac{|\Omega_c|^2}{|\mu_{13}|^2 N}$$

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Is it possible to stop the light ?

YES

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V_g decreases with Ω_C

Is it possible to stop the light ?

YES

What means a stopped light ??

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V_g decreases with Ω_C

Is it possible to stop the light ?

YES

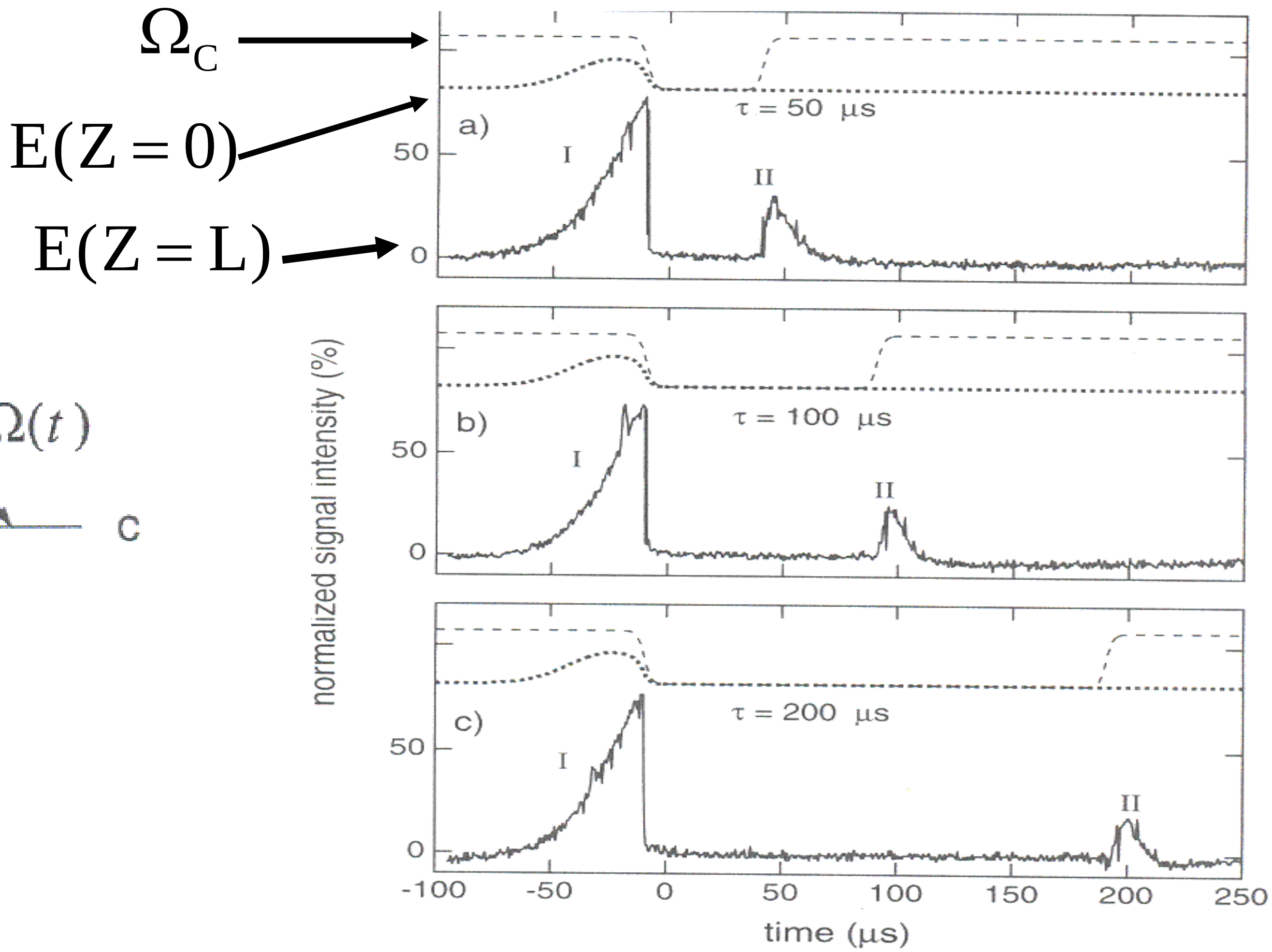
What means a stopped light ??

VOLUME 86, NUMBER 5 PHYSICAL REVIEW LETTERS 29 JANUARY 2001

Storage of Light in Atomic Vapor

D. F. Phillips, A. Fleischhauer, A. Mair, and R. L. Walsworth
Harvard-Smithsonian Center for Astrophysics, Cambridge, Massachusetts 02138

M. D. Lukin
ITAMP, Harvard-Smithsonian Center for Astrophysics, Cambridge, Massachusetts 02138
(Received 22 December 2000)



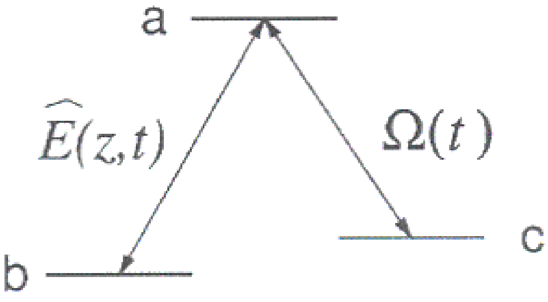
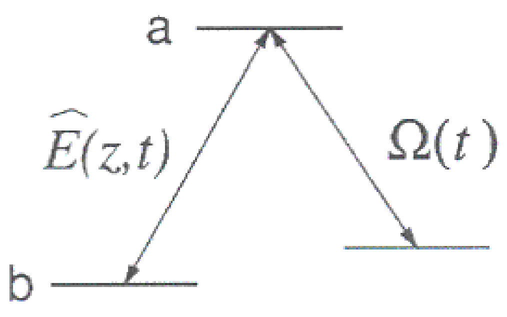


FIG. 2. Observed light pulse storage in a ^{87}Rb vapor cell. Examples are shown for storage times of (a) 50 μs, (b) 100 μs, and (c) 200 μs. (Background transmission from the control field, which leaks into the signal field detection optics, has been subtracted from these plots.) Signals are normalized to the peak intensity of the light transmitted through the stationary EIT medium. Shown above the data in each graph are calculated representations of the applied control field (dashed line) and input signal pulse (dotted line). Estimated peak $\Omega_c \sim 3$ MHz, $\Omega_s \sim 0.9$ MHz, and $\sqrt{\kappa} \sim 2000$ MHz.



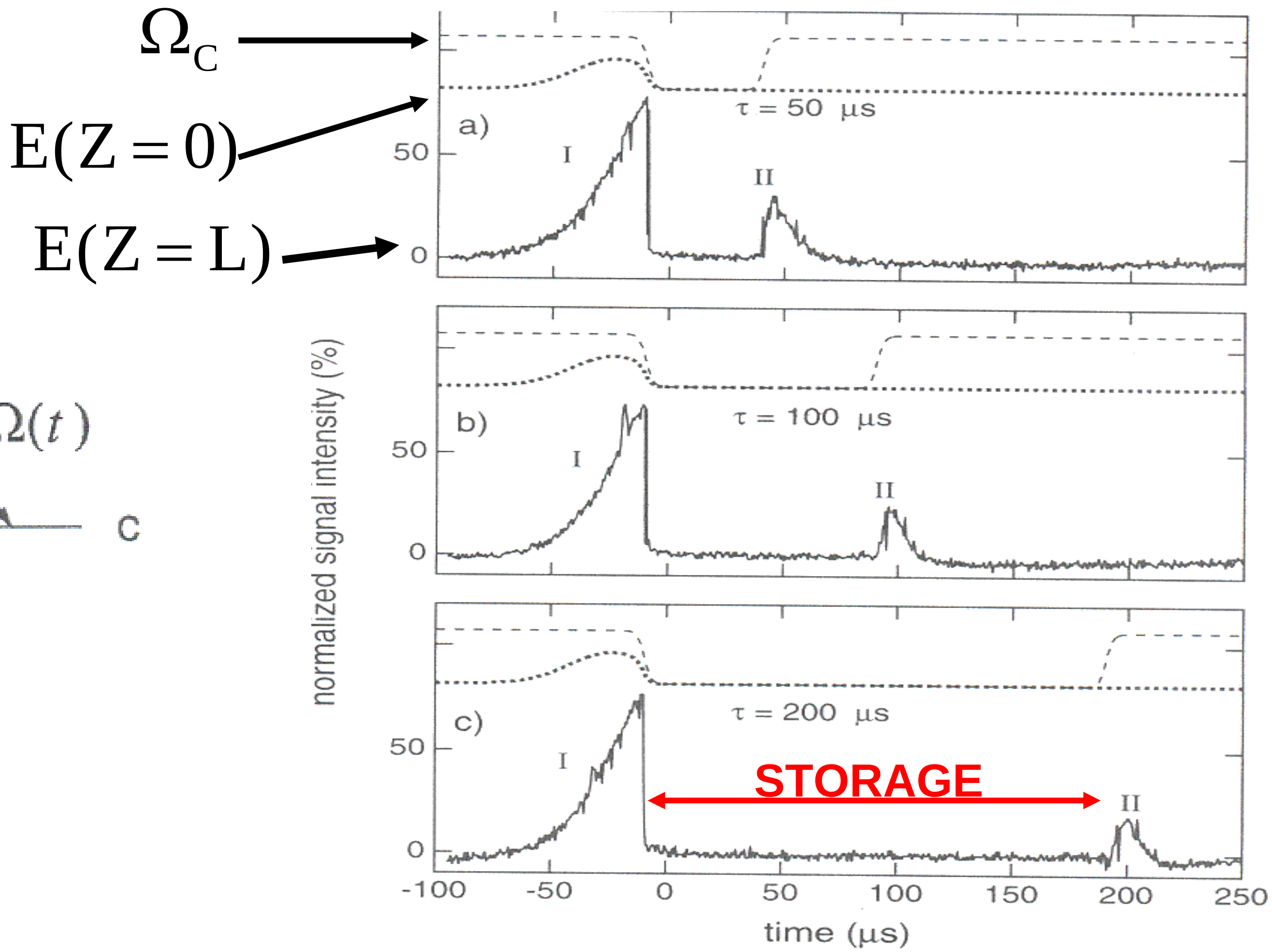
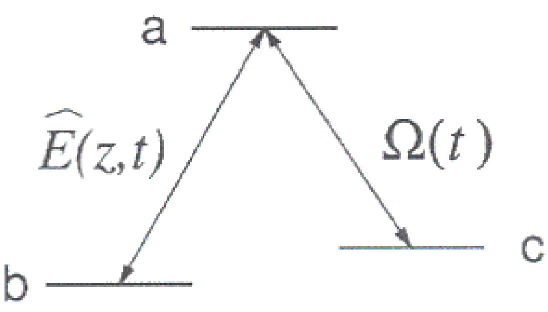
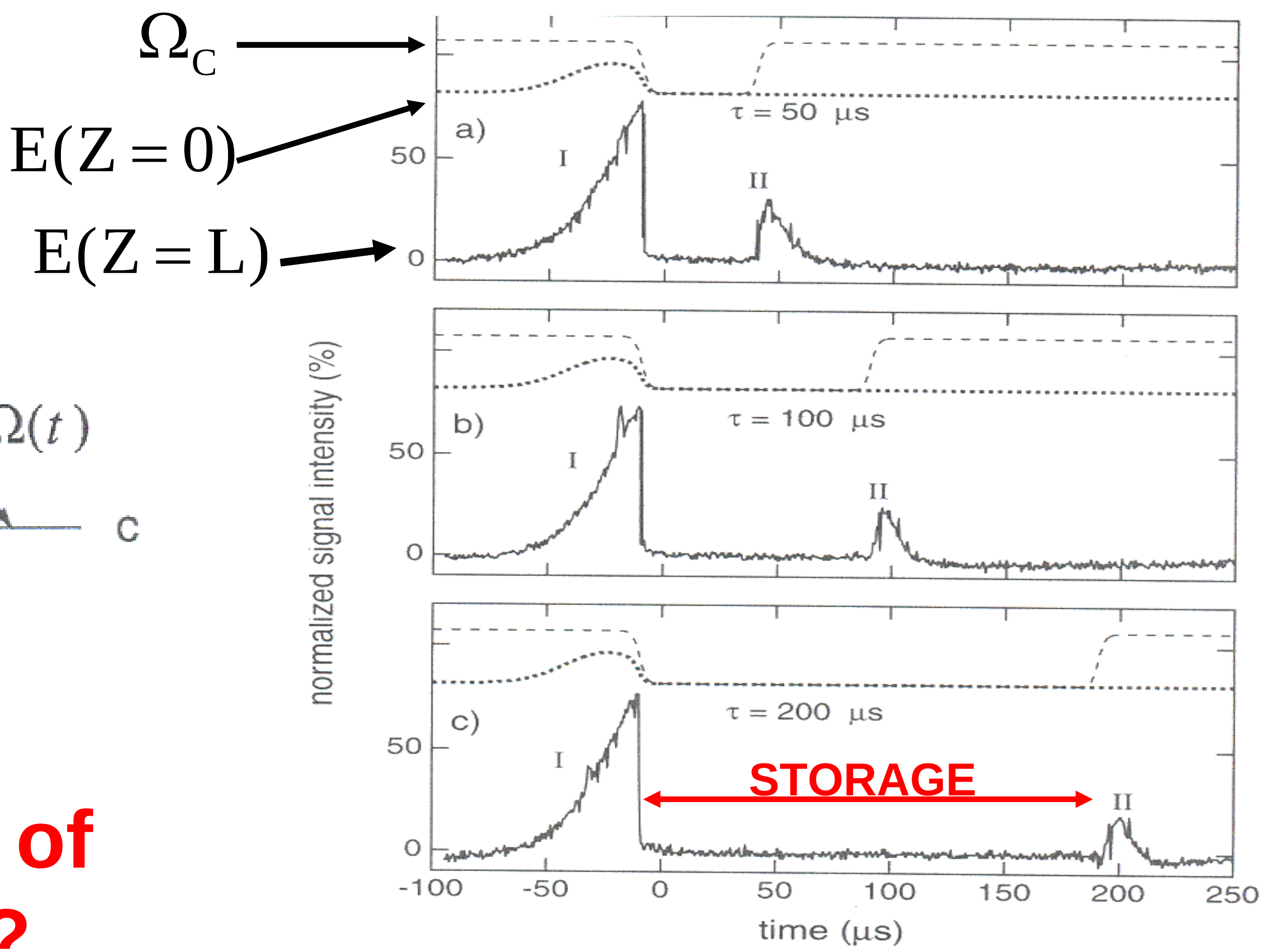


FIG. 2. Observed light pulse storage in a ^{87}Rb vapor cell. Examples are shown for storage times of (a) 50 μs, (b) 100 μs, and (c) 200 μs. (Background transmission from the control field, which leaks into the signal field detection optics, has been subtracted from these plots.) Signals are normalized to the peak intensity of the light transmitted through the stationary EIT medium. Shown above the data in each graph are calculated representations of the applied control field (dashed line) and input signal pulse (dotted line). Estimated peak $\Omega_c \sim 3$ MHz, $\Omega_s \sim 0.9$ MHz, and $\sqrt{\kappa} \sim 2000$ MHz.





What kind of storage?

FIG. 2. Observed light pulse storage in a ^{87}Rb vapor cell. Examples are shown for storage times of (a) 50 μs, (b) 100 μs, and (c) 200 μs. (Background transmission from the control field, which leaks into the signal field detection optics, has been subtracted from these plots.) Signals are normalized to the peak intensity of the light transmitted through the stationary EIT medium. Shown above the data in each graph are calculated representations of the applied control field (dashed line) and input signal pulse (dotted line). Estimated peak $\Omega_c \sim 3$ MHz, $\Omega_s \sim 0.9$ MHz, and $\sqrt{\kappa} \sim 2000$ MHz.

VOLUME 84, NUMBER 22 PHYSICAL REVIEW LETTERS 29 MAY 2000

Dark-State Polaritons in Electromagnetically Induced Transparency

M. Fleischhauer[1] and M. D. Lukin[2]
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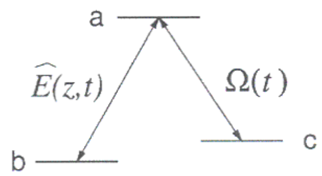
[2]*ITAMP, Harvard-Smithsonian Center for Astrophysics, Cambridge, Massachusetts 02138*
(Received 26 January 2000)



$$\cos\theta = \frac{\Omega(t)}{\sqrt{\Omega^2(t) + g^2 N}}$$

$$\psi(z,t) = \cos\theta(t)\, E(z,t) - \sin\theta(t)\, g\sqrt{N}\, \sigma_{bc}$$

Polariton (mixture field-atom)

$$\left(\frac{\partial}{\partial t} + c\cos^2\theta(t)\frac{\partial}{\partial z}\right)\psi(z,t) = 0$$

Shape-preserved propagation with

$$v_g(t) = c\cos^2\theta(t)$$

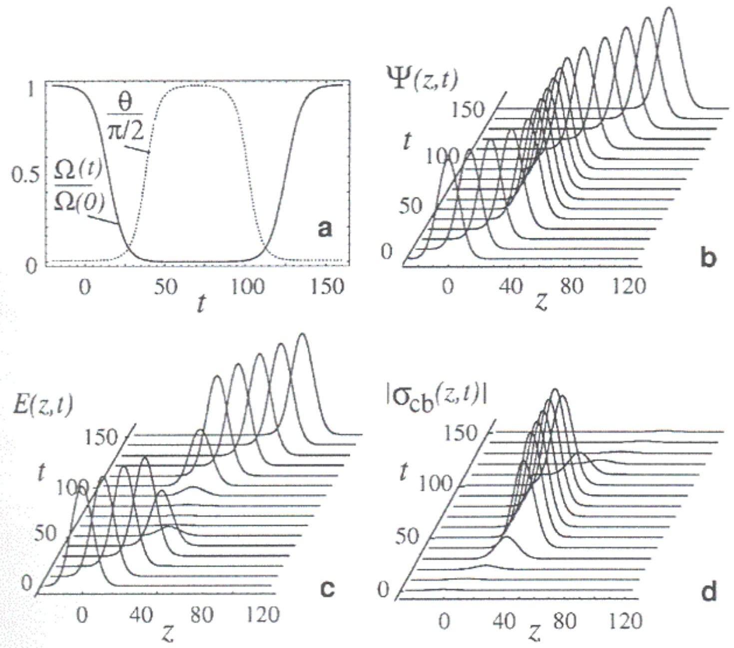
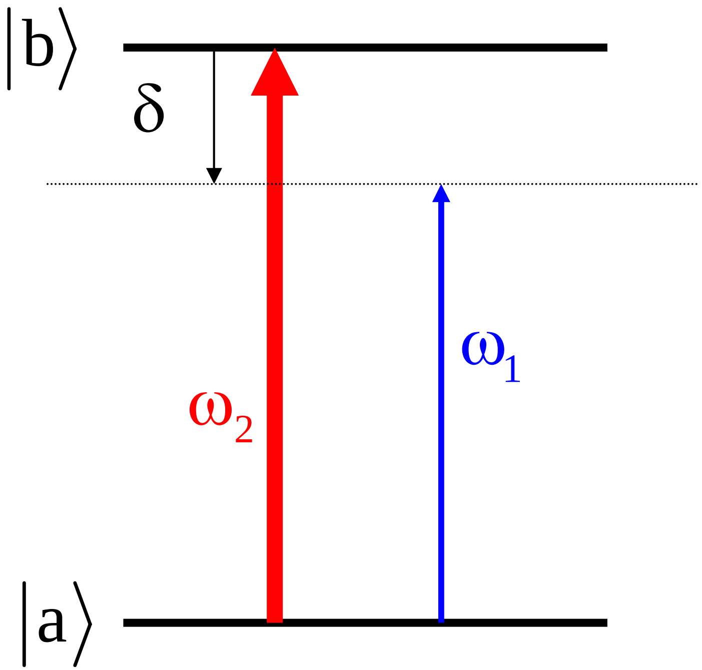


FIG. 2. Propagation of a dark-state polariton with envelope $\exp\{-(z/10)^2\}$. The mixing angle is rotated from 0 to $\pi/2$ and back according to $\cot\theta(t) = 100\{1 - 0.5\tanh[0.1(t-15)] + 0.5\tanh[0.1(t-125)]\}$ as shown in (a). The coherent amplitude of the polariton $\Psi = \langle\hat{\Psi}\rangle$ is plotted in (b), and the electric field $E = \langle\hat{E}\rangle$ and matter components $|\sigma_{cb}| = |\langle\hat{\sigma}_{cb}\rangle|$ in (c) and (d), respectively. Axes are in arbitrary units with $c = 1$.

