Control of the Pancharatnam phase of single q-bits

M. A. Bouchene¹, M. Abdelaty²

(1)Laboratoire « Collisions, Agrégats, Réactivité », Université Paul Sabatier, Toulouse, France
(2)Mathematic department, Sohag University, Egypt
• Phase in quantum mechanics

• Control of Pancharatnam phase with classical light: **Two-level system in semi-classical regime**

• Control of Pancharatnam phase with quantized field: **Trapped-ion**

• Control of Pancharatnam phase with static field: **Cooper pair box in a SQUID configuration**
Phase in quantum mechanics

- Relative phase between two states $|A\rangle$ and $|B\rangle$

$$\Phi_{PP} = \arg\langle A | B \rangle: \text{Pancharatnam phase}$$

$$I = |e^{i\phi_1} |A\rangle + e^{i\phi_2} |B\rangle|^2 = 2 + 2 |\langle A | B \rangle| \cos (\phi - \arg \langle A | B \rangle)$$

$$\phi = \phi_1 - \phi_2$$

$I$ max when $\phi = \Phi_{PP}$
Phase in quantum mechanics

- Dynamical phase of state $|\psi\rangle$

\[
\Phi_{dyn} = \int \text{Im} \left( \langle \psi | \frac{d}{dt} |\psi\rangle \right) dt = -\frac{i}{\hbar} \int \langle \psi | H |\psi\rangle dt
\]

If $|\psi\rangle(t) \approx |\psi_n\rangle(t)$, then $\Phi_{dyn} = -\frac{i}{\hbar} \int E_{\text{adiab.}}(t) dt$

$H(t)|\psi_n\rangle(t) = E_n(t)|\psi_n\rangle(t)$
Phase in quantum mechanics

- Geometric phase of state $|\psi\rangle$

\[ \Phi_{Geom} = \text{arg} \left( \langle \psi(0) | \psi(t) \rangle \right) - \int_0^t \text{Im} \left( \langle \psi | \frac{d}{dt} | \psi \rangle \right) dt = \Phi_{PP} - \Phi_{dyn}. \]

- Connection with Berry phase:

\[ H = H(\vec{R}(t)) \]
\[ H |\psi_n(\vec{R})\rangle = E_n |\psi_n(\vec{R})\rangle \]
\[ |\psi(0)\rangle = |\psi_n(\vec{R}(0))\rangle \]

If cyclic and adiabatic evolution

\[ |\psi(t)\rangle = e^{i\Phi_{dyn}} e^{i\Phi_{geom}} |\psi_n(\vec{R}(t) = \vec{R}(0))\rangle \]

\[ \Phi_{Geom} = \Phi_{Berry} = i \oint_C d\vec{R} \langle \psi_n(\vec{R}) | \vec{\nabla} R | \psi_n(\vec{R}) \rangle \]}
Phase in quantum mechanics

• Important properties of Berry phase:

\[ \Phi_{Berry} = i \oint_C d\vec{R} \left\langle \psi_n(\vec{R}) \right| \nabla_{\vec{R}} \psi_n(\vec{R}) \right\rangle \]

Depends only on the (close) path.

• Interest for quantum information:
Insensitive to parameters that makes this path unchanged

Ex:

\[ \vec{R} = (\Omega_x, \Omega_y, \Delta) \]

\[ \Phi_{Berry} = 2\pi \left(1 - \cos \theta \right) \]
Control of the Pancharatnam phase

- HERE:

Non cyclic regime.

Behaviour of the PP with excitation parameters and comparison with population dynamics

Investigation restricted to simple q-bits
Control of the Pancharatnam phase

- Simple case: two level system in semi-classical regime

$$n_b(t) = C(W_0, \Delta, t) + D(W_0, \Delta, t) \cos \phi_0$$

$$\tan \Phi_{PP} = \frac{A(W_0, \Delta, t) + B(W_0, \Delta, t) \cos \phi_0}{A'(W_0, \Delta, t) + B'(W_0, \Delta, t) \cos \phi_0}$$

- Interesting parameter: $$\phi_0 = \varphi - a \cdot \text{gr}(a(0)b^*(0))$$

$$\hat{W} = \hbar W_0 e^{-i\varphi} e^{i \omega_L t} |a\rangle \langle b| + \hbar c$$

$$D, B, B' \propto \sqrt{n_a(0)n_b(0)}$$

Needs initial coherence: Ramsey-like configuration
Two-level system in semi-classical regime

\[ n_a(0) = n_b(0) = 1/2; \quad \theta = \frac{\pi}{4} \]

