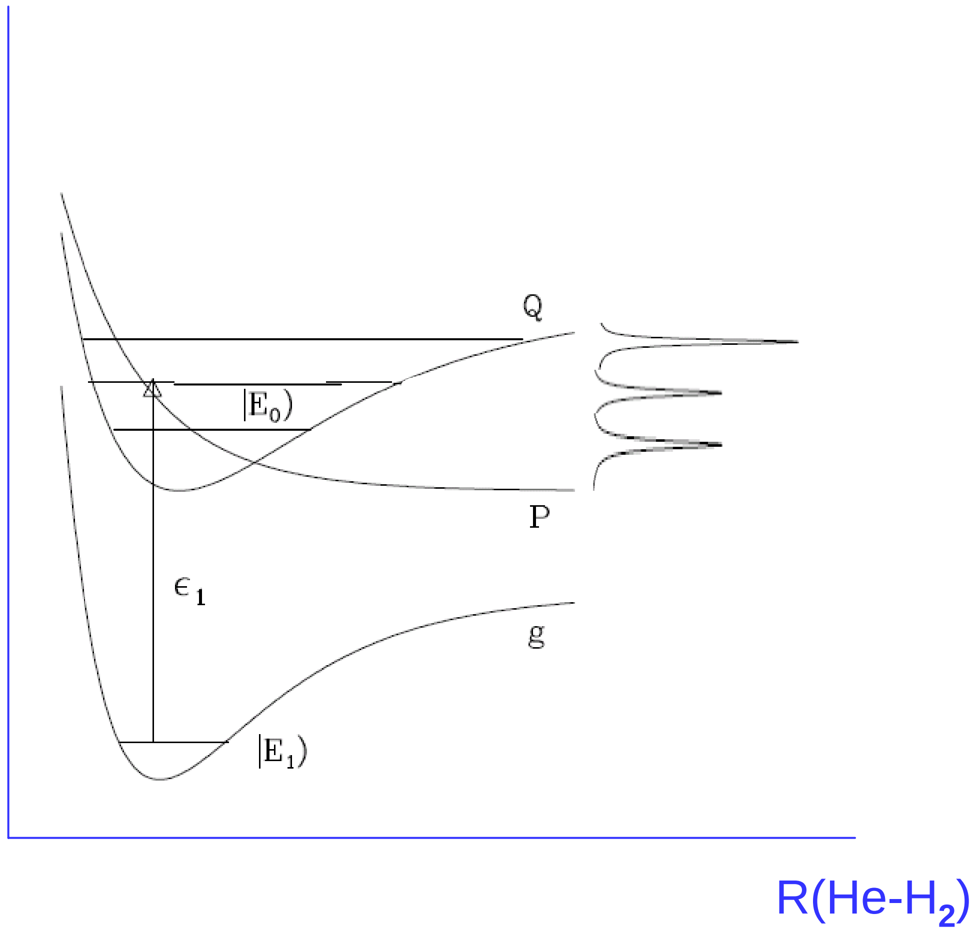
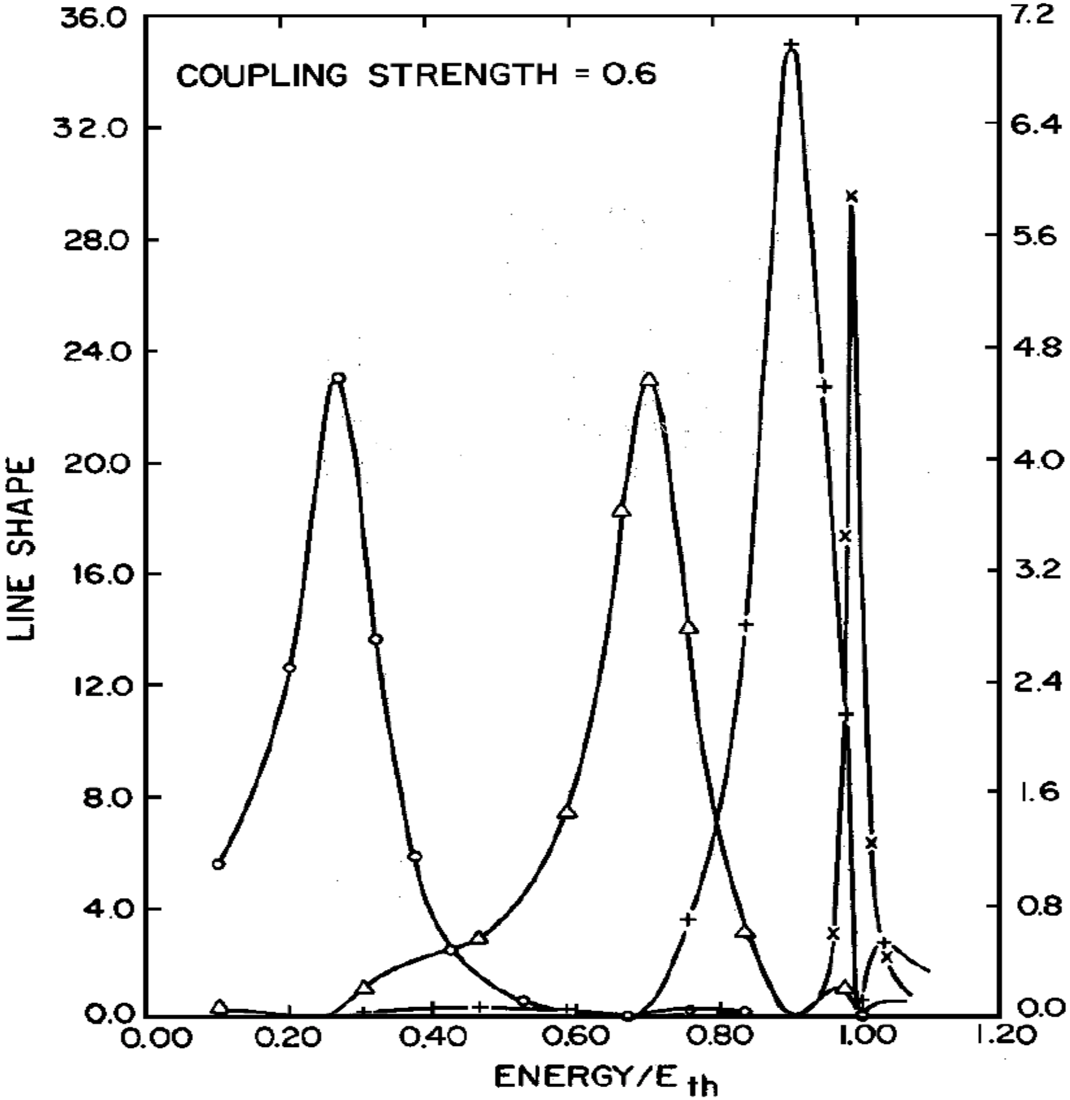
Quantum Theory of EIT and Slow Light