II-b Coherent Population Oscillations

Bichromatic field :

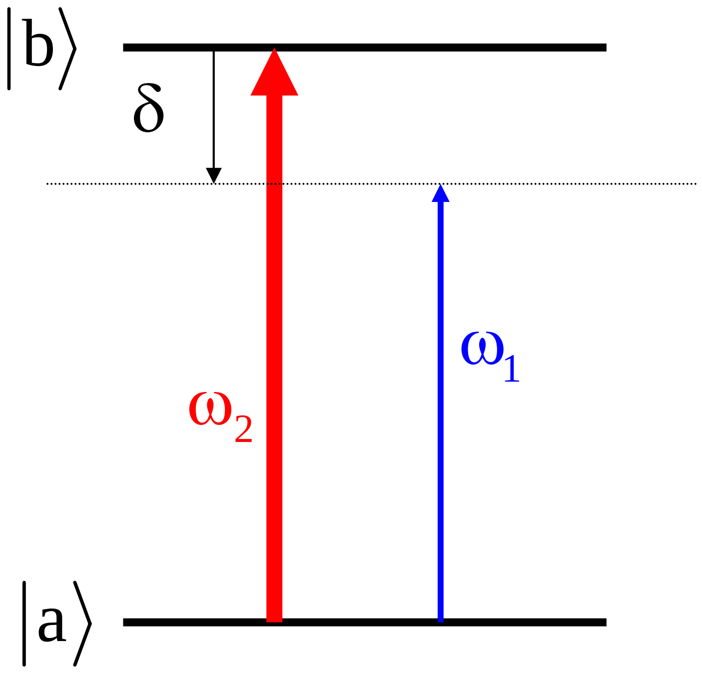


II-b Coherent Population Oscillations

Bichromatic field :

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos(\omega_2 - \omega_1)t$$

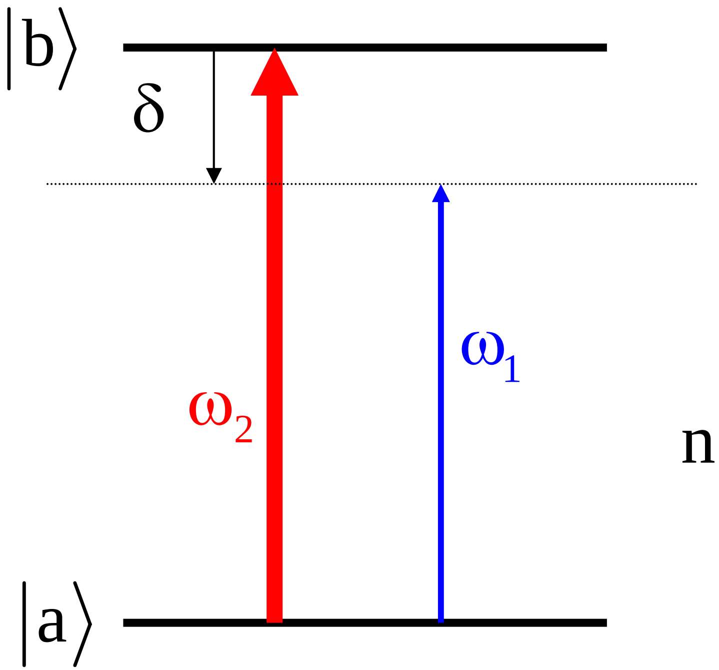
BEATING



II-b Coherent Population Oscillations

Bichromatic field :

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\left(\omega_1 - \omega_2\right)t$$

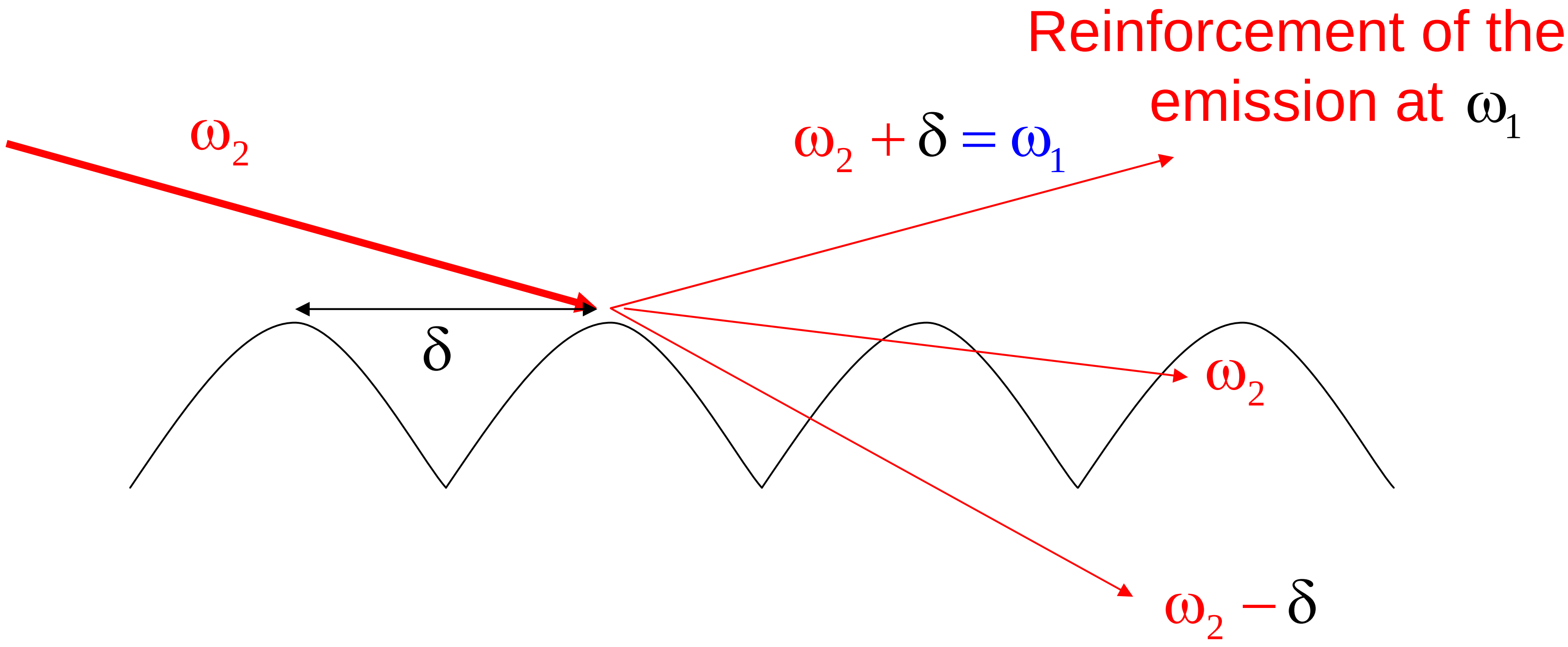


BEATING

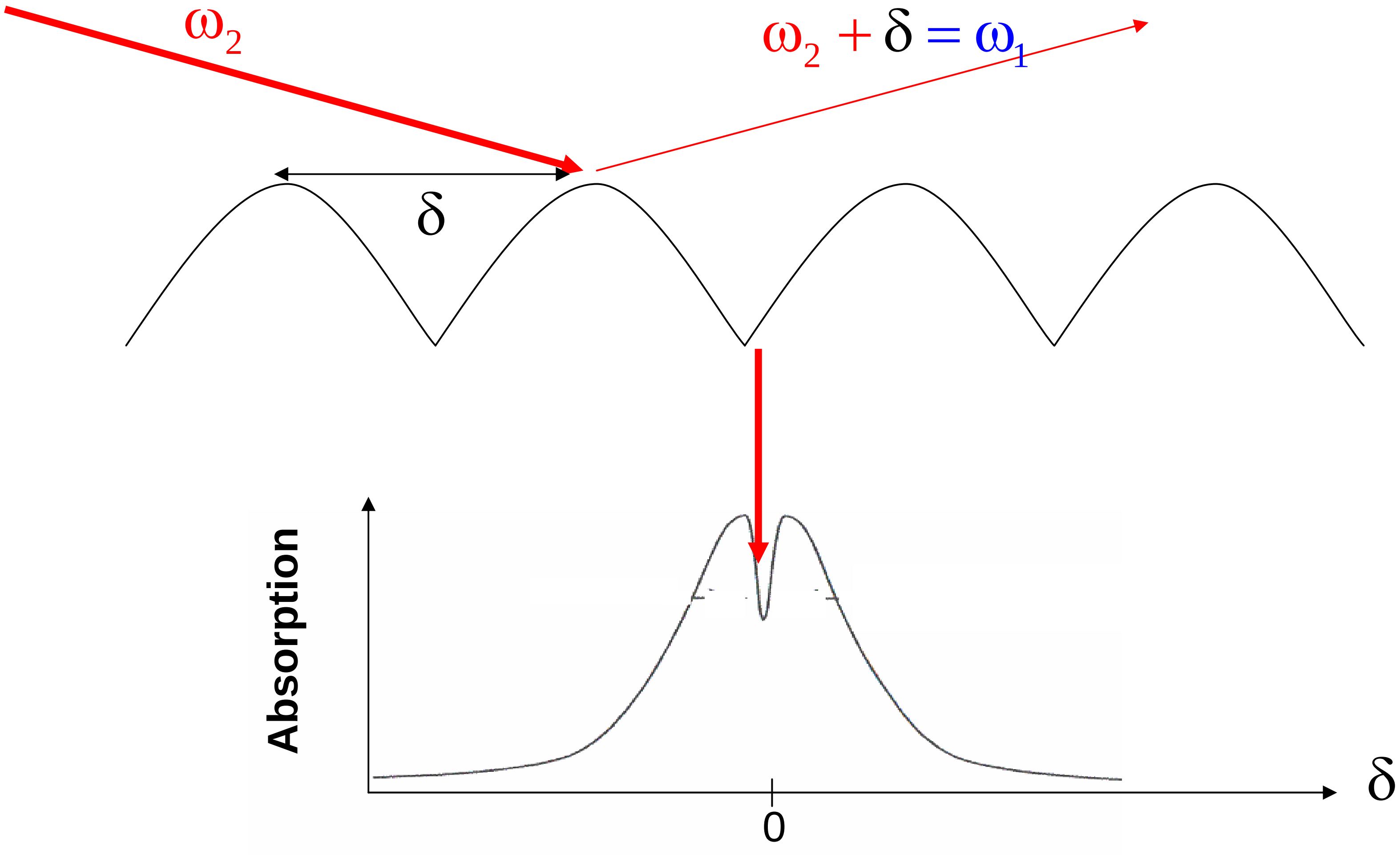
$$n_{ba} = n_b - n_a = n_{ba}^{(0)} + n_{ba}^{(+)} e^{i\delta t} + n_{ba}^{(-)} e^{-i\delta t}$$

TEMPORAL GRATING

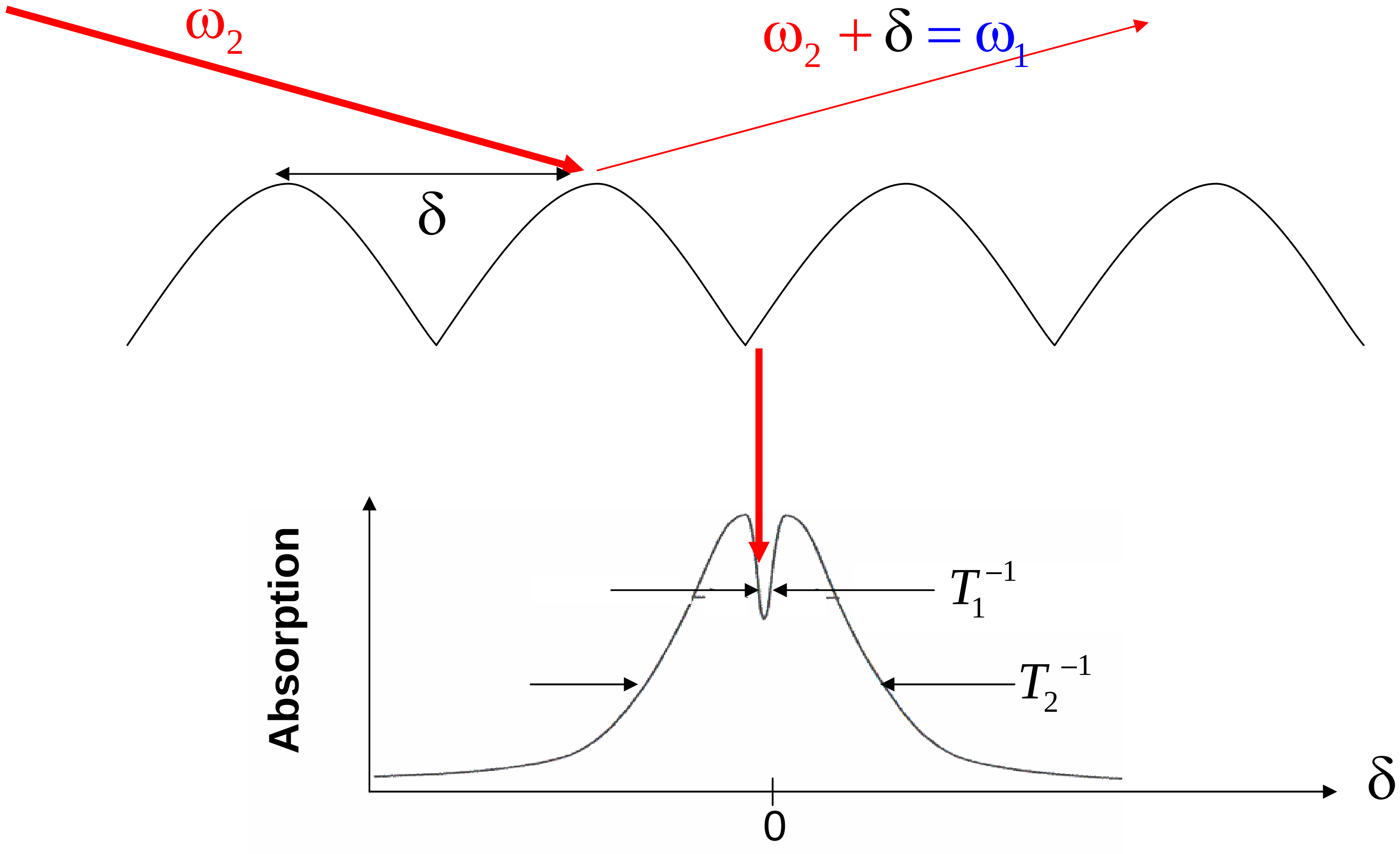
II-b Coherent Population Oscillations



II-b Coherent Population Oscillations



II-b Coherent Population Oscillations



$$\Delta t_{\mathrm{POPULATION}} = T_1 \leq \Delta t_{\mathrm{OSCILLATION}} = \delta^{-1}$$

Observation of Ultraslow Light Propagation in a Ruby Crystal at Room Temperature

Matthew S. Bigelow, Nick N. Lepeshkin, and Robert W. Boyd
The Institute of Optics, University of Rochester, Rochester, New York 14627
(Received 31 October 2002; published 21 March 2003)

We have observed slow light propagation with a group velocity as low as 57.5 ± 0.5 m/s at room temperature in a ruby crystal. A quantum coherence effect, coherent population oscillations, produces a very narrow spectral "hole" in the homogeneously broadened absorption profile of ruby. The resulting rapid spectral variation of the refractive index leads to a large value of the group index. We observe slow light propagation both for Gaussian-shaped light pulses and for amplitude modulated optical beams in a system that is much simpler than those previously used for generating slow light.

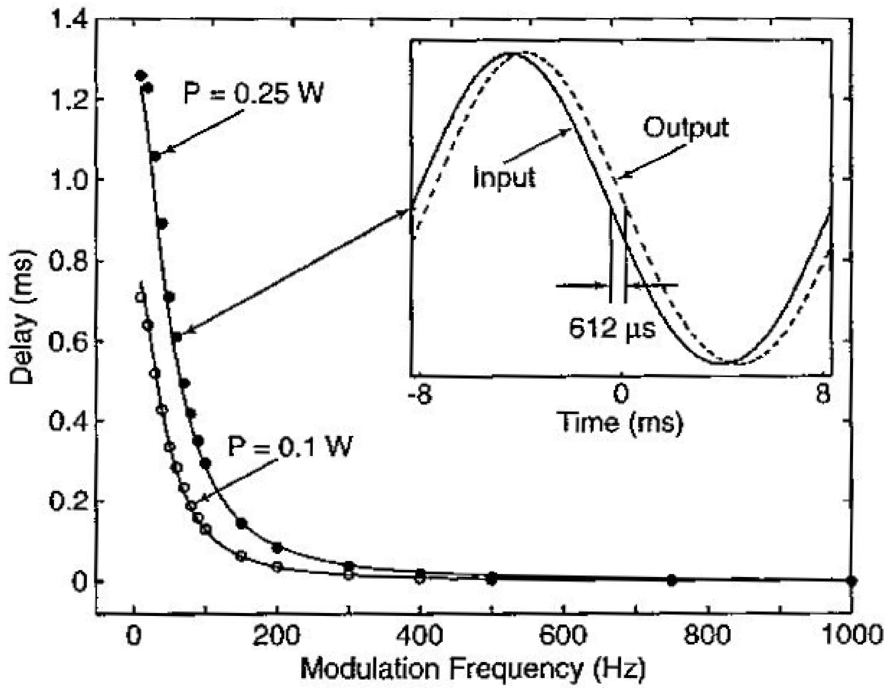
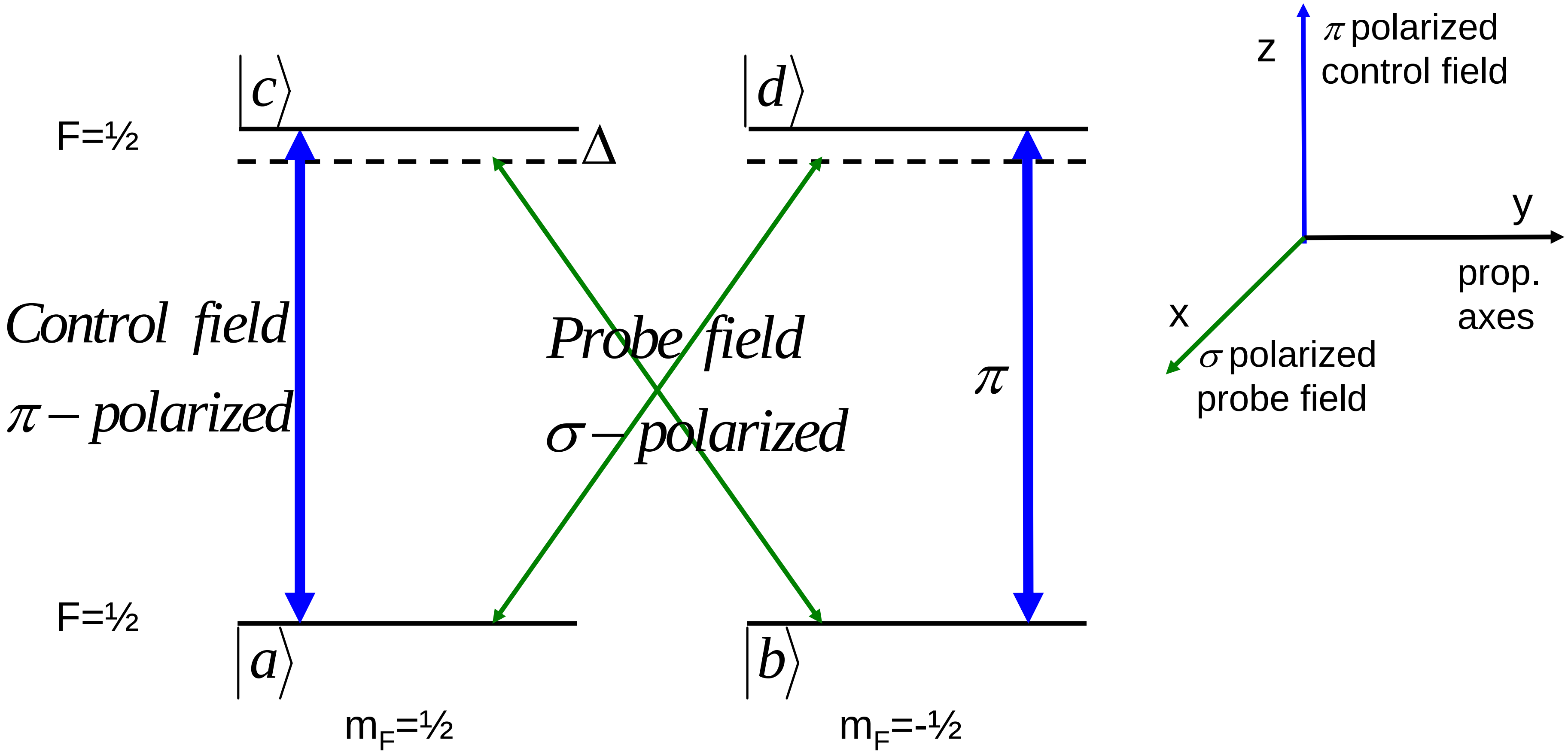


Figure 10. Observed time delay as a function of the modulation frequency for input pump powers of 0.1 and 0.25 W. The inset shows the normalized 60 Hz input (solid curve) and output (dashed curve) signal at 0.25 W. The 60 Hz signal was delayed 612 μs corresponding to an average group velocity of 118 m s^{-1}.