Highly non-linear dependence
(Attosecond resolution in optical domain)

\[ W_0 t = 0.05 \]

\[ W_0 t = 1 \]

\[ W_0 t = 10. \]
Trapped-ion

Schematic drawing of the electrodes for a linear rf trap. A common rf potential $U^- \cos(\nu ft)$ is applied to the dark electrodes; the other electrodes are held at rf ground through capacitors (not shown) connected to ground. The lower right portion of the figure shows the $x$-$y$ electric fields from the applied rf potential at an instant when the rf potential is positive relative to the ground. A static electric potential well is created (for positive ions) along the $z$ axis by applying a positive potential to the outer segments (gray) relative to the center segments (white).

$$\hat{H}_{\text{trans.ion}} \approx \hbar \omega_y \left( \hat{a}^+ \hat{a} + 1/2 \right)$$

laser: $E = \varepsilon e^{i(kz-\Omega t)} e^{i\phi} + cc$
Trapped-ion

\[ \hat{H}_{\text{int}} = \hbar g \left( \sigma^+ e^{-i(\Omega t - k \hat{z})} + HC \right) \]

\[ \sigma^+ = |e\rangle \langle g| \quad \text{Doppler effect: coupling between electronic and vibrational motion} \]

\[ |e\rangle \quad \text{Side bands} \quad |e\rangle \quad \text{Lamb-Dicke regime} \quad |e\rangle \quad \text{RWA} \]

\[ \Delta \quad \omega_v \quad \hbar \omega_v \gg E_{\text{recoil}} \]

\[ \hat{H}_{\text{int}} = \hbar g'(\Omega) \left( \sigma^+ \hat{a} e^{i\Delta t} + \hat{a}^+ e^{-i\Delta t} \right) : \text{Jaynes–Cummins model for atom–phonon interaction assisted by laser} \]
Trapped-ion

More complex system

$|e\rangle$

$|g\rangle$

$\hat{H}_{\text{eff}} = \begin{pmatrix} \Delta + \beta n & \lambda_{eg} e^{-2i\phi} \sqrt{(n+1)(n+2)} \\ \lambda_{eg} e^{2i\phi} \sqrt{(n+1)(n+2)} & -\beta(n+2) \end{pmatrix}$

Parameters: Laser intensity; Laser phase; Laser frequency; Phonon number
Trapped-ion

Coherent state $\bar{n} = 10$, $\Delta t = 1$, $n_g(0) = n_e(0) = 1/2$
Trapped-ion

Coherent state $\bar{n} = 10$, $\Delta t = 1$, $n_g(0) = n_e(0) = 1/2$

$\beta / \Delta = 0.6$

$\phi_0 = 1.1\pi$

$\beta / \Delta$ constant
Trapped-ion

Coherent state $\bar{n} = 10$, $\Delta t = 1$, $n_g(0) = n_e(0) = 1/2$

Highly non harmonic

$\beta/\Delta = 0.6$ $\phi_0 = 1.1\pi$ constant

Still vary

$\Phi_{PP}$

$\beta/\Delta$ $\phi_0$

$n_e$

$\beta/\Delta$ $\phi_0$

Highly non harmonic
Single Cooper-pair box

Cooper-pair box with a SQUID loop.

$E_{J0}$ is the Josephson energy,

$V_g$ is the voltage gate and $\Phi_{ext}$ is the magnetic flux

$$\hat{H} = 4E_c \sum_n (n-n_g)^2 |n\rangle\langle n| - \frac{E_J}{2} \sum_n (|n+1\rangle\langle n| + |n\rangle\langle n+1|)$$

$$E_c = \frac{e^2}{2(C_g + C_J)} \quad n_g = C_g V_g / 2e \quad E_J = E_{J0} \cos \left( \frac{\pi \Phi_{ext}}{\Phi_q} \right) \quad \Phi_q = h / 2e$$

$E_g \approx 1/2$\hspace{1cm} $E_J / 2$\hspace{1cm} Effective Josephson coupling energy

$|n = 1\rangle; \quad E_1 = 4E_c (1 - 2n_g)$

$|n = 0\rangle; \quad E_0 = 0$

Effective two-level system

Control parameter: $\phi_0 = \pi \Phi_{ext} / \Phi_q$
Single Cooper-pair box

\[ \Delta n_1 / \Delta \varphi_0 \propto E_{J0} t \]

Higher sensitivity to the magnetic field

\[ \Delta \Phi_{PP} / \Delta \varphi_0 \propto E_{J0} t \left( E_{J0} / E_1 \right) \gg 1 \]
Conclusion

• Comparison of PP with population: greater sensitivity to control parameters

• Applications:
  - Phase, flux: pertinent parameters for q-bits manipulation
  - Detection of small magnetic flux (Cooper-box).
  - Stabilisation of interferometers.
Two-level system in semi-classical regime

\[ n_a(0) = n_b(0) = 1/2; \quad \theta = \frac{\pi}{4} \]

Highly non-linear dependence
(Attosecond resolution in optical domain)

\[ W_0 \ t = 0.05 \]
\[ W_0 \ t = 1 \]
\[ W_0 \ t = 10. \]