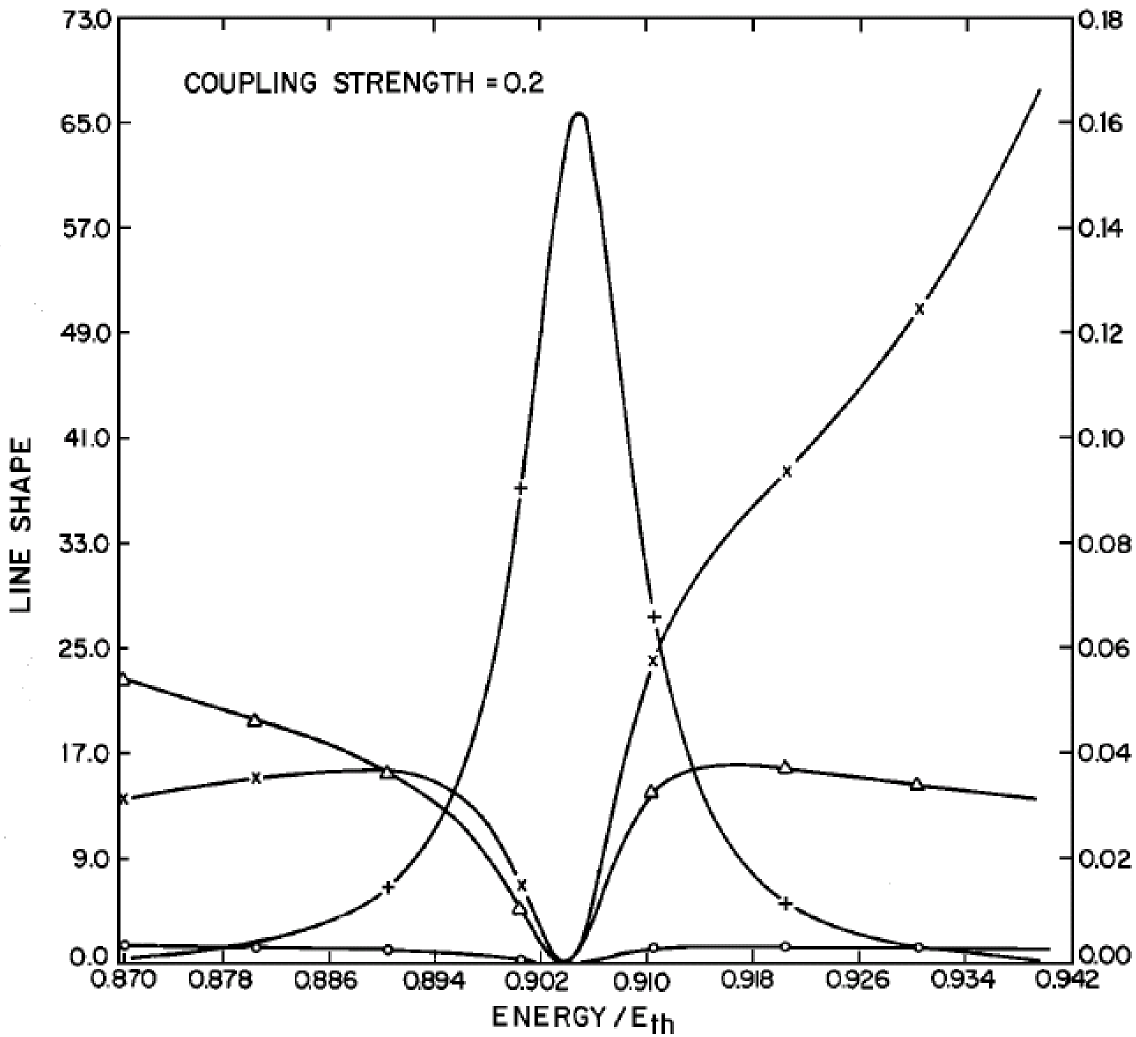
M.S., J. Chem. Phys. **56**, 2582, 1972 (received June 1970)