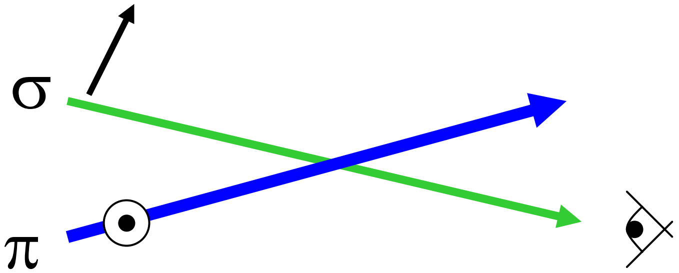
II-c **Coherent Zeeman Oscillations**

A double two level system



II-c Coherent Zeeman Oscillations

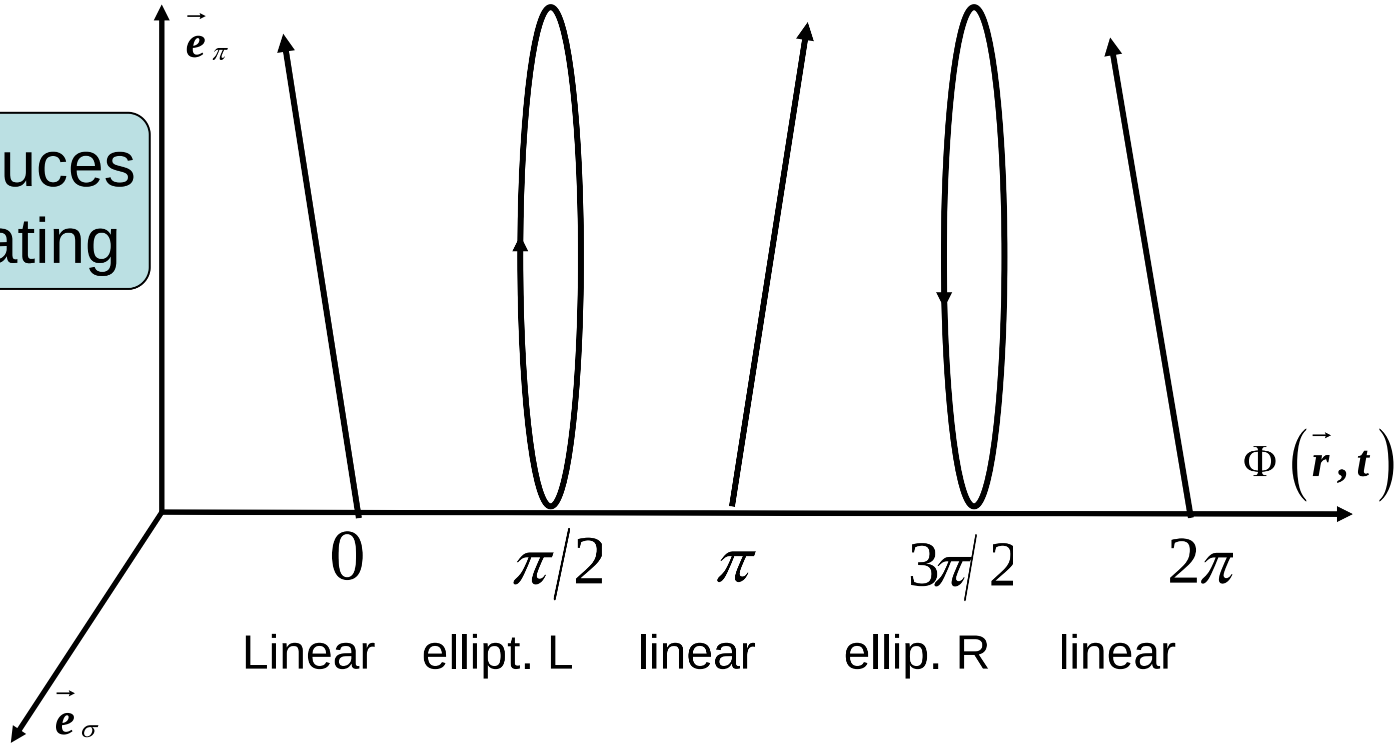
Bichromatic field, with strong π resonant pulse and σ detuned pulse



$$\Phi\left(\vec{r},t\right) = \Delta\, t - \delta\vec{k}.\vec{r}$$

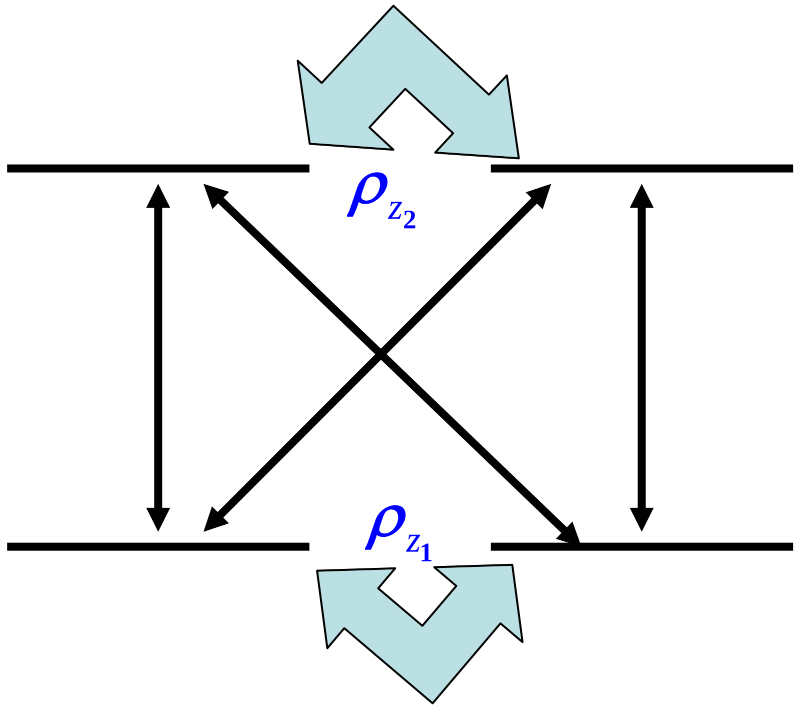
$$\vec{e}_T = \vec{e}_\pi + \vec{e}_\sigma \frac{E_\sigma}{E_\pi} e^{-i\Phi\left(\vec{r},t\right)}$$

polarization produces a time /space grating



grating imprints on Zeeman coherences

$$\rho_Z = \rho_{z_1} + \rho_{z_2} = \sum_{n=-1}^{n=1} \rho^{(n)} e^{in\left(\Delta t - \delta \vec{k}.\vec{r}\right)}$$



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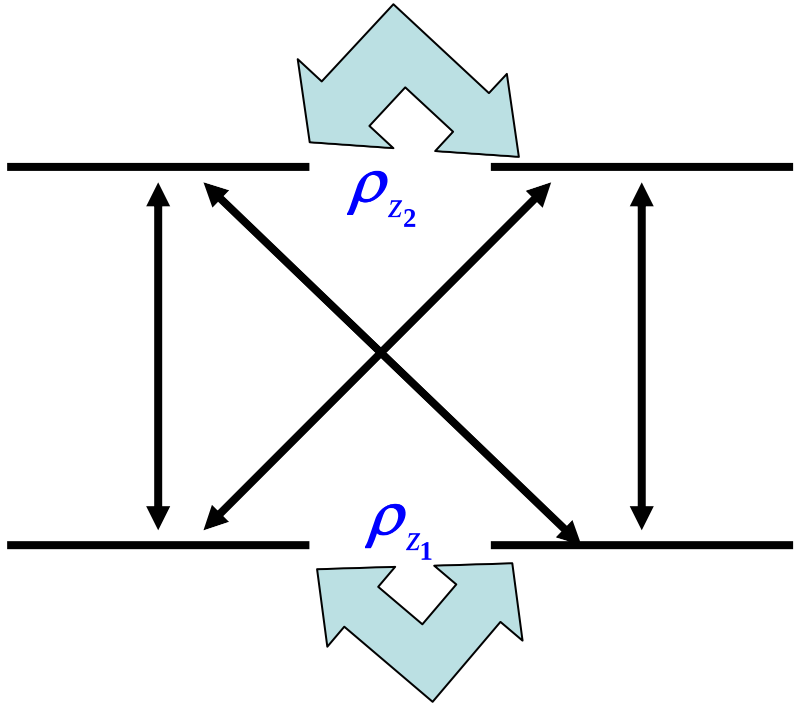
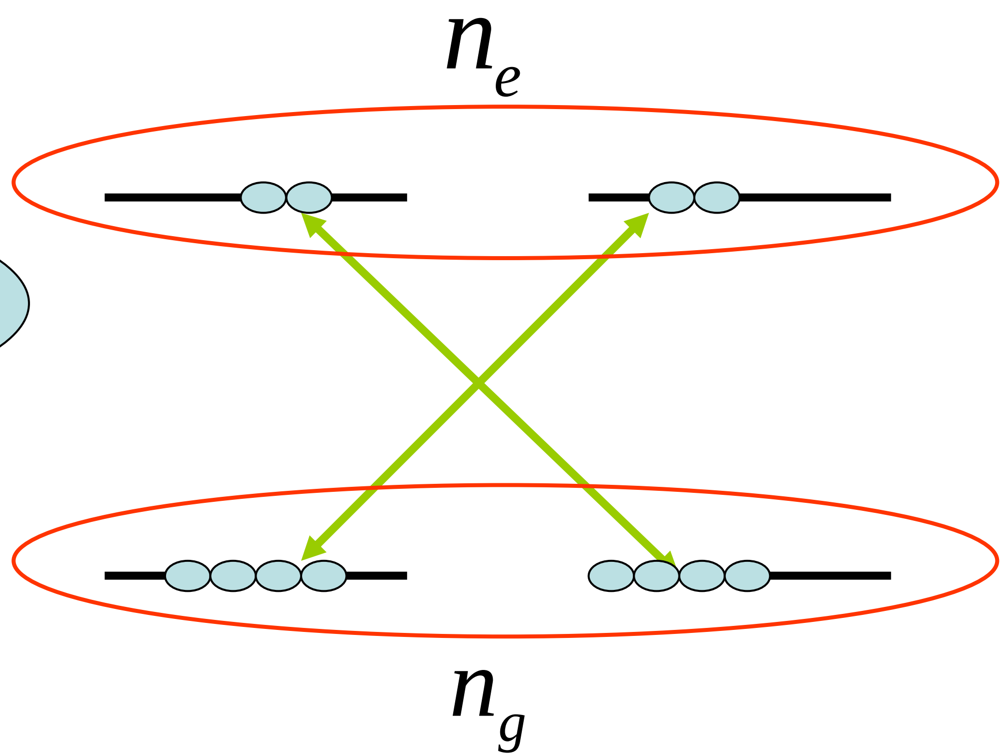


$$\dot{\rho}_\sigma = -\theta_\pi \, \mathrm{Im}\, \rho_Z - i\theta_\sigma e^{-i\Phi\left(\vec{r},t\right)} \left(n_e - n_g\right) - \frac{\rho_\sigma \Gamma}{2}$$

$$\theta_\pi = \frac{dE_\pi}{\hbar\Gamma} ; \theta_\sigma = \frac{dE_\sigma}{\hbar\Gamma}$$

grating imprints on Zeeman coherences

$$\rho_Z = \rho_{z_1} + \rho_{z_2} = \sum_{n=-1}^{n=1} \rho^{(n)} e^{in\left(\Delta t - \delta \vec{k}.\vec{r}\right)}$$



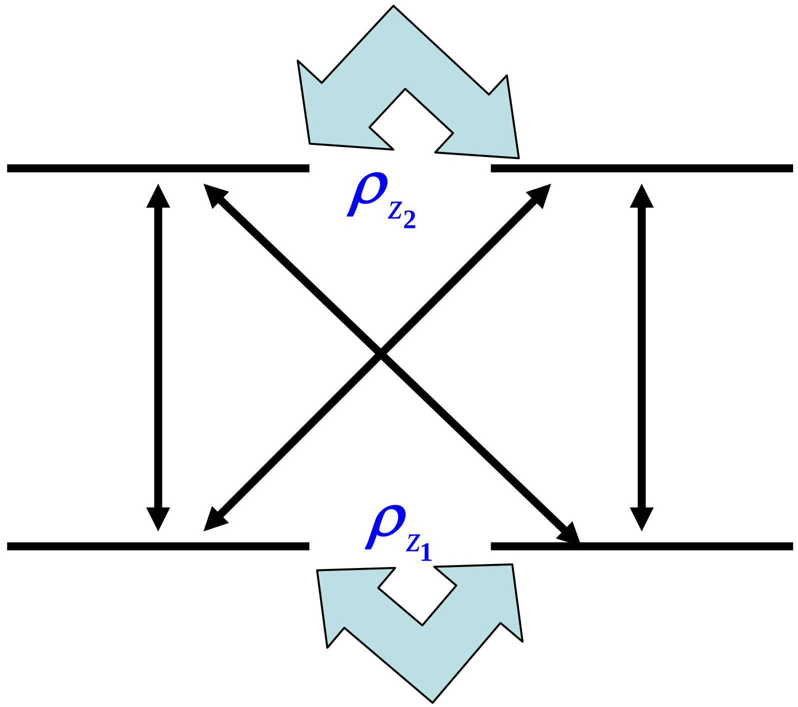
$$\dot{\rho}_\sigma = -\theta_\pi \, \mathbf{Im}\, \rho_Z \left(-i\theta_\sigma e^{-i\Phi\left(\vec{r},t\right)}\left(n_e - n_g\right)\right) - \frac{\rho_\sigma \Gamma}{2}$$

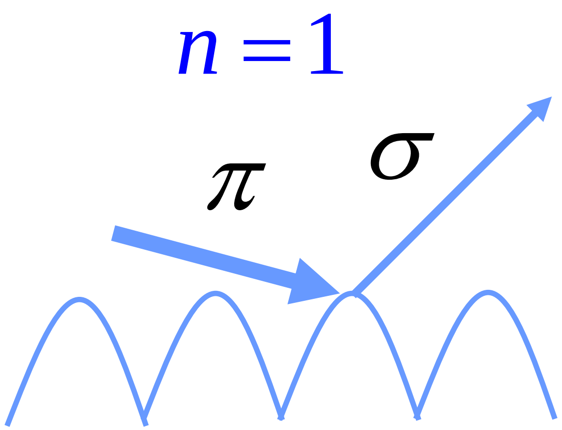
$$\theta_\pi = \frac{dE_\pi}{\hbar\Gamma}; \theta_\sigma = \frac{dE_\sigma}{\hbar\Gamma}$$

Absorption by population

grating imprints on Zeeman coherences

$$\rho_Z = \rho_{z_1} + \rho_{z_2} = \sum_{n=-1}^{n=1} \rho^{(n)} e^{in\left(\Delta t - \delta\vec{k}.\vec{r}\right)}$$



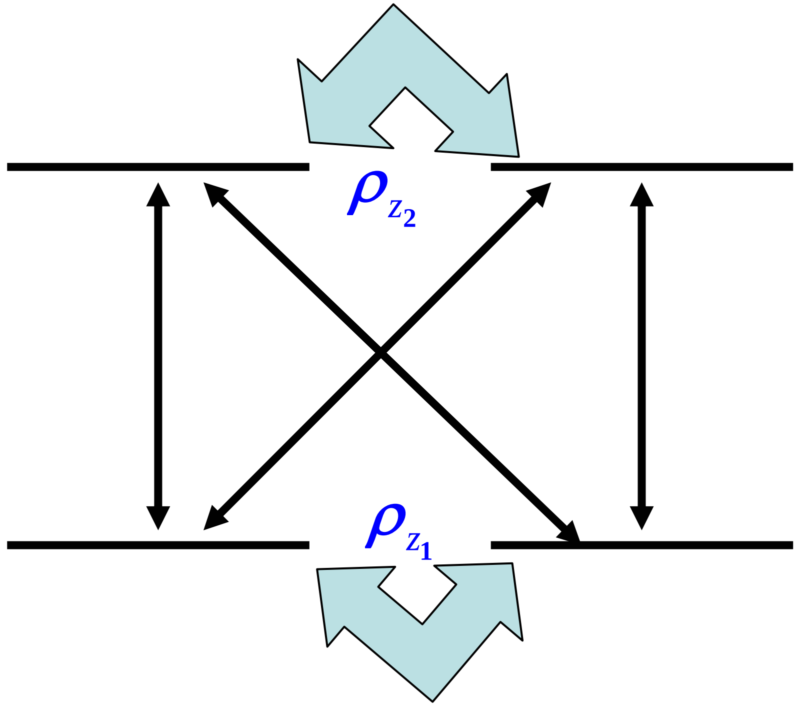


$$\dot{\rho}_\sigma = -\theta_\pi \,\mathbf{Im}\, \rho_Z - i\theta_\sigma e^{-i\Phi\left(\vec{r},t\right)} \left(n_e - n_g\right) - \frac{\rho_\sigma \Gamma}{2}$$

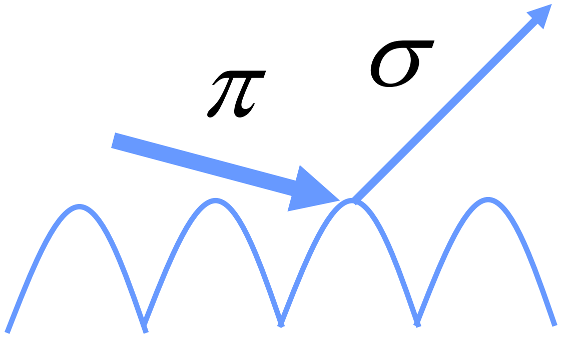
Diffraction of strong field by Zeeman coherence

grating imprints on Zeeman coherences

$$\rho_Z = \rho_{z_1} + \rho_{z_2} = \sum_{n=-1}^{n=1} \rho^{(n)} e^{in\left(\Delta t - \delta\vec{k}.\vec{r}\right)}$$



$n = 1$



$$\dot{\rho}_\sigma = -\theta_\pi \,\mathbf{Im}\, \rho_Z - i\theta_\sigma e^{-i\Phi\left(\vec{r},t\right)}\left(n_e - n_g\right) - \frac{\rho_\sigma \Gamma}{2}$$

Diffraction of strong field by Zeeman coherence

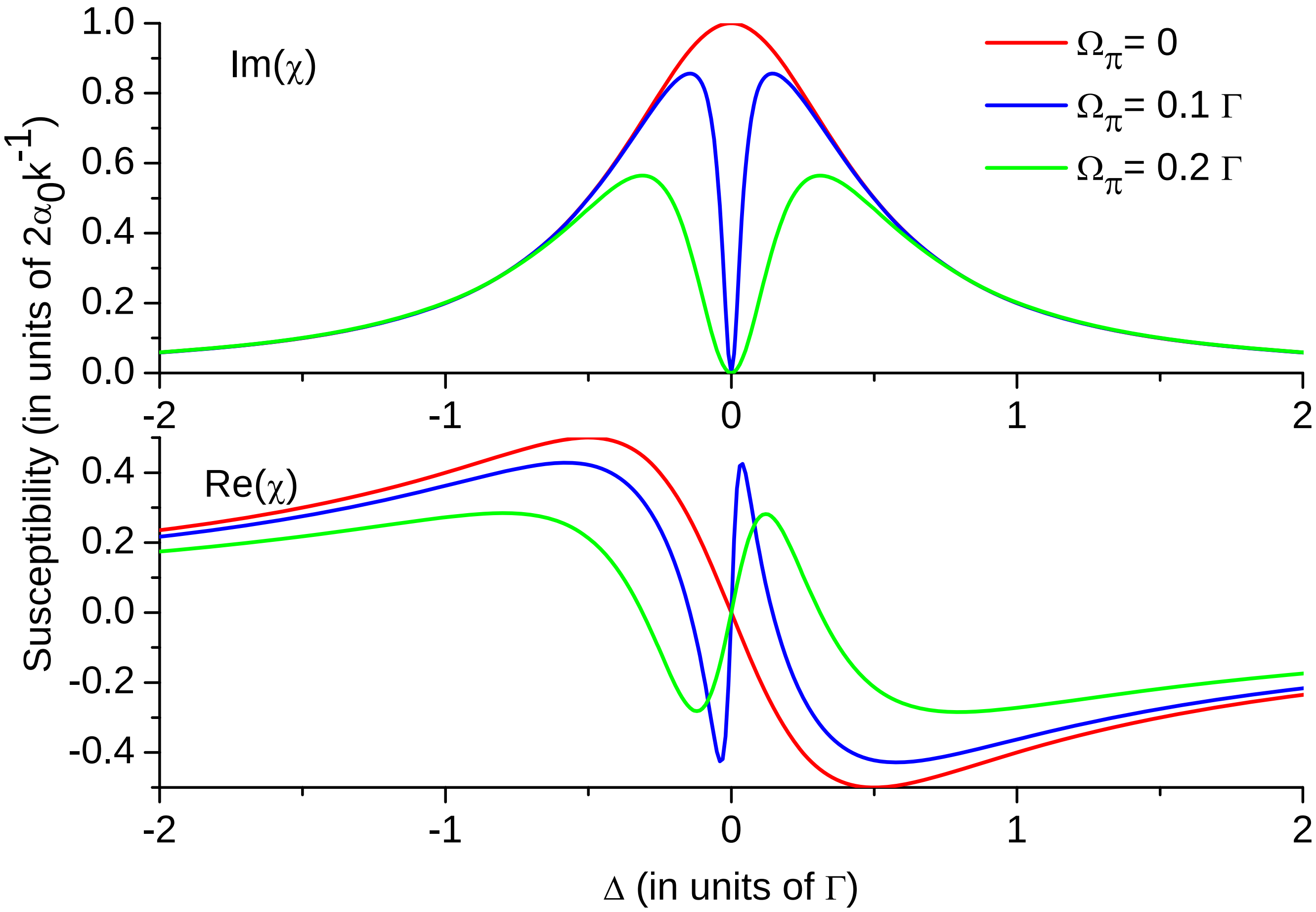
Cancel exactly

at $\Delta = 0$

Absorption by population

II-c **Coherent Zeeman Oscillations**

$\text{Width} \propto (\chi_\pi / \Gamma)^2$



F. A. Hashmi and M. A. Bouchene Phy. Rev. A 77, 051803(R) 2008

II-c **Coherent Zeeman Oscillations**

- Pure Non-linear effect
- **No dark state**
- Hybrid properties between CPO and EIT