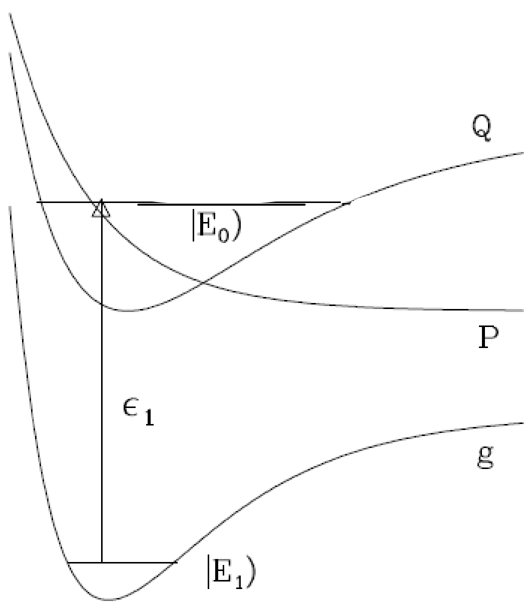
Overlapping resonances in the predissociation of the He-H_2 van der Waals complex



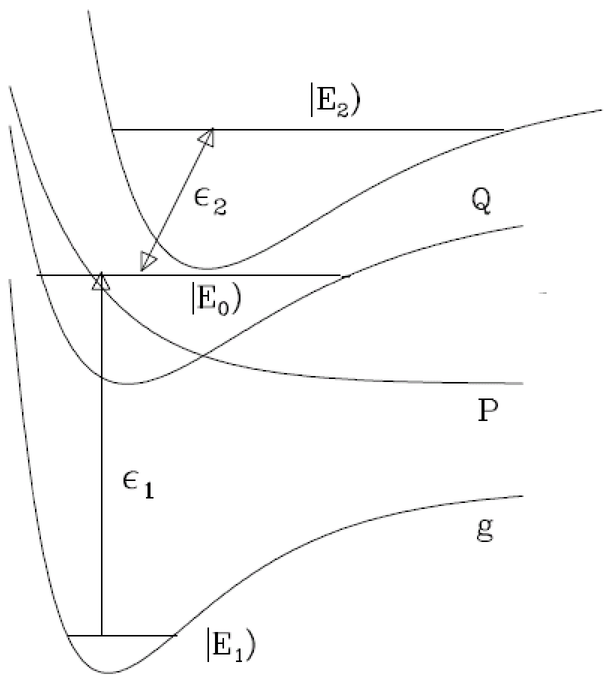
J. Chem. Phys. **56**, 2582, 1972



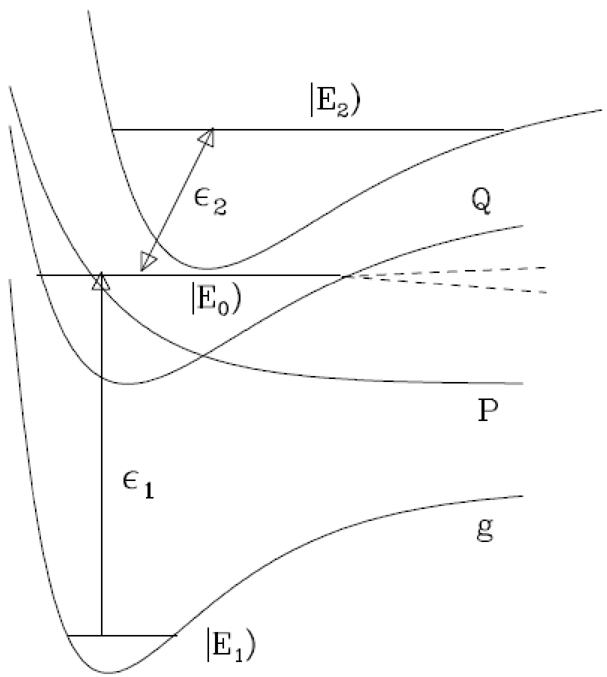

COUPLING STRENGTH = 0.2
LINE SHAPE
ENERGY / E_{th}
73.0
65.0
57.0
49.0
41.0
33.0
25.0
17.0
9.0
0.0
0.18
0.16
0.14
0.12
0.10
0.08
0.06
0.04
0.02
0.00
0.870
0.878
0.886
0.894
0.902
0.910
0.918
0.926
0.934
0.942

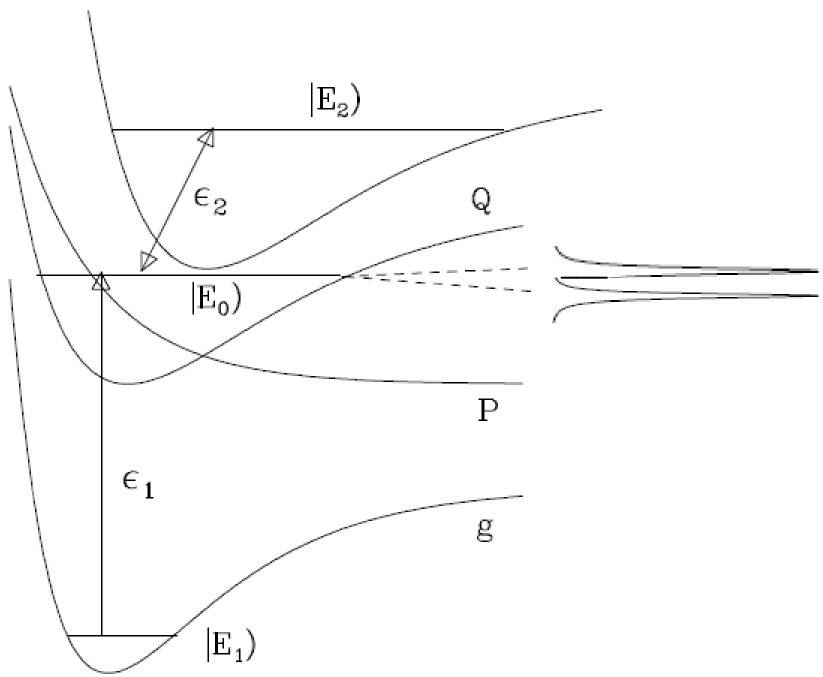


Q
$|E_0)$
P
ϵ_1
g
$|E_1)$

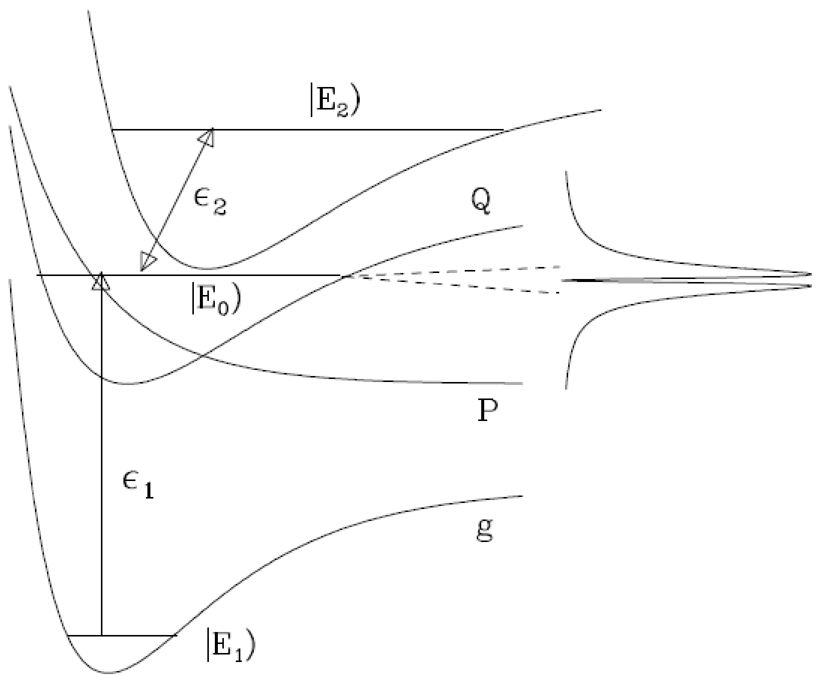


$|E_2)$
ϵ_2
Q
$|E_0)$
P
ϵ_1
g
$|E_1)$

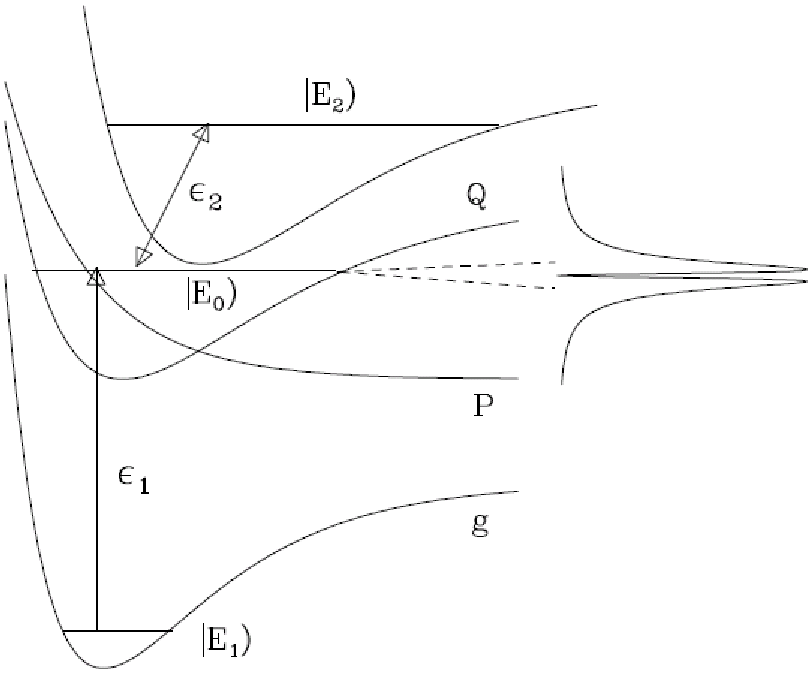

|E2)
ϵ2
Q
|E0)
P
ϵ1
g
|E1)



|E₂)
ε₂
Q
|E₀)
P
ε₁
g
|E₁)