IId- Coherent Control of the Susceptibility

- **Giant Non-linearity**:

$$P = \chi_{lin} E + \chi^{(2)} E^2 + \chi^{(3)} E^3 + \ldots$$

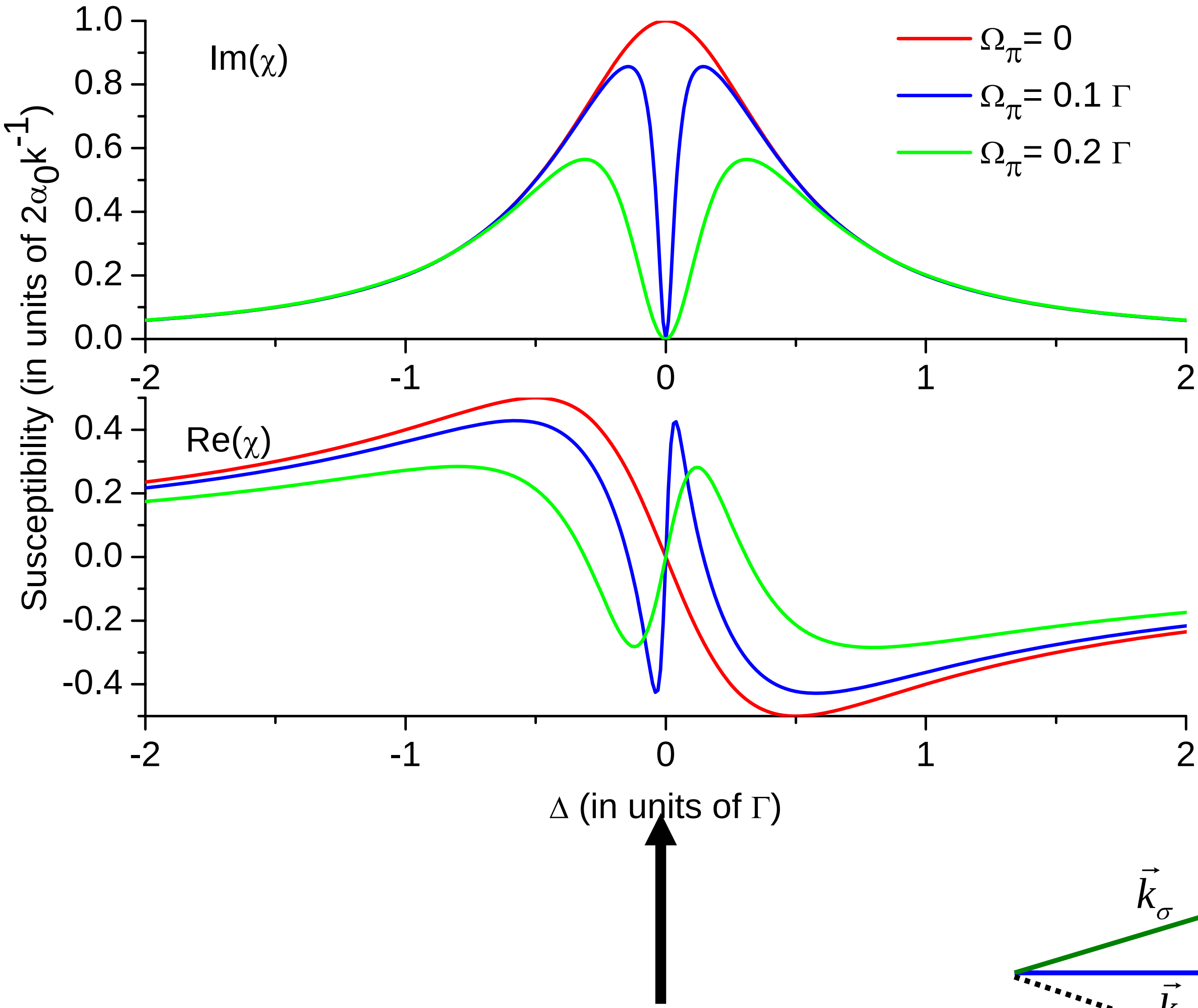
Increase NL effects ⟶ increase *E* or χ

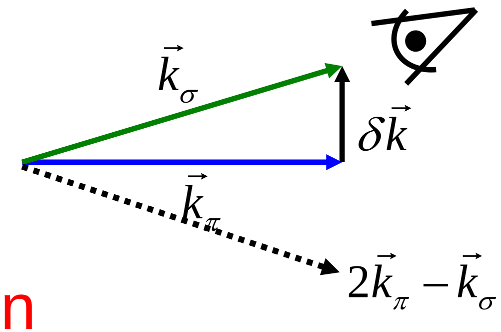
If *E* is strong : ionisation

If resonance : absorption

Slow light techniques: reduction of absorption AND resonance

⟶ Effects of giant non linearities observable





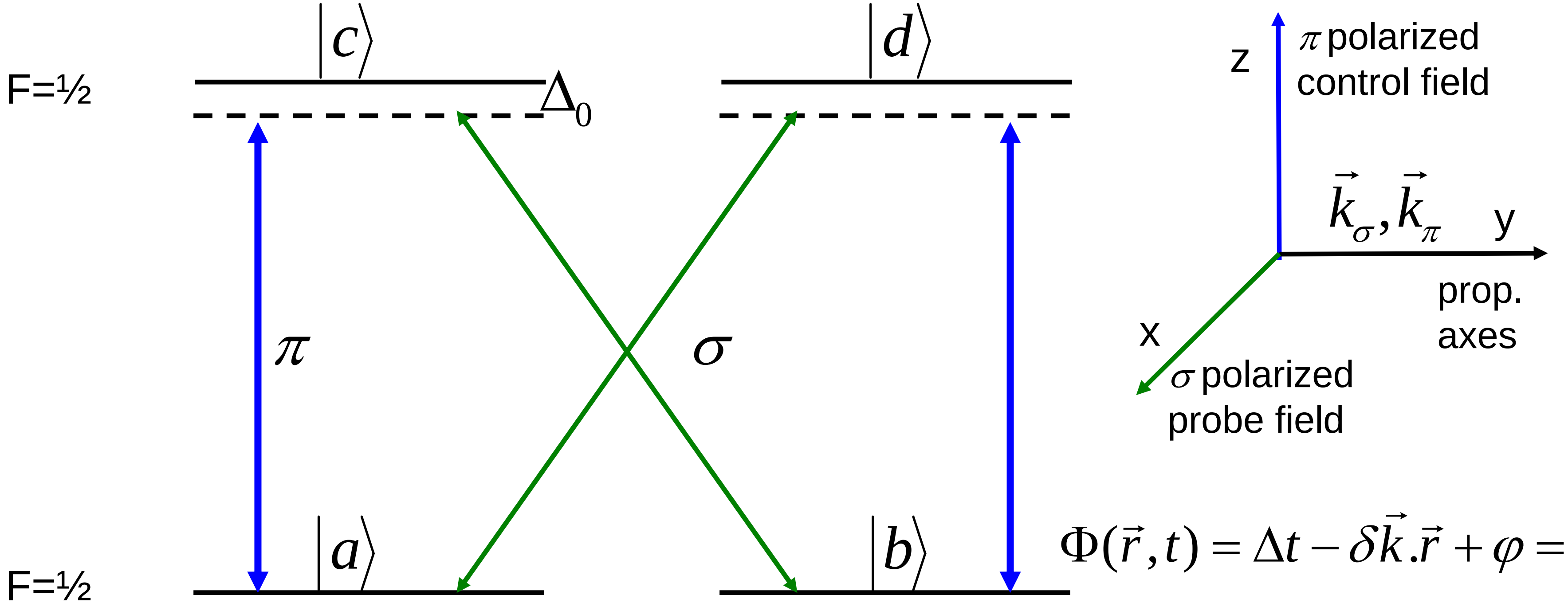
$\Delta = 0$:

Optical response canceled in the $\vec{k}_\sigma$ direction

Optical response determined by other emitted waves

II-d Coherent Control of Susceptibility

$\Delta = 0, \delta\vec{k} = 0$



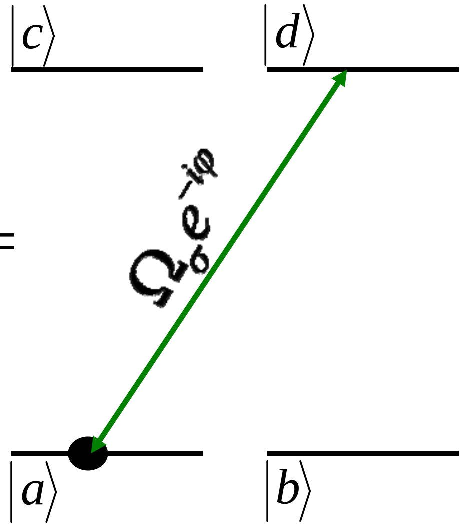
$$\Phi(\vec{r},t) = \Delta t - \delta\vec{k}.\vec{r} + \varphi = \varphi$$

$$\rho_\sigma = \rho_{da} + \rho_{cb} = \rho_\sigma(\varphi) :$$ Coherent Control

II-d Coherent Control of Susceptibility

$$\rho_\sigma = \rho_{da} + \rho_{cb}$$

$\rho_{da} =$

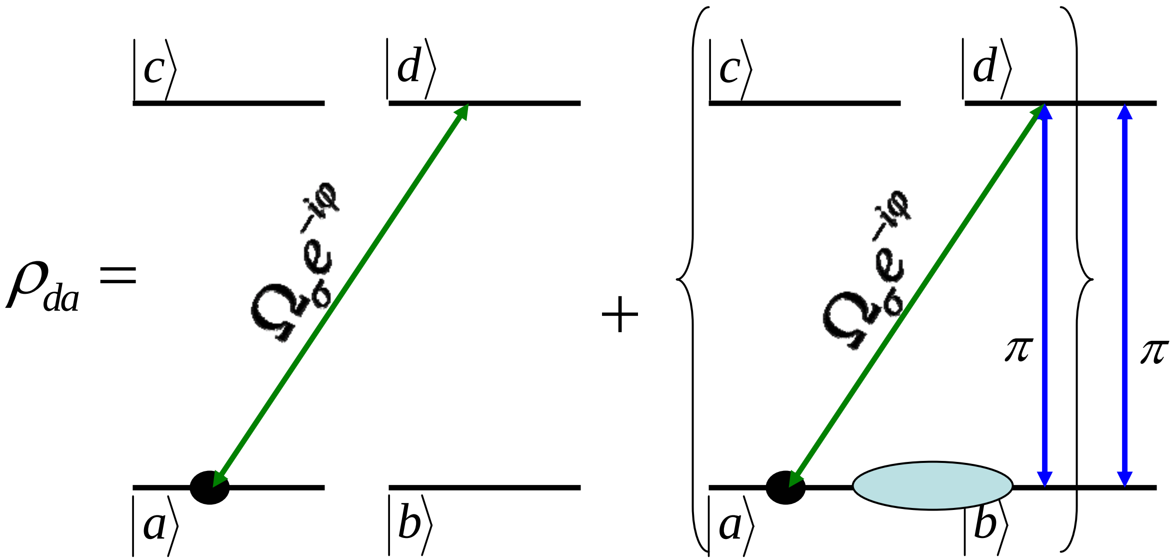


Absorption path

$\vec{k}_\sigma$

II-d Coherent Control of Susceptibility

$$\rho_{\sigma} = \rho_{da} + \rho_{cb}$$



Absorption path

$\vec{k}_{\sigma}$

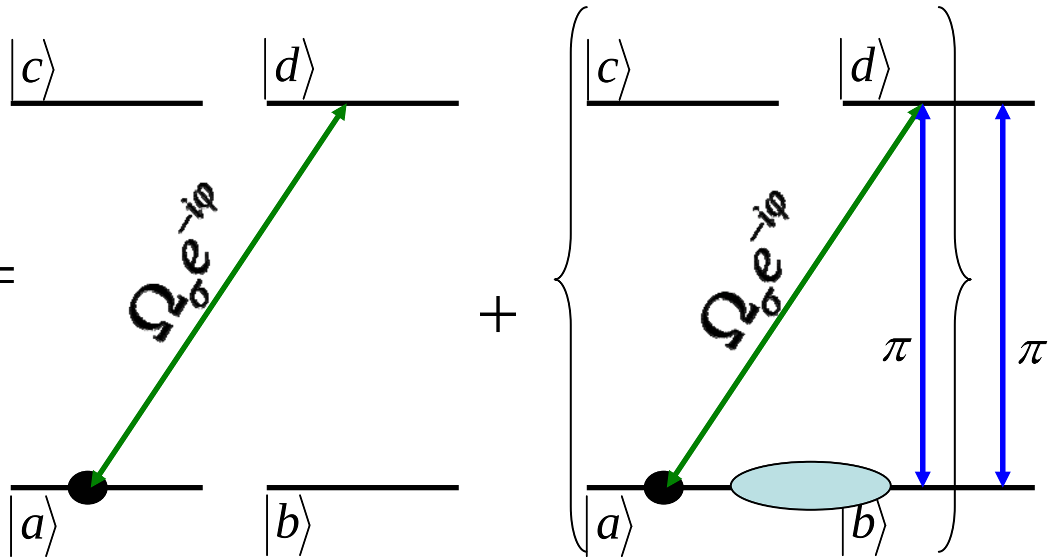
Cross- Kerr type path

$\vec{k}_{\pi} + \left(\vec{k}_{\sigma} - \vec{k}_{\pi}\right) = \vec{k}_{\sigma}$

II-d Coherent Control of Susceptibility

$$\rho_\sigma = \rho_{da} + \rho_{cb}$$

$\rho_{da} =$



Absorption path $\vec{k}_\sigma$

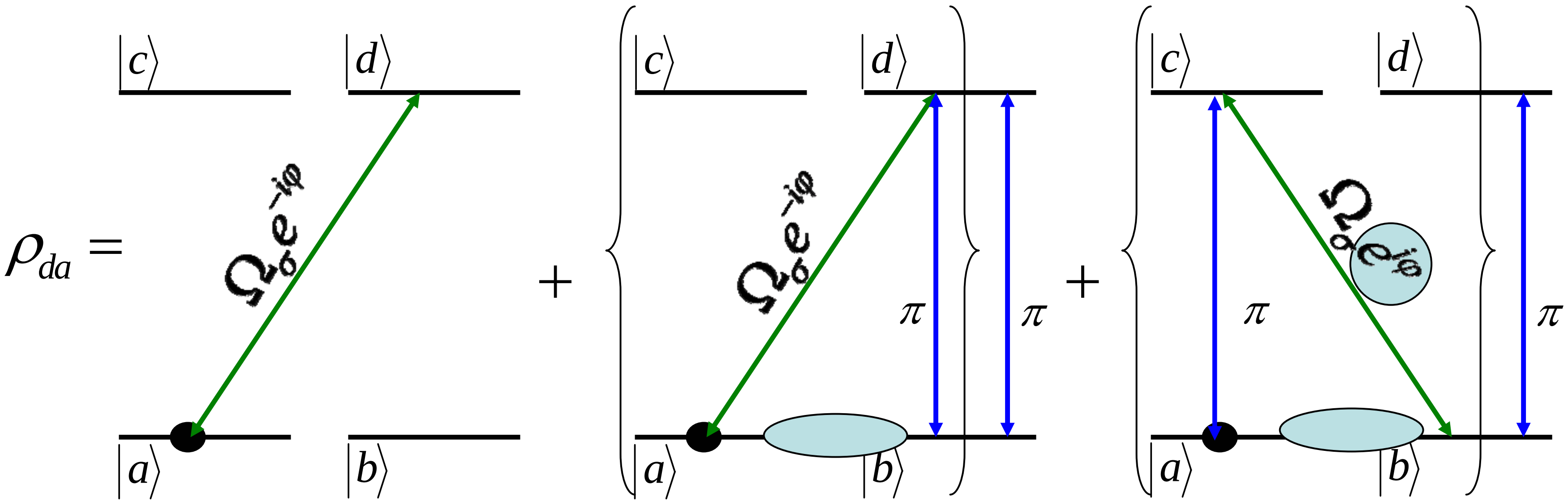
Cross- Kerr type path $\vec{k}_\pi + \left(\vec{k}_\sigma - \vec{k}_\pi\right) = \vec{k}_\sigma$

Cancel each other

$\Delta = 0$

II-d Coherent Control of Susceptibility

$$\rho_\sigma = \rho_{da} + \rho_{cb}$$



Absorption path

$\vec{k}_\sigma$

Cross- Kerr type path

$\vec{k}_\pi + \left(\vec{k}_\sigma - \vec{k}_\pi\right) = \vec{k}_\sigma$

Phase Conjugate type path

$\vec{k}_\pi - \left(\vec{k}_\sigma - \vec{k}_\pi\right) = 2\vec{k}_\pi - \vec{k}_\sigma$

$= \vec{k}_\sigma$

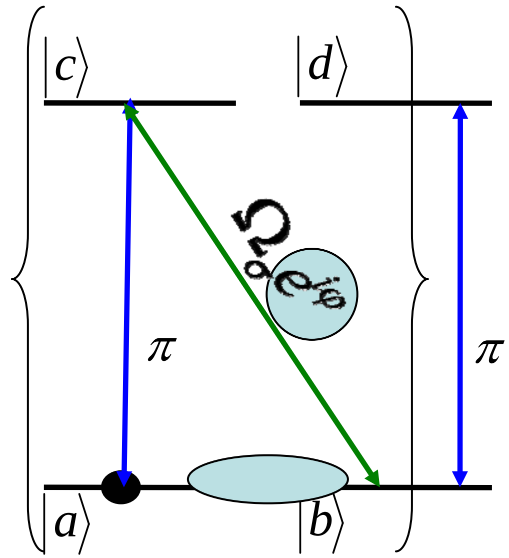
Cancel each other

$\Delta = 0$

Define the optical response

II-d Coherent Control of Susceptibility

$if\ \Omega_{\pi} << \Gamma\ and\ \alpha_0 L << 1(thin\ sample):$

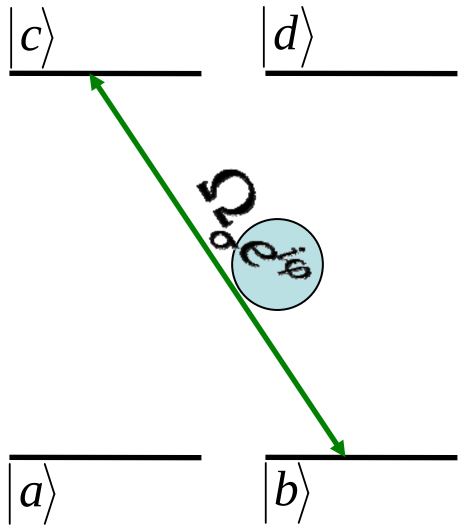


II-d Coherent Control of Susceptibility

$if\ \Omega_\pi << \Gamma\ and\ \alpha_0 L << 1\ (thin\ sample):$

Problem equivalent to linear propagation of field

$\Omega_\sigma e^{i\varphi}$ *in an assembly of two − level atoms :*



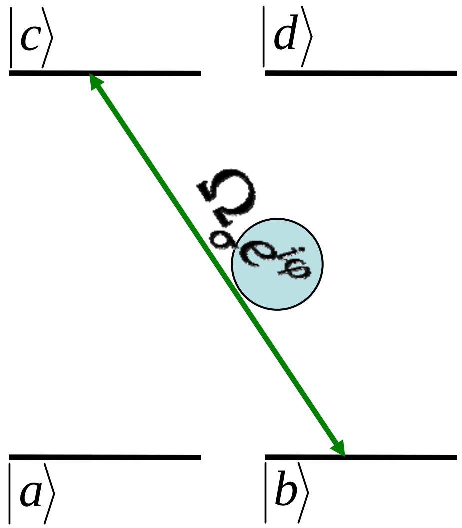
II-d Coherent Control of Susceptibility

if $\Omega_\pi << \Gamma$ *and* $\alpha_0 L << 1$ *(thin sample):*

Problem equivalent to linear propagation of field

$\Omega_\sigma e^{i\varphi}$ *in an assembly of two-level atoms:*

$$P_\sigma \left(\propto \rho_\sigma\right) \simeq \chi_{lin}\left(\Omega_\sigma e^{i\varphi}\right) = \left(\chi_{lin} e^{2i\varphi}\right)\Omega_\sigma e^{-i\varphi}$$

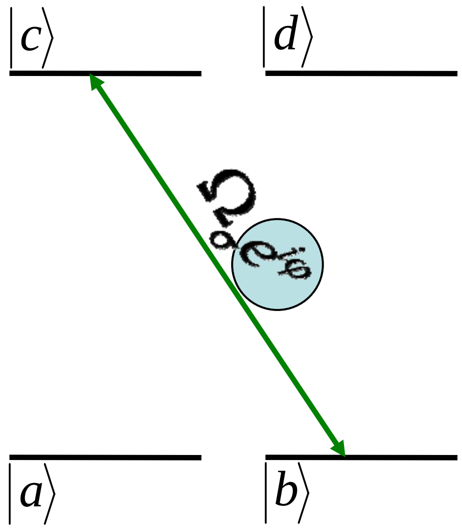


II-d Coherent Control of Susceptibility

if $\Omega_\pi << \Gamma$ *and* $\alpha_0 L << 1$ *(thin sample):*

Problem equivalent to linear propagation of field $\Omega_\sigma e^{i\varphi}$ *in an assembly of two-level atoms:*

$$P_\sigma \left(\propto \rho_\sigma\right) \simeq \chi_{lin}\left(\Omega_\sigma e^{i\varphi}\right) = \left(\chi_{lin} e^{2i\varphi}\right)\Omega_\sigma e^{-i\varphi}$$



$$\longrightarrow \quad \chi_{eff} \approx \chi_{lin} e^{2i\varphi}\,!$$

Independent from pump field characteristics!

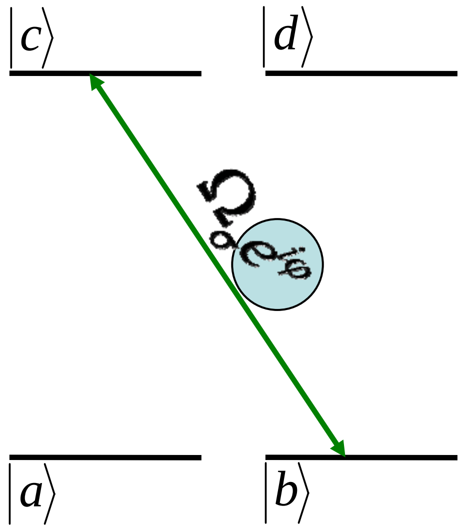
II-d Coherent Control of Susceptibility

$if\ \Omega_\pi << \Gamma\ and\ \alpha_0 L << 1\ (thin\ sample):$

Problem equivalent to linear propagation of field

$\Omega_\sigma e^{i\varphi}$ *in an assembly of two − level atoms :*

$$P_\sigma\left(\propto \rho_\sigma\right) \simeq \chi_{lin}\left(\Omega_\sigma e^{i\varphi}\right) = \left(\chi_{lin} e^{2i\varphi}\right)\Omega_\sigma e^{-i\varphi}$$



$$\longrightarrow \quad \chi_{eff} \approx \chi_{lin} e^{2i\varphi}\ !$$

Independent from pump field characteristics!

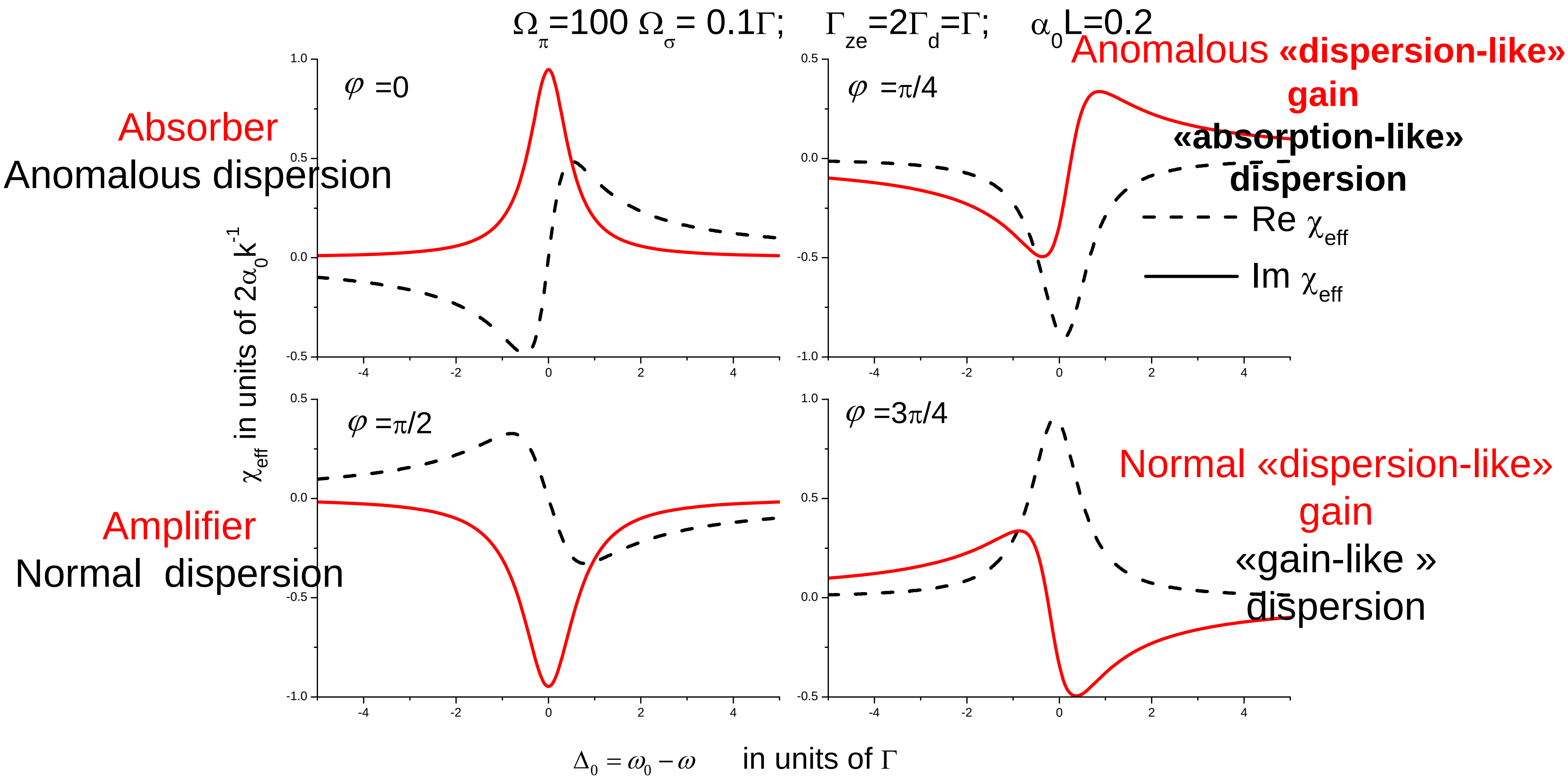
-The system behaves as a linear medium with a modified (linear) suceptibility controlled by the phase shift

-Giant non-linearity

F. A. Hashmi and M. A. Bouchene Phys. Rev. Letters 101, 21306 (2008)

II-d Coherent Control of Susceptibility

$$\chi_{eff} \approx \chi_{lin} e^{2i\varphi} : gain-dispersion\ coupling$$

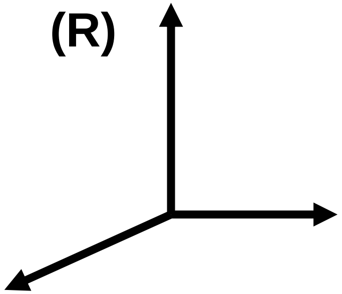


F. A. Hashmi and M. A. Bouchene, Phys. Rev. Letters 101, 21306 (2008)

Superluminal propagation

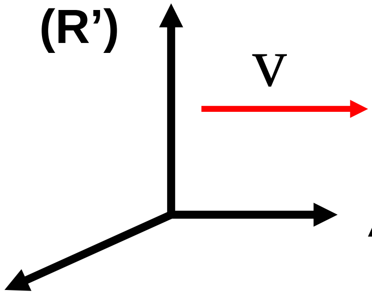
CAUSALITY AND RELATIVITY

(R)



(1) $\otimes$ (x, t) → (2) $\otimes$ $(x+\Delta x, t+\Delta t)$

(R')



$$\Delta t' = \frac{\Delta t - \frac{v}{c^2}\Delta x}{\sqrt{1-(v/c)^2}} = \frac{\Delta t}{\sqrt{1-(v/c)^2}}\left(1-\frac{uv}{c^2}\right); \quad u = \frac{\Delta x}{\Delta t}$$

RELATIVITY (implicit) → $v < c$

CAUSALITY → $\Delta t' \Delta t \geq 0$ $\xrightarrow[v<c]{}$ $u \leq c$

u: signal velocity

WHAT IS A SIGNAL?

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In Optics :

Wave packet with sharp front (Sommerfeld 1910)

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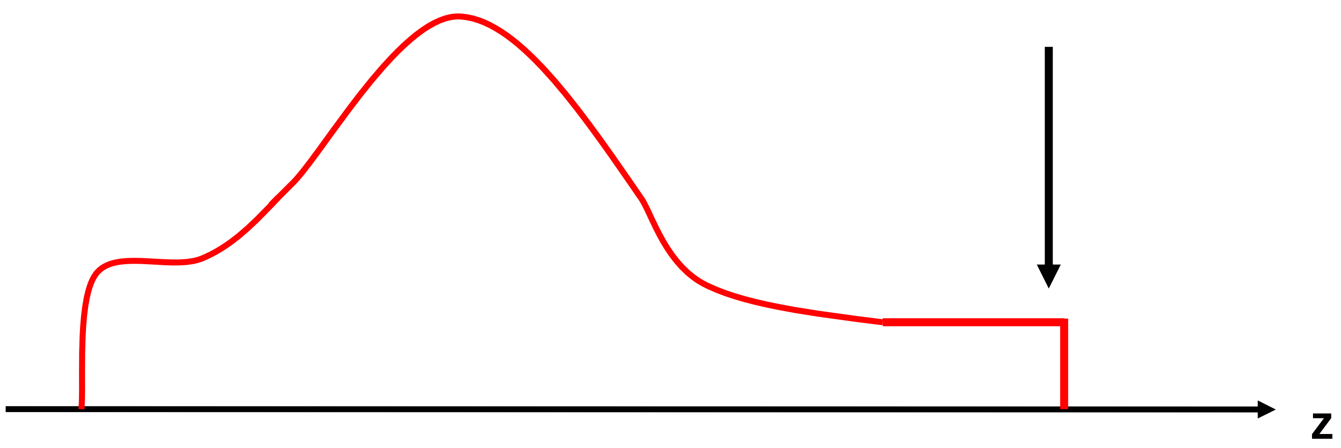
In Optics :

Wave packet with <u>sharp front</u> (Sommerfeld 1910)



WHAT IS A SIGNAL?

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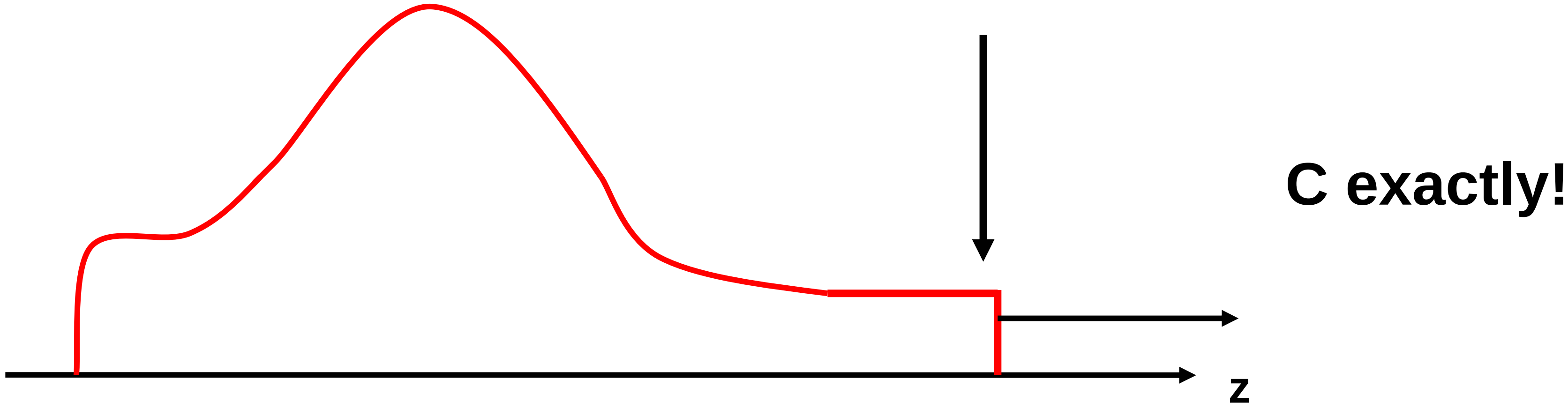
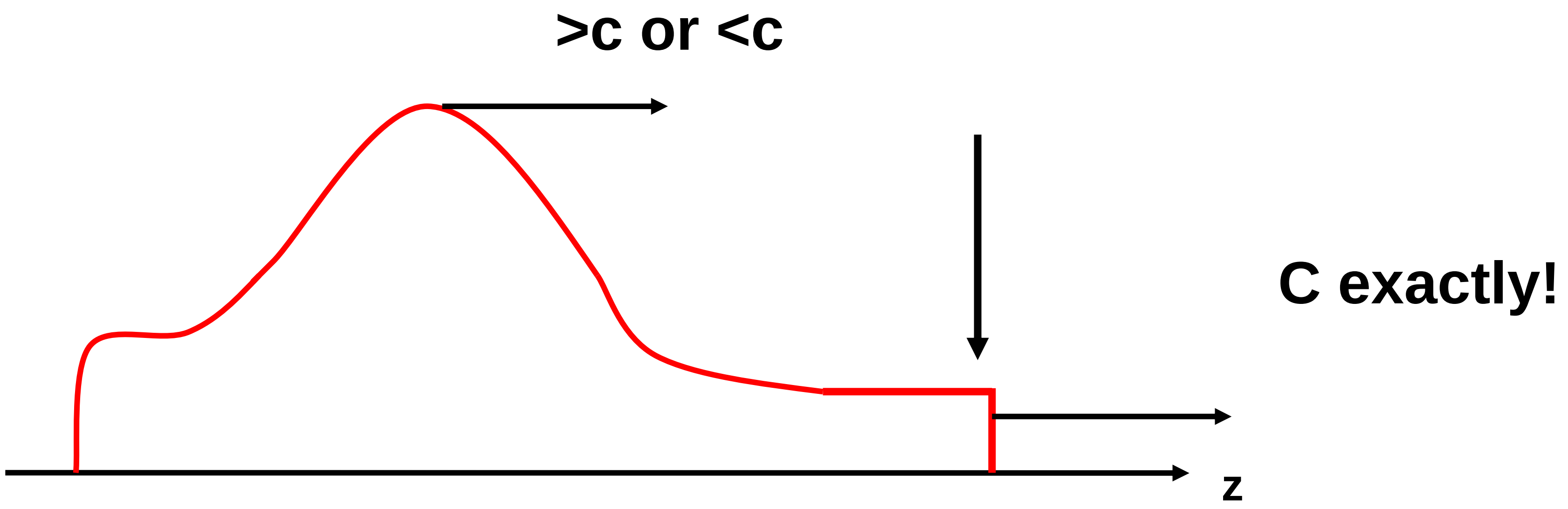


WHAT IS A SIGNAL?

In Optics :

Wave packet with <u>sharp front</u> (Sommerfeld 1910)

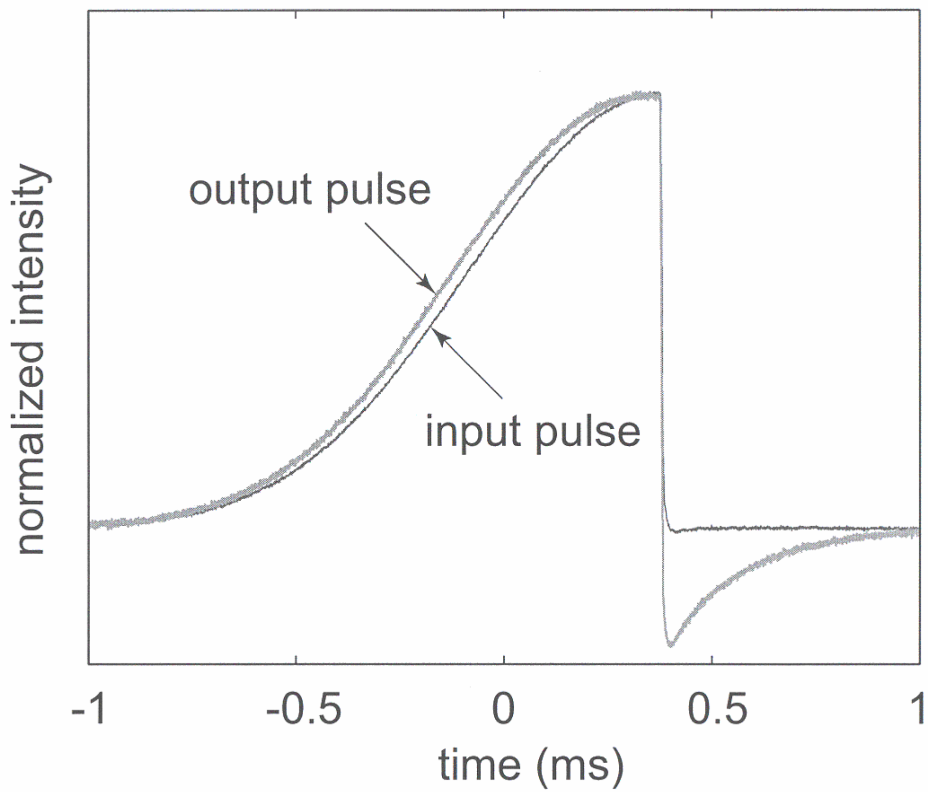


- Envelope faster than c ! (Vg > c), *<u>Vg still meaningful</u>*

-Pulse distortion occurs if the back part of the pulse catch up the sharp front

<u>Extension: non analyticity (Chiao et al. 2000)</u>

Propagation of a Truncated Pulse through Alexandrite as a Fast-Light Medium



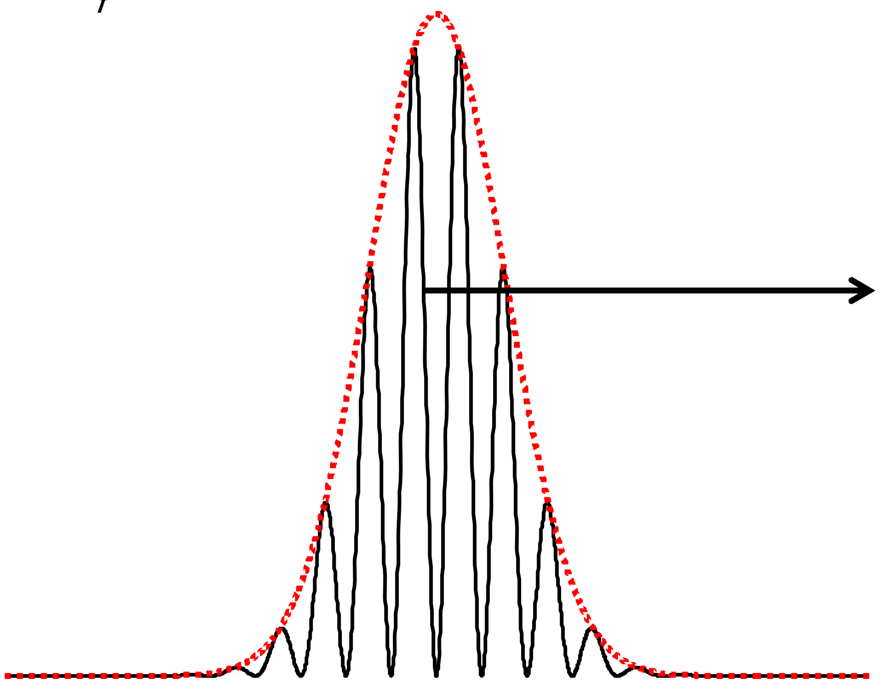
Smooth part of pulse propagates at group velocity