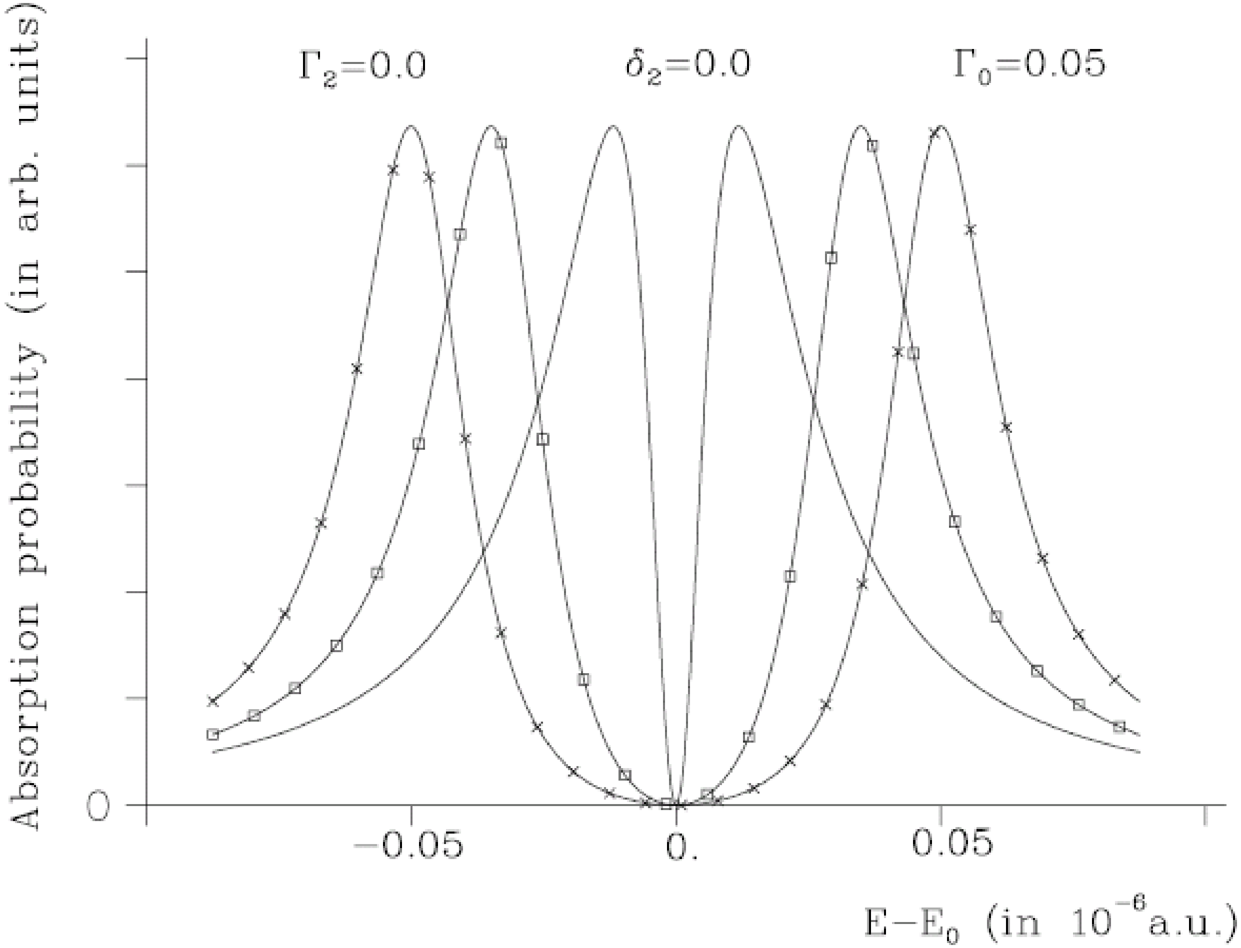
|E₂)
ε₂
Q
|E₀)
P
ε₁
g
|E₁)