Discontinuity propagates at c

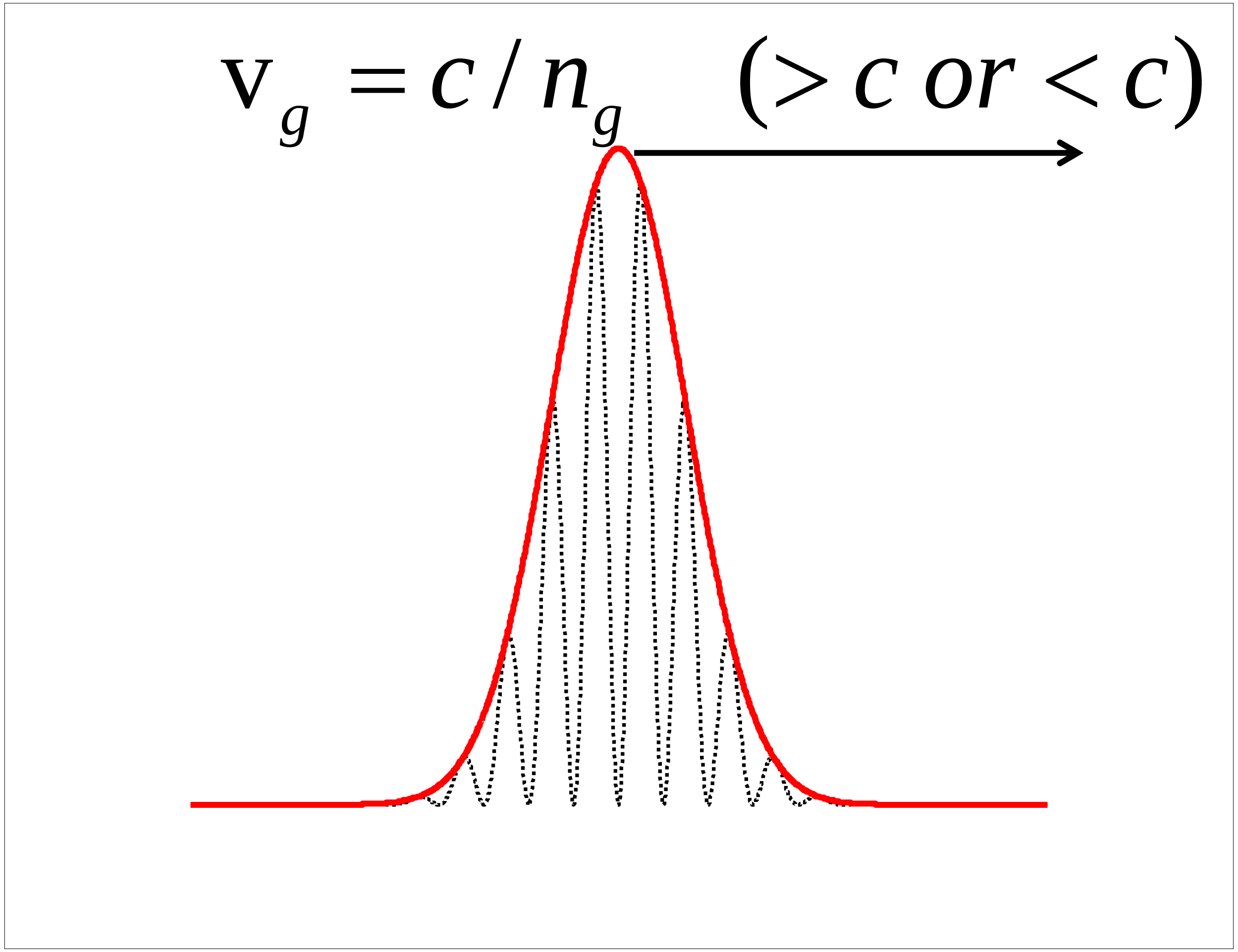
Five velocities for a light pulse:

Phase velocity

$$\mathrm{v}_{\varphi} = c/n \quad (> c \text{ or } < c)$$



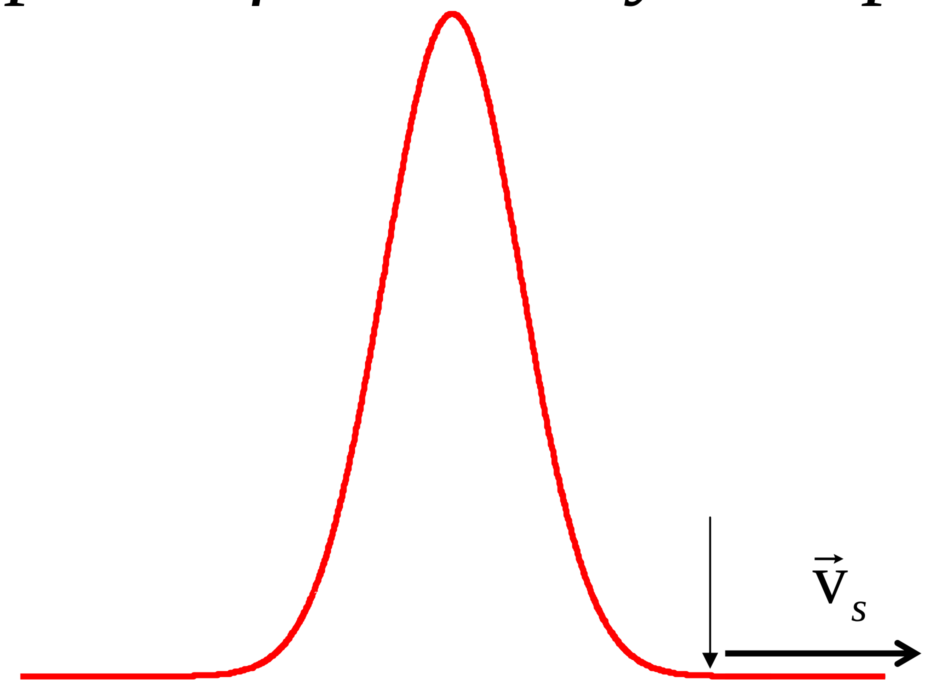
Five velocities for a light pulse: Group velocity



Meaningful in a linear dispersive medium with constant spectral gain

Five velocities for a light pulse: Signal velocity

$$\vec{v}_s : speed\ of\ non\ analytical\ point = c$$



Five velocities for a light pulse:

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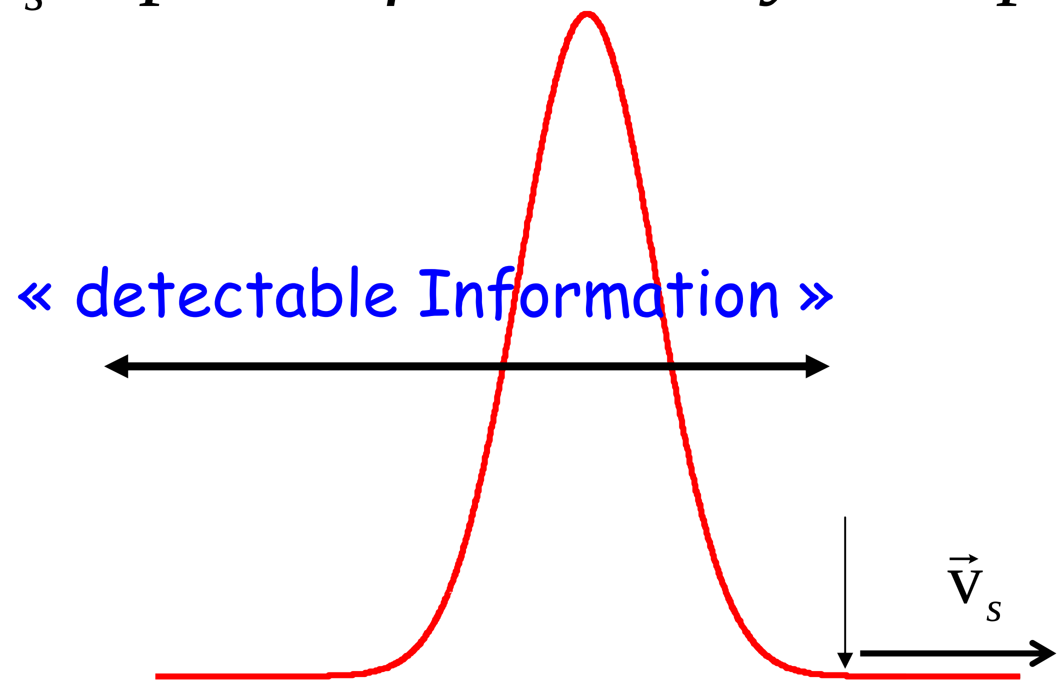
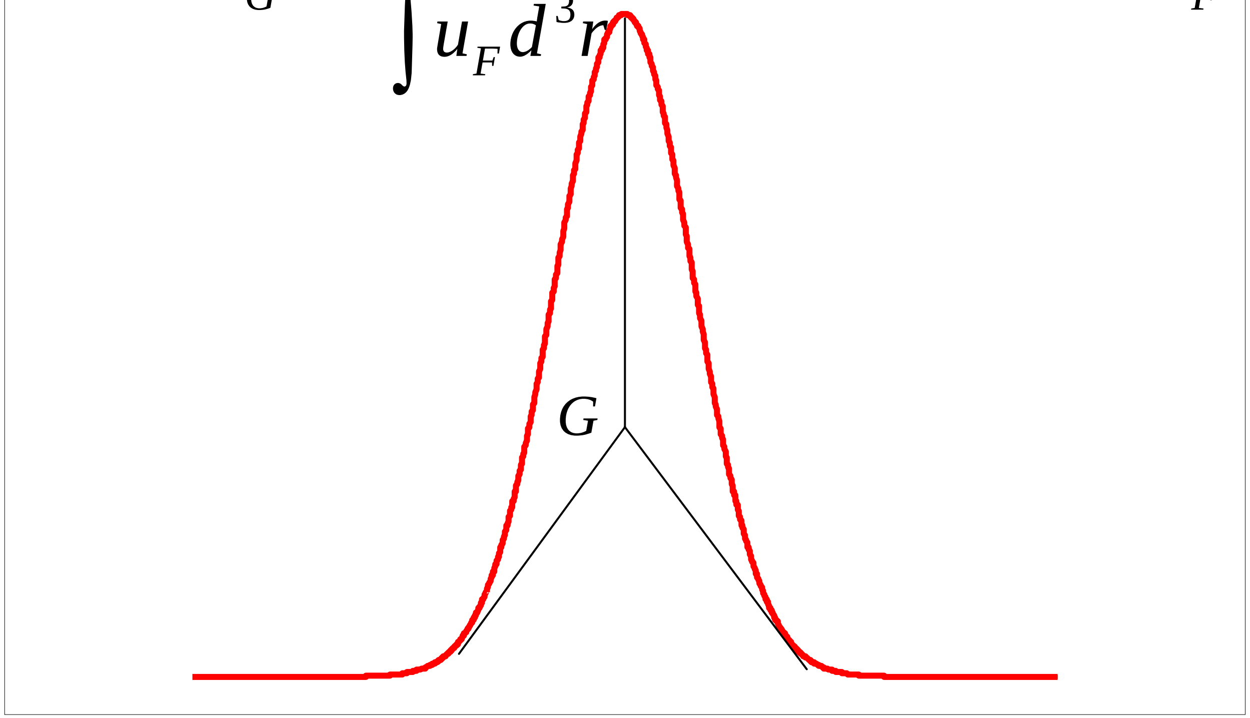


Five velocities for a light pulse:
centroid of field energy

$$\vec{r}_G = \frac{\int \vec{r}\, u_F\, d^3 r}{\int u_F\, d^3 r}$$

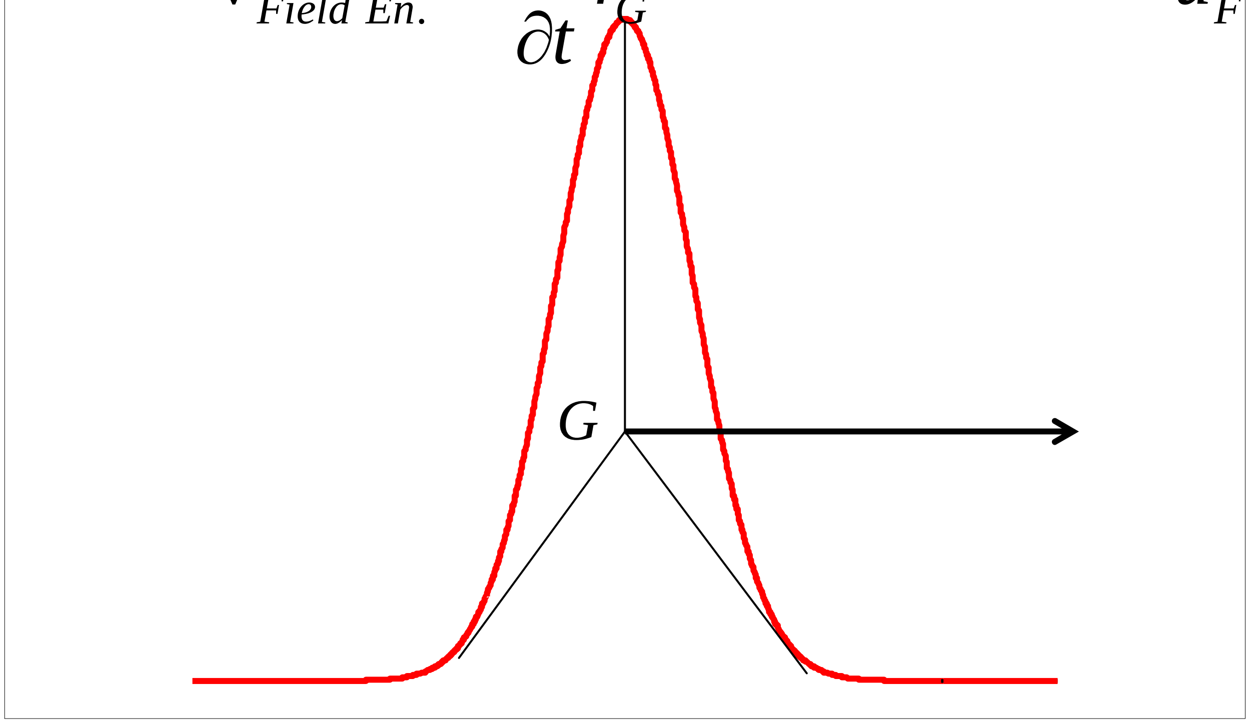
$$u_F = \frac{\varepsilon_0 E^2}{2} + \frac{B^2}{2\mu_0}$$



Five velocities for a light pulse: centroid of field energy

$$\vec{v}_{Field\ En.} = \frac{\partial}{\partial t}\vec{r}_G$$

$$u_F = \frac{\varepsilon_0 E^2}{2} + \frac{B^2}{2\mu_0}$$



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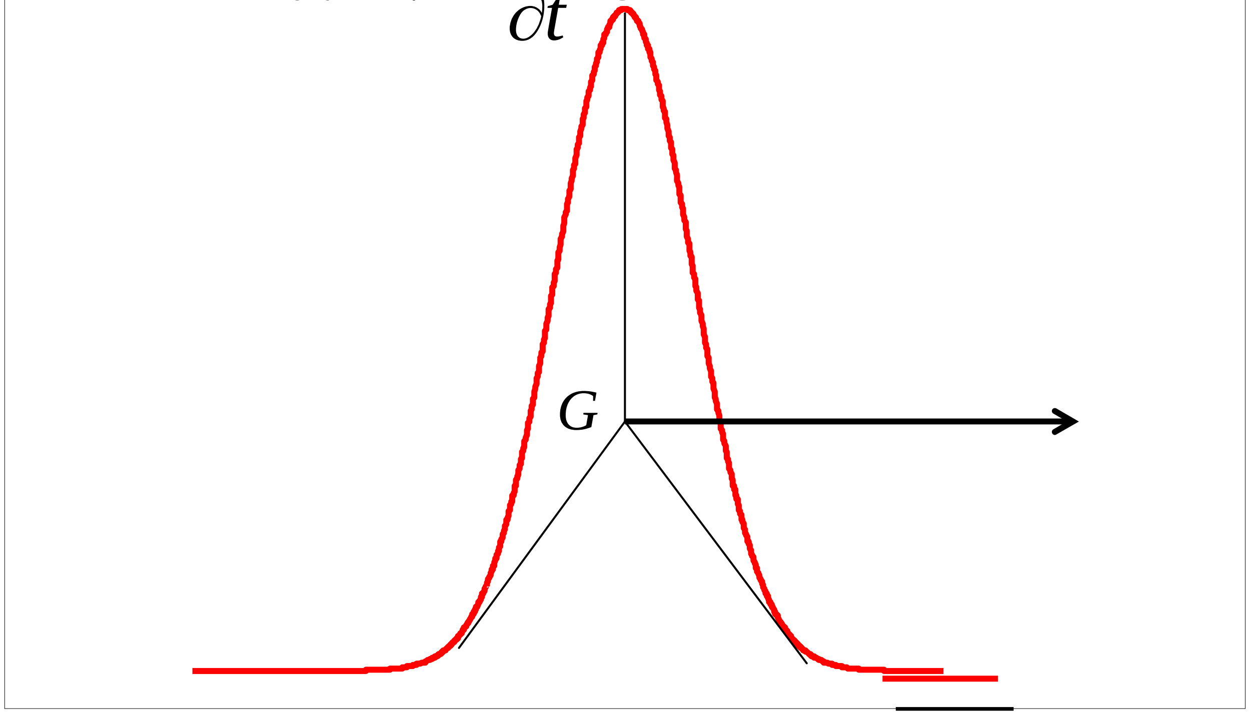


$\vec{v}_{\text{Field En.}}$: "*watched*" *velocity in pulse propagation experiment*

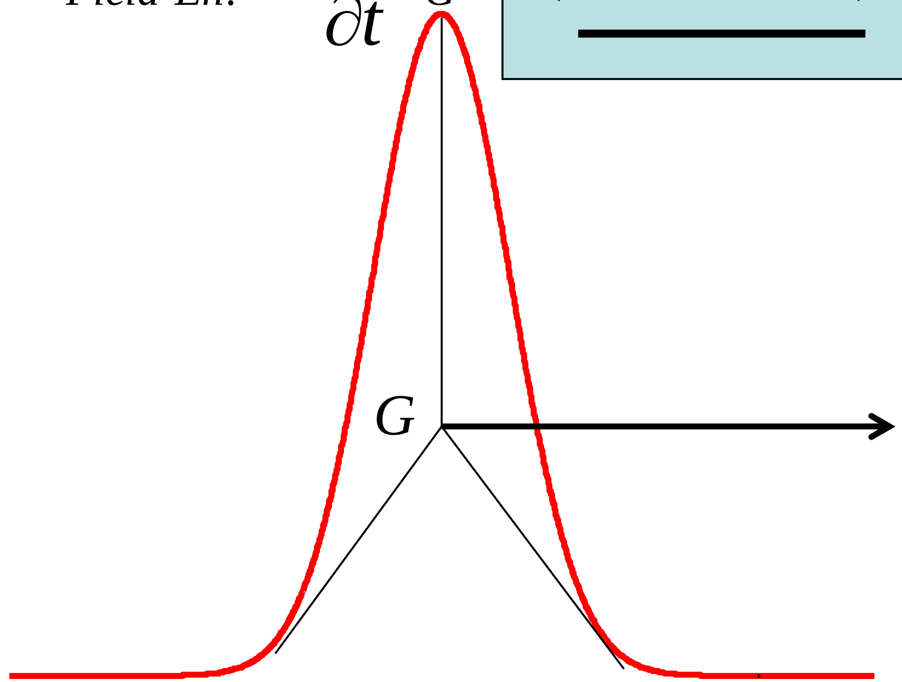
$\vec{v}_{\text{Field En.}} = \vec{v}_g + \vec{v}(\partial\alpha / \partial\omega) = \vec{v}_g$ *if dispersive medium with cte gain*

Five velocities for a light pulse: centroid of field energy

$$\vec{v}_{Field\ En.} = \frac{\partial}{\partial t}\vec{r}_G$$

(> c or < c), __________

$$u_F = \frac{\varepsilon_0 E^2}{2} + \frac{B^2}{2\mu_0}$$



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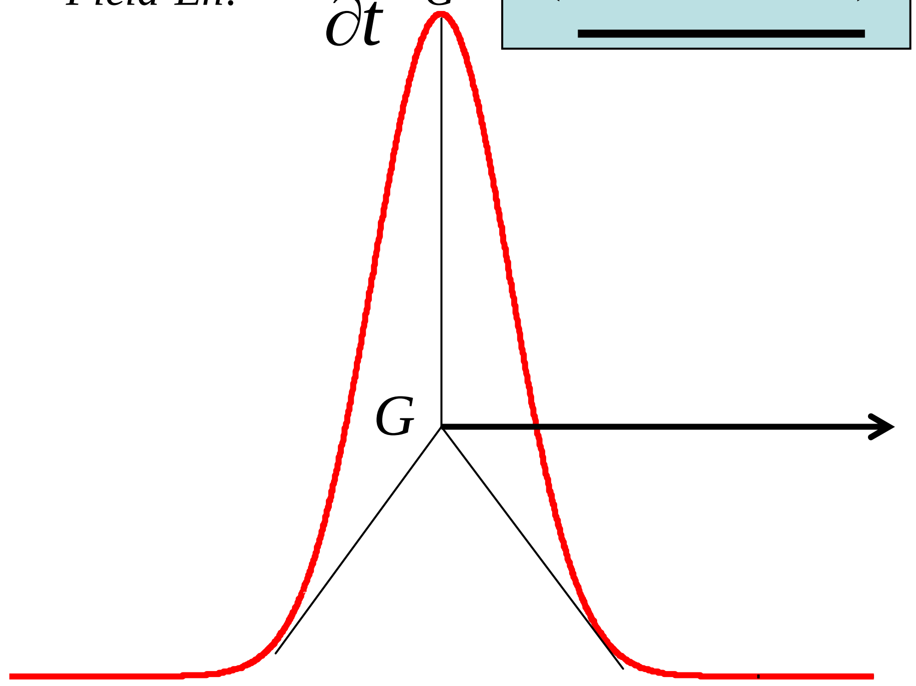
$$\vec{v}_{Field\ En.} = \frac{\partial}{\partial t}\vec{r}_G$$

$(> c \text{ or } < c)$,

$$u_F = \frac{\varepsilon_0 E^2}{2} + \frac{B^2}{2\mu_0}$$



$$\vec{v}_{Field.En.} \neq \frac{\int \vec{S}\, d^3 r}{\int u_F\, d^3 r}$$

$\vec{v}_{Field\ En.}$: "*watched*" *velocity in pulse propagation experiment*

$\vec{v}_{Field\ En.} = \vec{v}_g + \vec{v}(\partial\alpha/\partial\omega) = \vec{v}_g$ *if dispersive medium with cte gain*

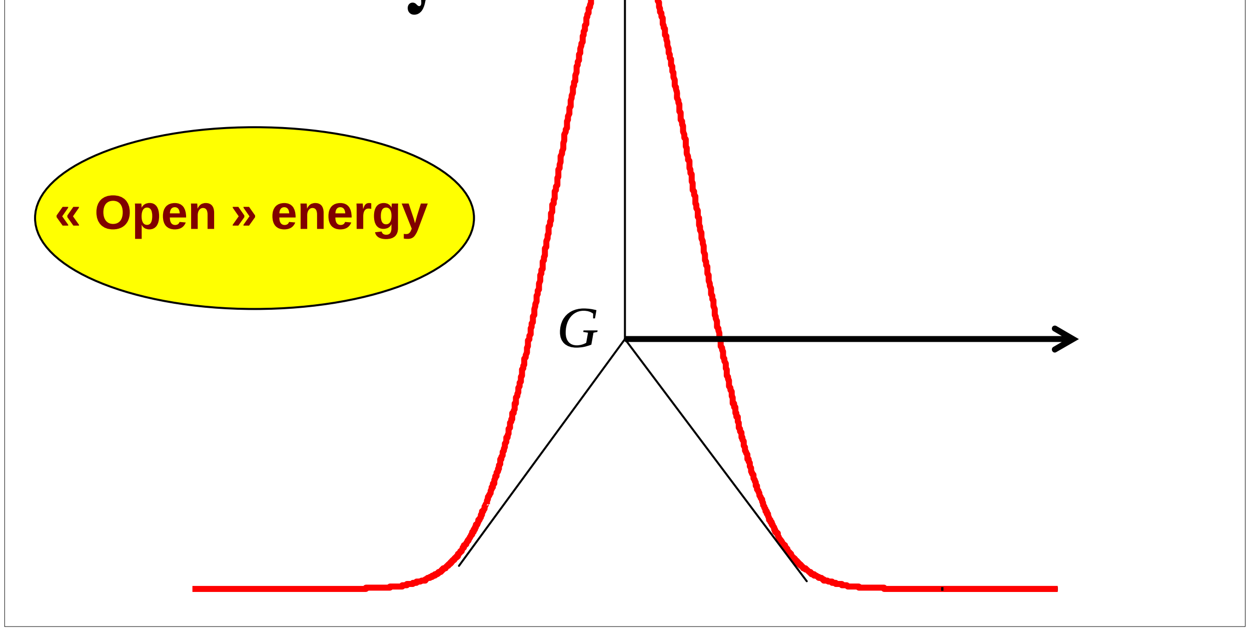
- **Can the energy go faster than c ???**
- **relativity: energy $\equiv$ matter and v < c!!**

How to solve the paradox ??

Energy has to be defined as the <u>total</u> energy

Five velocities for a light pulse: centroid of field energy

$$\vec{v}_{Field\ En.} = \frac{\partial}{\partial t}\frac{\int \vec{r}\, u_F d^3r}{\int u_F d^3r} \quad (> c \text{ or } < c), \qquad u_F = \frac{\varepsilon_0 E^2}{2} + \frac{B^2}{2\mu_0}$$



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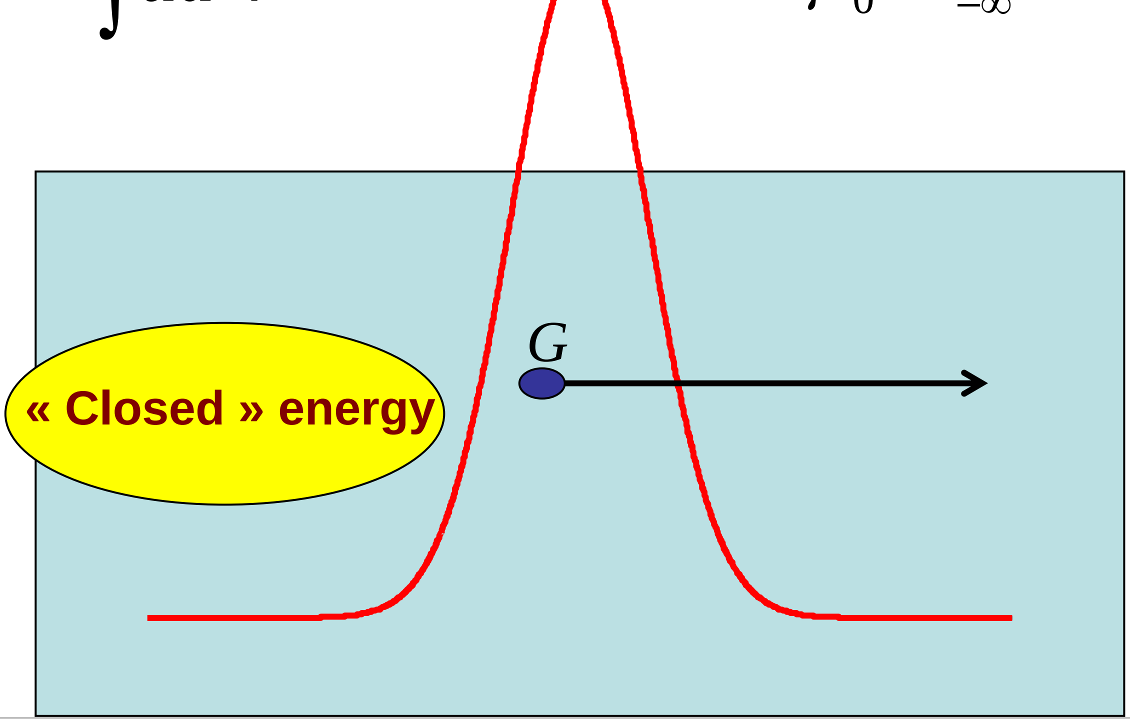
$\vec{v}_{\text{Field En.}}$: "*watched*" *velocity in pulse propagation experiment*

Five velocities for a light pulse: centroid of total energy

$$\vec{v}_{Tot.En.} = \frac{\partial}{\partial t}\frac{\int \vec{r}\,u\,d^3r}{\int u\,d^3r}, \qquad u = \frac{\varepsilon_0 E^2}{2} + \frac{B^2}{2\mu_0} + \int_{-\infty}^{t} \vec{E}\,\frac{\partial \vec{P}}{\partial t}dt' + u(t \to -\infty)$$



Five velocities for a light pulse: centroid of total energy

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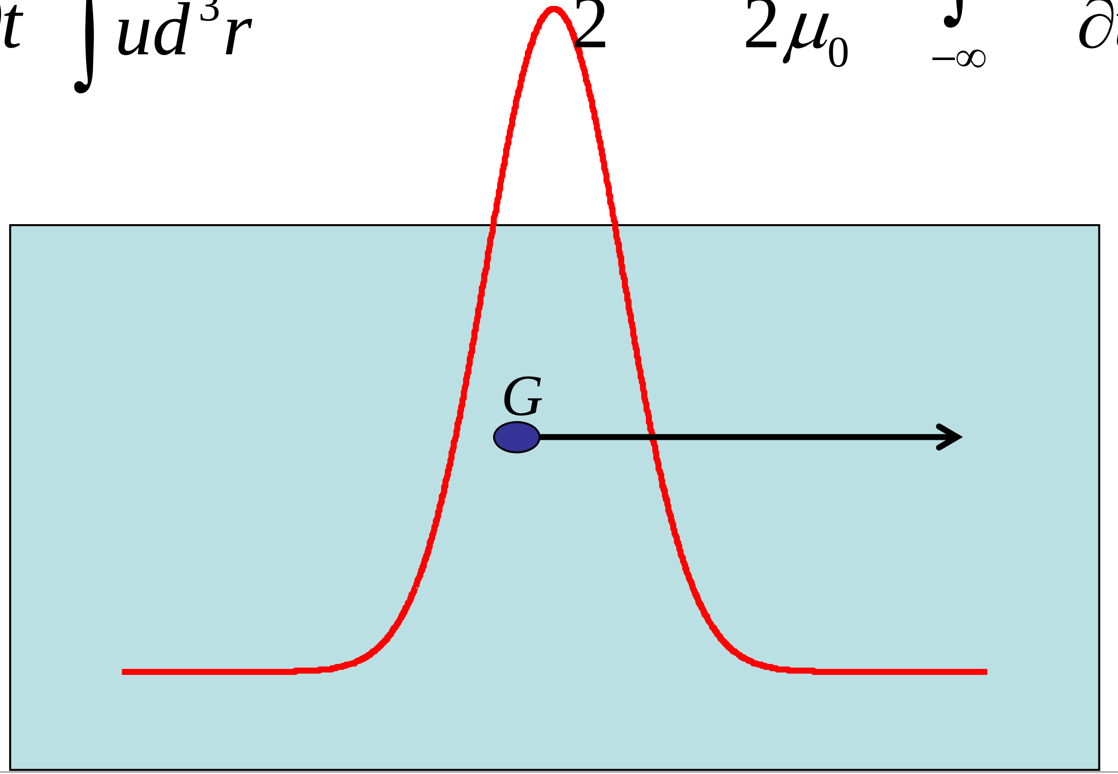
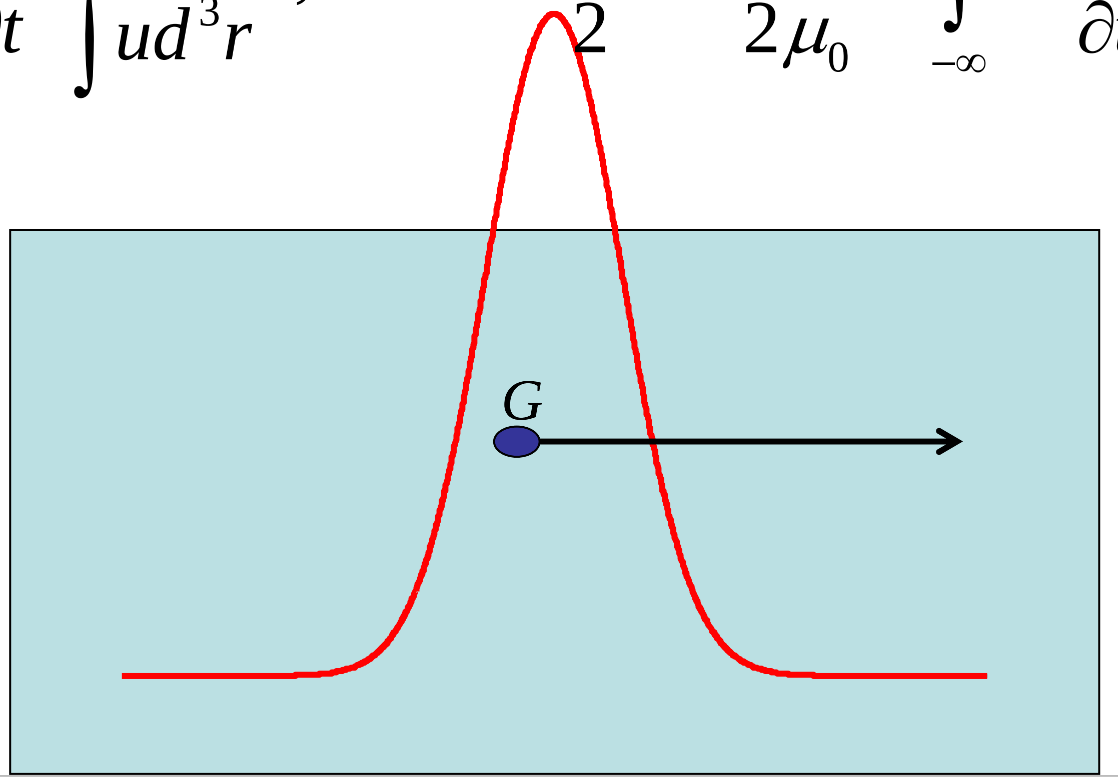


$v_{Tot.En.} < c$: **Subluminal transport of the total energy**

Five velocities for a light pulse: centroid of total energy

$$\vec{\mathrm{v}}_{Tot.En.} = \frac{\partial}{\partial t} \frac{\int \vec{r}\, u d^3 r}{\int u d^3 r}, \qquad u = \frac{\varepsilon_0 E^2}{2} + \frac{B^2}{2\mu_0} + \int_{-\infty}^{t} \vec{E}\, \frac{\partial \vec{P}}{\partial t} dt' + u(t \to -\infty)$$



$$\vec{\mathrm{v}}_{Tot.En.} = \frac{\int \vec{S}\, d^3 r}{\int u d^3 r}$$

J. Peatross et al.
PRL 84, 2370 (2000)

S. Glasgow et al.
PRE 64, 046610 (2001)

$\mathrm{v}_{\mathrm{Tot.En.}} < \mathrm{c}$: **Subluminal transport of the total energy**

BORN and WOLF: 5th edition.

without appreciable "diffusion". In such circumstances, the group velocity, which may be considered as the velocity of the propagation of the group as a whole, will also represent the velocity at which the energy is propagated.* This, however, is not true in general. In particular, in regions of anomalous dispersion (cf. § 2.3.4) the group velocity may exceed the velocity of light or become negative, and in such cases it has no longer any appreciable physical significance.

1.4 VECTOR WAVES

1.4.1 The general electromagnetic plane wave

The simplest electromagnetic field is that of a plane wave; then each Cartesian component of the field vectors and consequently $\mathbf{E}$ and $\mathbf{H}$ are, according to §1.3.1, functions of the variable $u = \mathbf{r}\,.\,\mathbf{s} - vt$ only:

$$\mathbf{E} = \mathbf{E}(\mathbf{r}\,.\,\mathbf{s} - vt), \qquad \mathbf{H} = \mathbf{H}(\mathbf{r}\,.\,\mathbf{s} - vt), \tag{1}$$

$\mathbf{s}$ denoting as before a unit vector in the direction of propagation.

Denoting by a dot differentiation with respect to t, and by a prime differentiation with respect to the variable u, we have

$$\left.\begin{aligned} \dot{\mathbf{E}} &= -v\mathbf{E}' \\ (\text{curl } \mathbf{E})_x &= \frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = E_z' s_y - E_y' s_z = (\mathbf{s} \wedge \mathbf{E}')_x. \end{aligned}\right\} \tag{2}$$

Substituting these expressions into MAXWELL's equations § 1.1 (1), (2) with $\mathbf{j} = 0$, and using the material equations § 1.1 (10), (11) we obtain

$$\left.\begin{aligned} \mathbf{s} \wedge \mathbf{H}' + \frac{\varepsilon v}{c}\mathbf{E}' &= 0, \\ \mathbf{s} \wedge \mathbf{E}' - \frac{\mu v}{c}\mathbf{H}' &= 0. \end{aligned}\right\} \tag{3}$$

If we set the additive constants of integration equal to zero (i.e. neglect a field constant in space) and set, as before, $v/c = 1/\sqrt{\varepsilon\mu}$, (3) gives, on integration,

$$\left.\begin{aligned} \mathbf{E} &= -\sqrt{\frac{\mu}{\varepsilon}}\,\mathbf{s} \wedge \mathbf{H}, \\ \mathbf{H} &= \sqrt{\frac{\varepsilon}{\mu}}\,\mathbf{s} \wedge \mathbf{E}. \end{aligned}\right\} \tag{4}$$

Scalar multiplication with $\mathbf{s}$ gives

$$\mathbf{E}\,.\,\mathbf{s} = \mathbf{H}\,.\,\mathbf{s} = 0. \tag{5}$$

This relation expresses the "transversality" of the field, i.e. it shows that the electric and magnetic field vectors lie in planes normal to the direction of propagation.

* See, for example, F. BORGNIS, *Z. f. Phys.*, **117** (1941), 642; L. J. F. BROER, *Appl. Sci. Res.*, **A2** (1951), 329.

JACKSON: 2nd Edition.

For light waves the relation between ω and k is given by

$$\omega(k) = \frac{ck}{n(k)} \tag{7.87}$$

where c is the velocity of light in vacuum, and $n(k)$ is the index of refraction expressed as a function of k. The phase velocity is

$$v_p = \frac{\omega(k)}{k} = \frac{c}{n(k)} \tag{7.88}$$

and is greater or smaller than c depending on whether $n(k)$ is smaller or larger than unity. For most optical wavelengths $n(k)$ is greater than unity in almost all substances. The group velocity (7.86) is

$$v_g = \frac{c}{[n(\omega) + \omega(dn/d\omega)]} \tag{7.89}$$

In this equation it is more convenient to think of n as a function of ω than of k. For normal dispersion $(dn/d\omega) > 0$, and also $n > 1$; then the velocity of energy flow is less than the phase velocity and also less than c. In regions of anomalous dispersion, however, $dn/d\omega$ can become large and negative as can be inferred from Fig. 7.8. Then the group velocity differs greatly from the phase velocity, often becoming larger than c.* The behavior of group and phase velocities as a

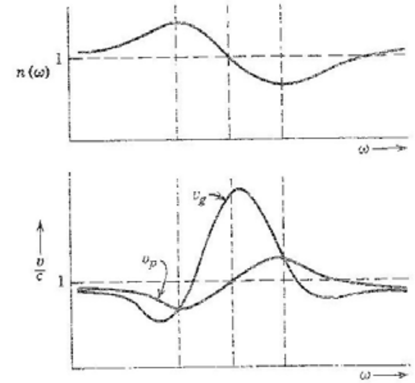


Fig. 7.13 Index of refraction $n(\omega)$ as a function of frequency ω at a region of anomalous dispersion; phase velocity v_p and group velocity v_g as functions of ω.

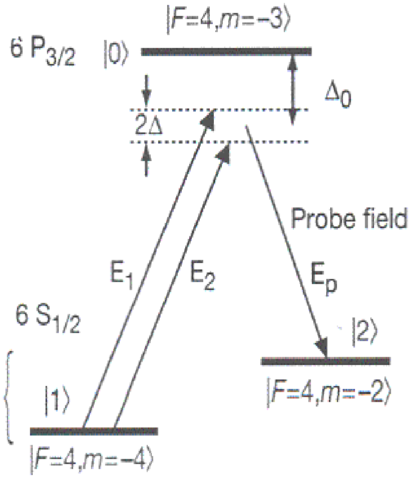
* There is no cause for alarm that our ideas of special relativity are violated; group velocity is just not a useful concept here. A large value of $dn/d\omega$ is equivalent to a rapid variation of ω as a function of k. Consequently the approximations made in (7.83) ff. are no longer valid. The behavior of the pulse is much more involved.

Gain-assisted superluminal light propagation

L. J. Wang, A. Kuzmich & A. Dogariu

NATURE, 406 (2000)

NEC Research Institute, 4 Independence Way, Princeton, New Jersey 08540, U.