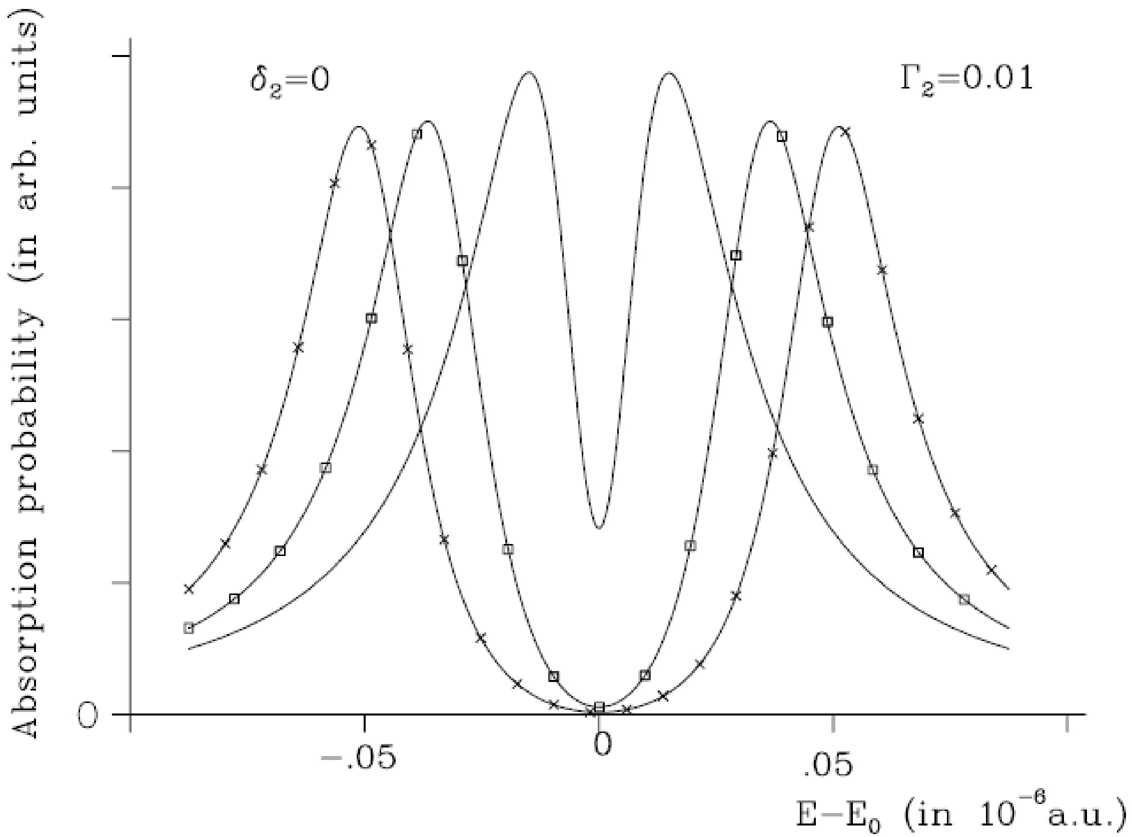
Electromagnetically Induced Transparency

S.E. Harris, Phys. Rev. Lett., 1989

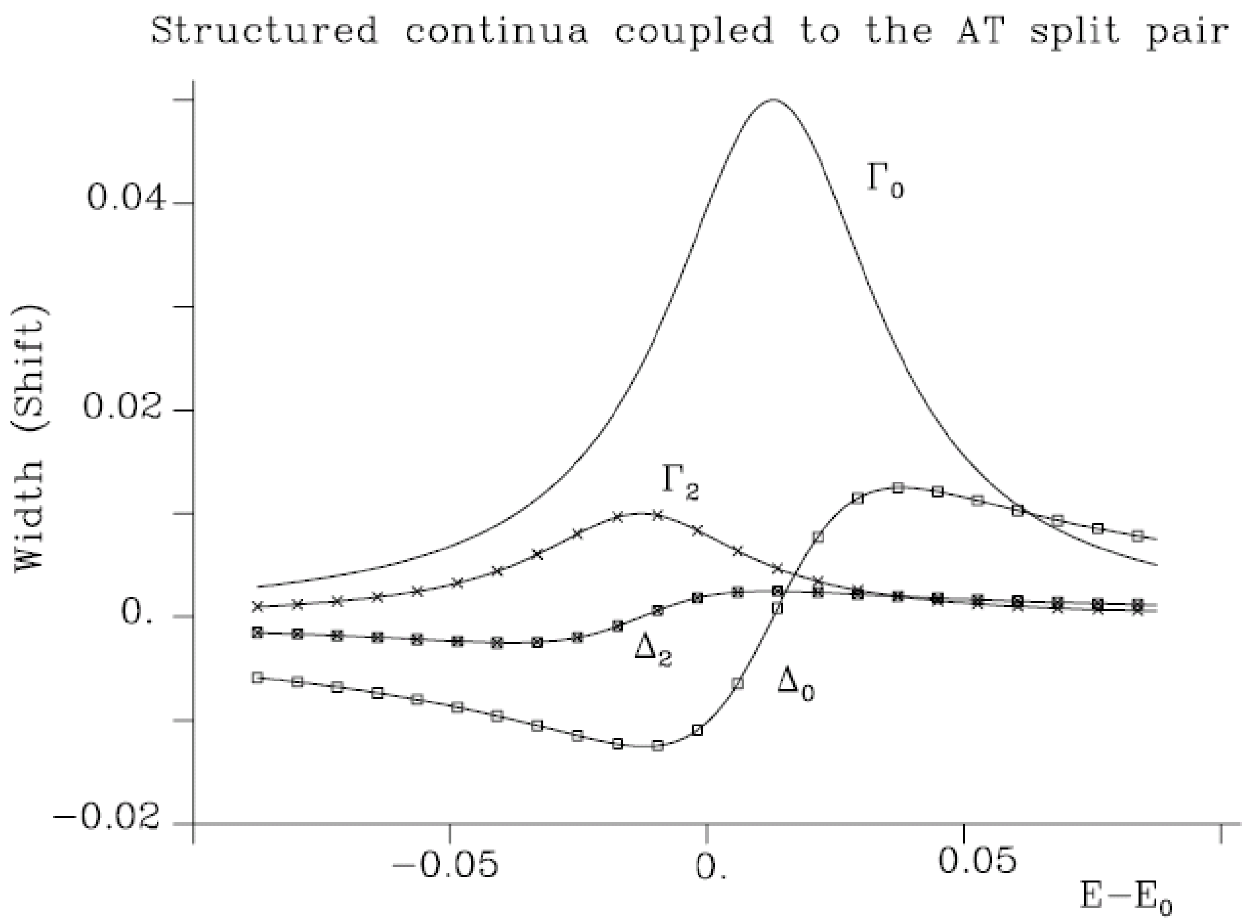
$$P_{\mathbf{n}}(E) = \frac{|2\pi\mu_{1,0}V_{0,\mathbf{n}}\epsilon_1(\omega_{E,1})|^2\left[(E-E_0)^2+\Gamma_2^2/4)\right]}{\left[(E-E_0)^2-|\Omega_2(t)|^2-\Gamma_0\Gamma_2/4\right]^2+\left[(E-E_0)(\Gamma_0+\Gamma_2)/2\right]^2}$$