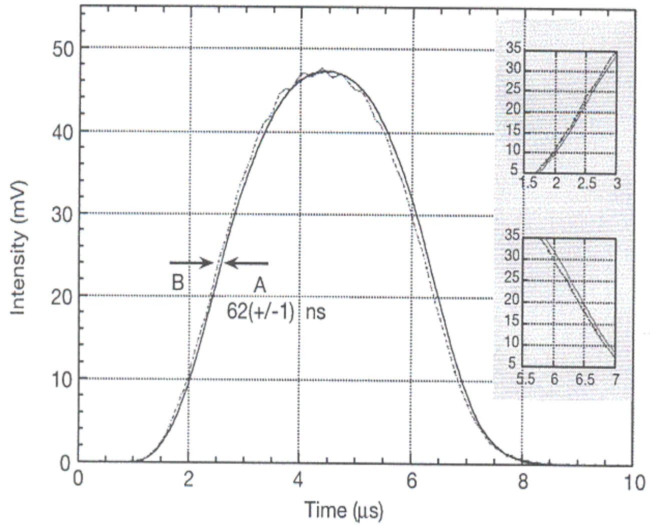


Figure 4 Measured pulse advancement for a light pulse traversing through the caesium vapour in the gain-assisted superluminality state. A indicates a light pulse far off-resonance from the caesium D_2 transitions propagating at speed c through 6 cm of vacuum. B shows the same light pulse propagating through the same caesium-cell near resonance with a negative group velocity $-c/310$. Insets show the front and trailing portions of the pulses. Pulses A and B are both the average of 1,000 pulses. The off-resonance pulse (A) is normalized to the magnitude of B.

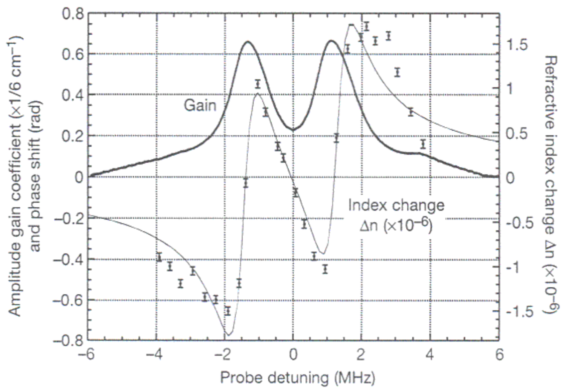
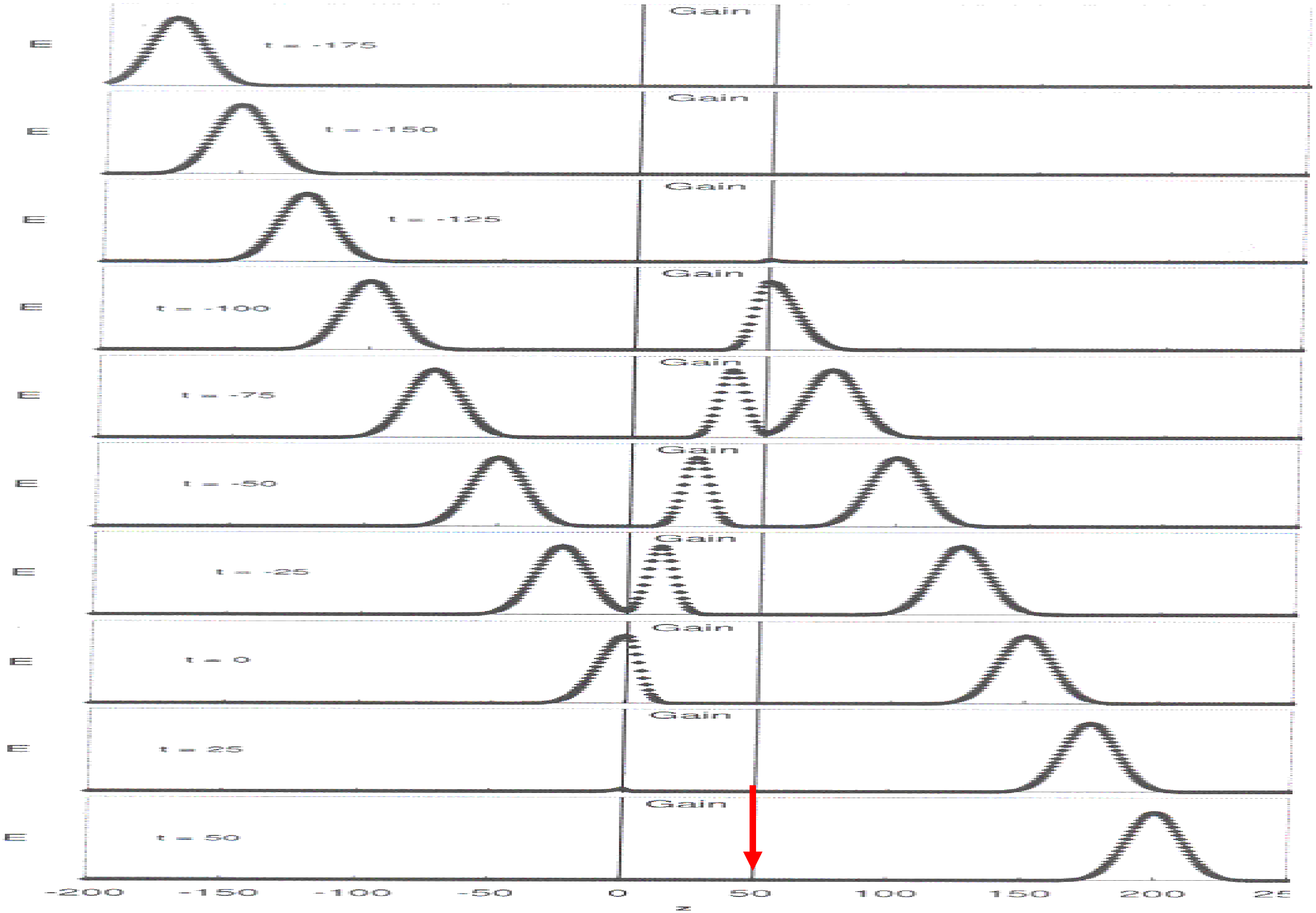


Figure 3 Measured refractive index and gain coefficient. The superimposed curve over the index data is obtained using equation (1) with parameters ν_1, ν_2 and γ obtained experimentally.

Supraluminal propagation: Backward propagation. PROPAGATION THROUGH NEGATIVE MEDIUM (n_g=-2)



Observation of Backward Pulse Propagation Through a Medium with a Negative Group Velocity

George M. Gehring,[1*] Aaron Schweinsberg,[1] Christopher Barsi,[3] Natalie Kostinski,[1,4] Robert W. Boyd[1,2]

SCIENCE, 312 (2006)

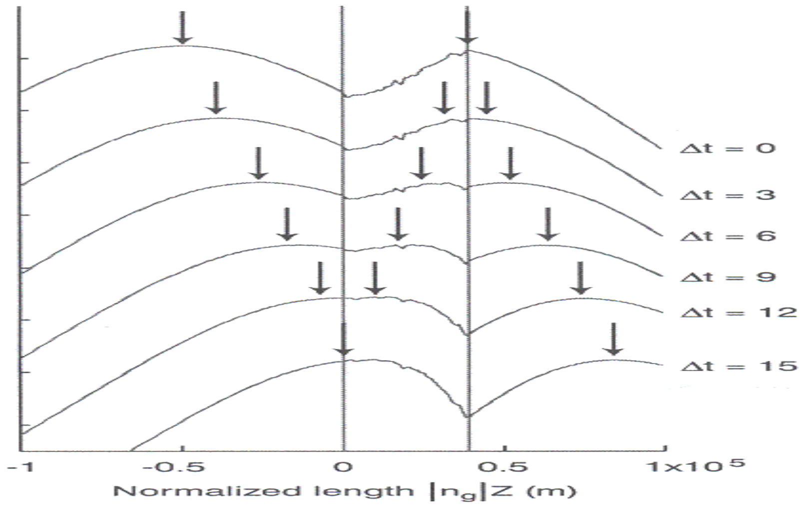


Fig. 3. Time evolution of the pulse as it propagates through the fiber. The data have been normalized at each point in the fiber to remove the effects of gain. The peak of the transmitted pulse is seen to exit the fiber before the peak of the incident pulse enters the fiber, and inside the fiber the peak moves from right to left as time increases. The arrows mark the peak of the pulse before entering the fiber (**left**), within the fiber (**center**), and after leaving the fiber (**right**). The time intervals are in milliseconds.

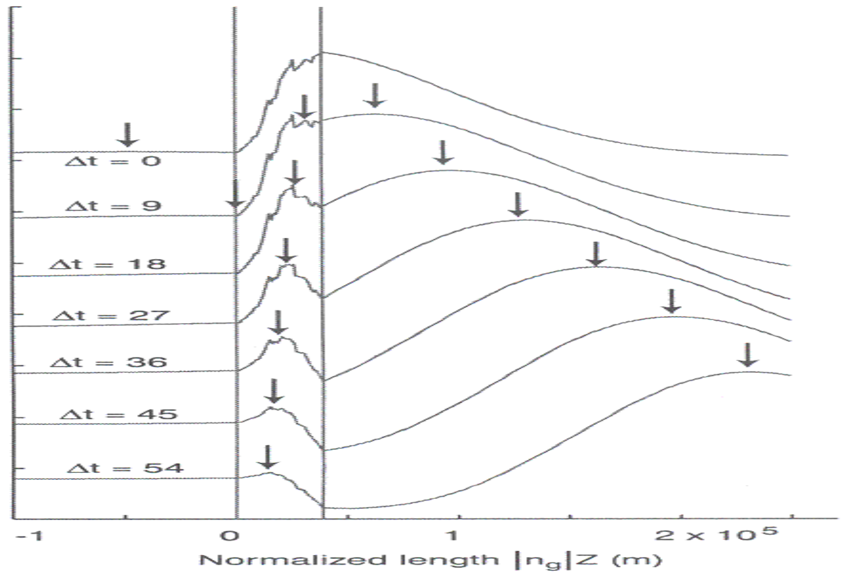


Fig. 4. Same as Fig. 3, except that the data have not been normalized to remove the effects of gain. Thus, the output pulse is much larger than the input pulse. The pulse is still seen to propagate in the backward direction, but with a different value of the pulse velocity from that of Fig. 3 as a consequence of the influence of gain. The time intervals are in milliseconds.

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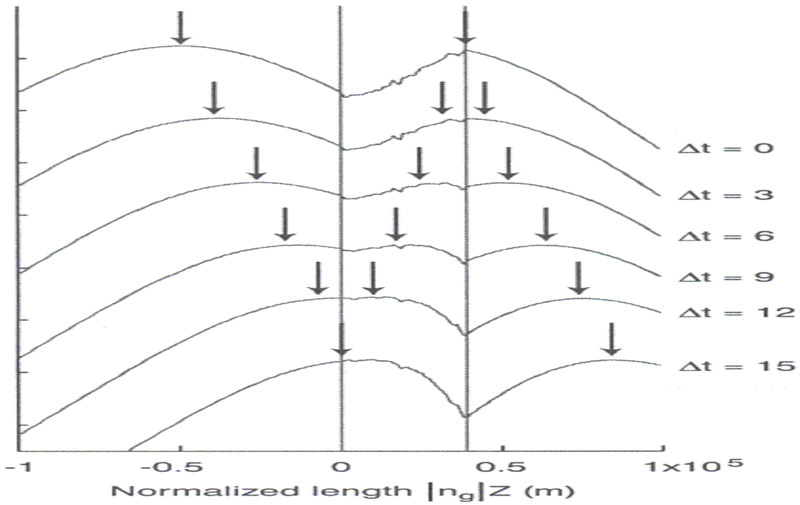

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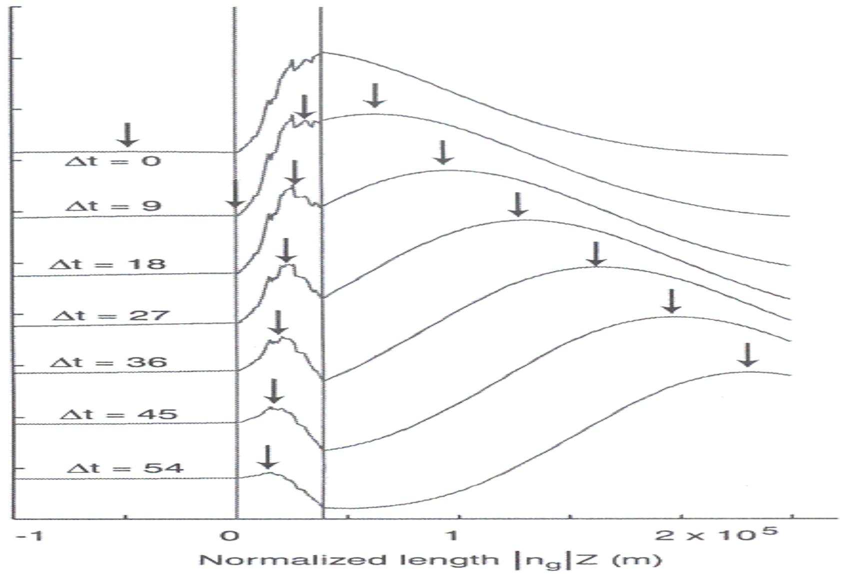

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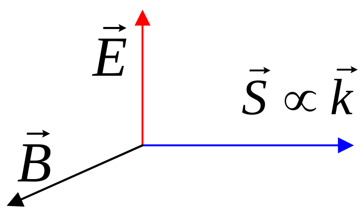
Energy flow in the forward direction

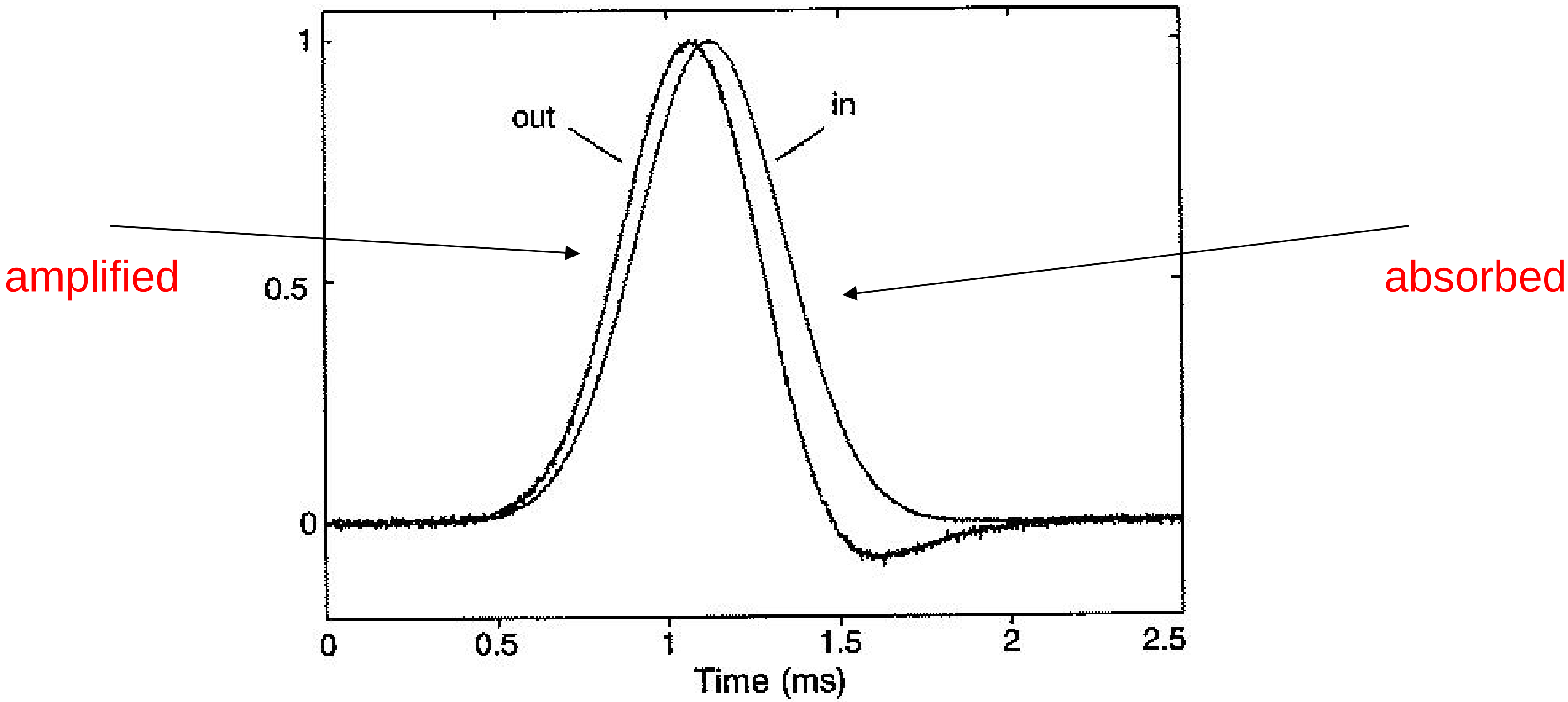


Paradox:

v_g and $v_{Tot. En.}$ not only differ by their magnitude but also by their sign!!

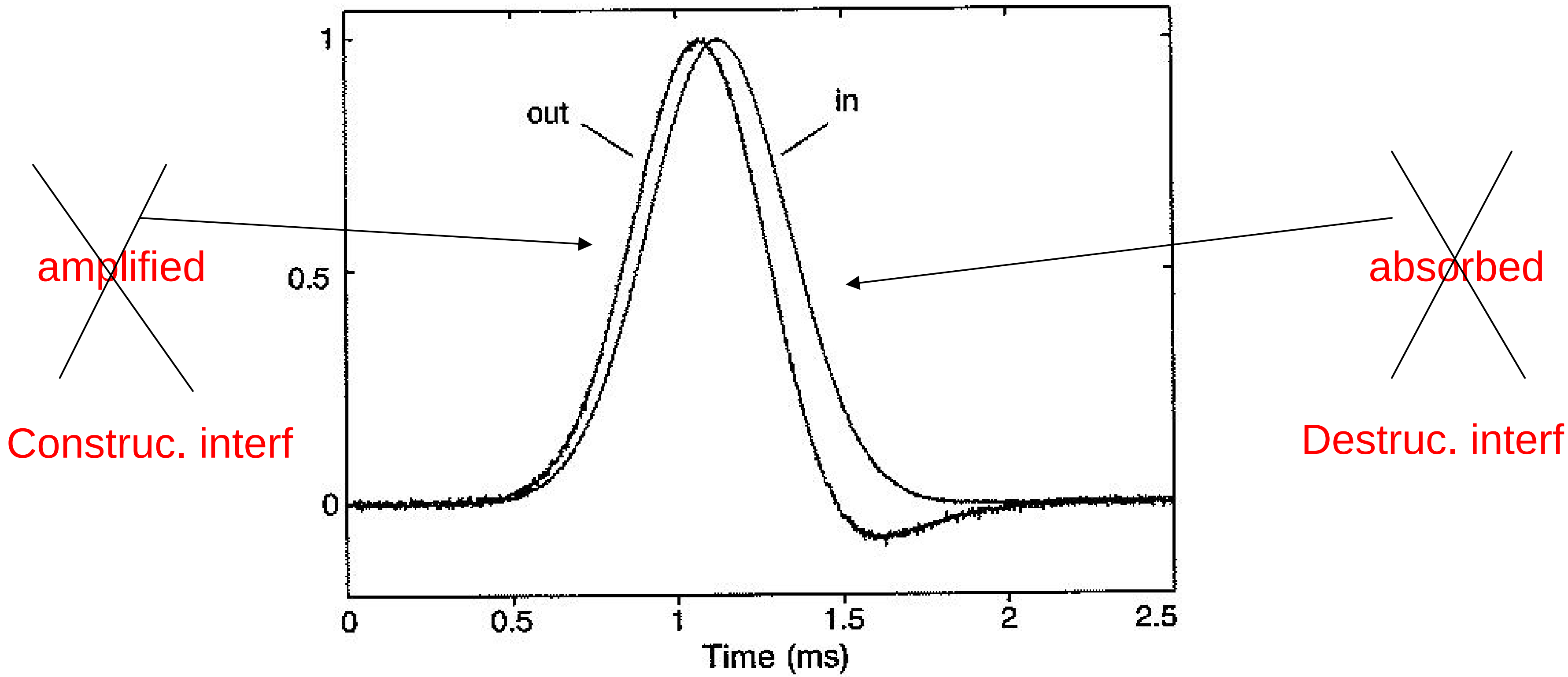
Pulse envelope displacement: spatial pulse shaping

Alternative description



Any saturable absorber may re-shape (distort) the pulse
Here (constant spectral gain), the re-shaping is <u>symmetric!,</u> Vg meaningful

Alternative description



Any saturable absorber may re-shape (distort) the pulse
Here (constant spectral gain), the re-shaping is symmetric!, Vg meaningful

Conclusion

- **Fast light: better understanding of relativity implications and light pulse velocities.**
- **Relativity implications on signal and information propagation:**

  ***Signal: non analyticity**

  ***transmission of (detectable) Information faster than c possible!**

- **Relativity implications on energy propagation:**

  ***Group velocity characterizes the displacement of the envelope but is not representative of the (total) energy flux!**

  ***Group velocity is significative in a constant spectral gain medium: displacement of the EM energy, watched velocity.**

  ***Superluminal pulse envelope propagation is the result of <u>both temporal and spatial amplitude pulse shaping</u> by the medium.**

  ***Superluminal propagation appears when truncated balance of energy**

- **Challenge in slow light: extension to short pulses (higher bandwidth): SRS, SBS techniques.**
- **Development of quantum optical memories**