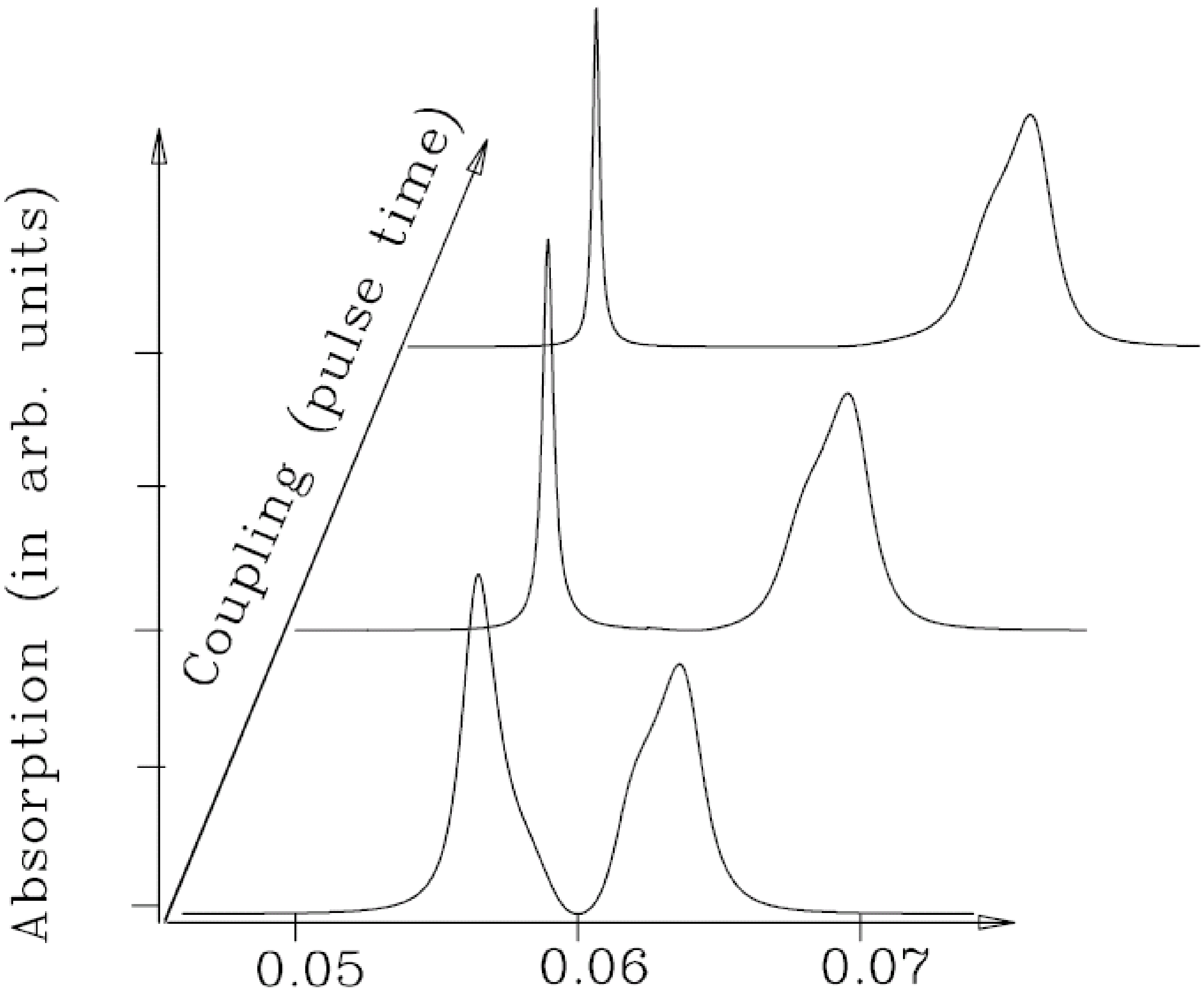


$$P_{\mathbf{n}}(E) = \frac{\left|2\pi\mu_{1,0}V_{0,\mathbf{n}}\epsilon_1(\omega_{E,1})\right|^2\left[(E-E_0)^2+\Gamma_2^2/4)\right]}{\left[(E-E_0)^2-|\Omega_2(t)|^2-\Gamma_0\Gamma_2/4\right]^2+\left[(E-E_0)\left(\Gamma_0+\Gamma_2\right)/2\right]^2}$$



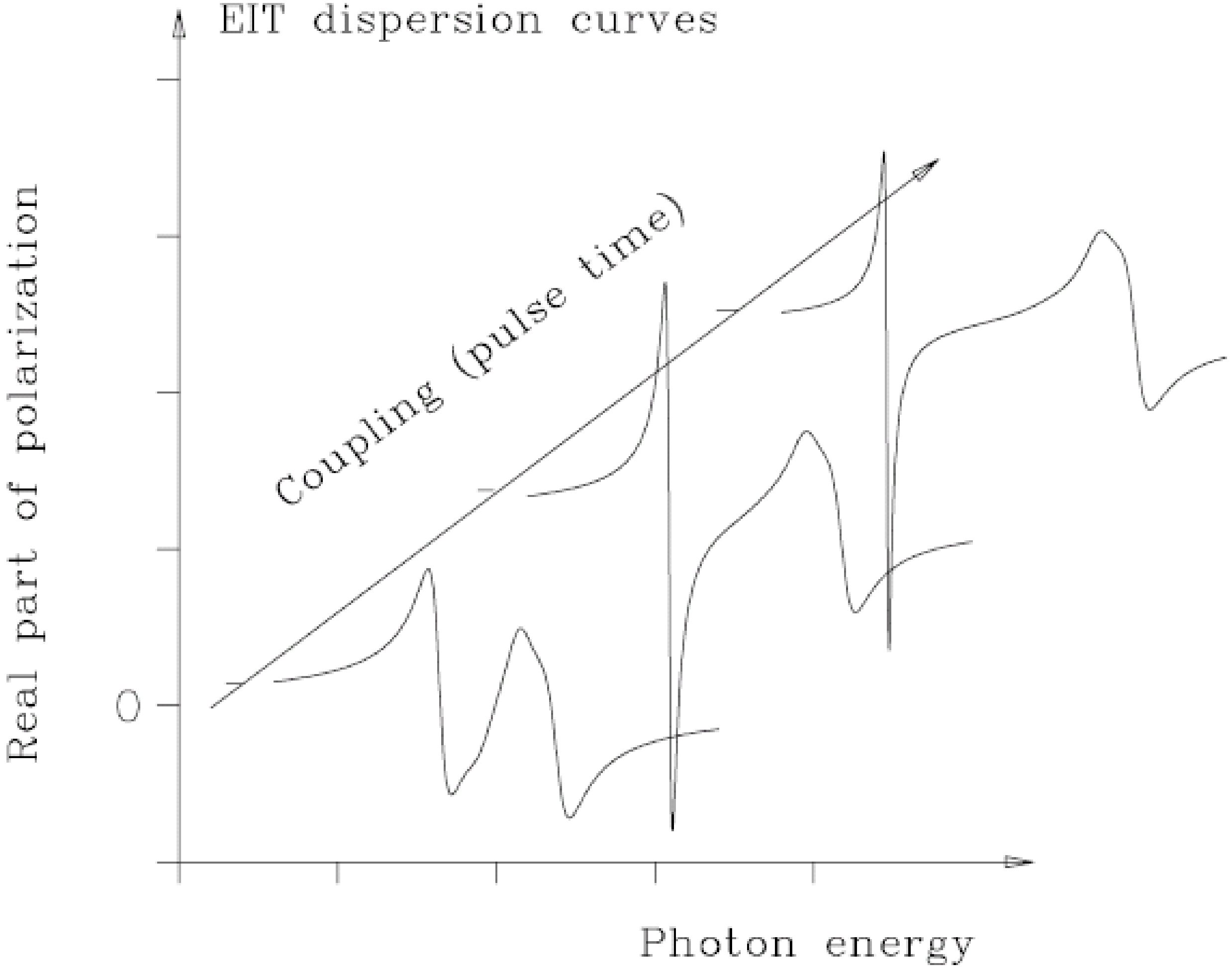
Structured continuum





Absorption (in arb. units)
Coupling (pulse time)
0.05
0.06
0.07

The phenomenon of slow light



S. Harris, L. Hau, M. Lukin

A Quantum Theory of Slow Light

Going beyond the weak field limit

$$|\,\Psi(t)\,\rangle = b_1(t)|\,E_1\,\rangle e^{-iE_1t/\hbar} + \sum_{\mathbf{n}} \int dE b_{E,\mathbf{n}}(t)|\,E, \mathbf{n}^-\,\rangle e^{-iEt/\hbar}\,.$$

We obtain

$$\frac{d}{dt}b_1(t) \;=\; (i/\hbar)\sum_n \int dE b_{E,n}(t)\mu_{1,\mathbf{n}}(E)\mathcal{E}_1^*(t)e^{-i(\omega_{E,1}-\omega_1)t}$$

$$\frac{d}{dt}b_{E,\mathbf{n}}(t) = (i/\hbar)b_1(t)\mu_{\mathbf{n},1}(E)\mathcal{E}_1(t)e^{i(\omega_{E,1}-\omega_1)t}\;.$$

Eliminate the continuum,

$$b_{E,\mathbf{n}}(t) = (i/\hbar) \int_{-\infty}^{t} dt' b_1(t') \mu_{\mathbf{n},1}(E) \mathcal{E}_1(t') e^{i\delta_E t'} \ ,$$

where $\delta_E \equiv \omega_{E,1} - \omega_1$.

Eliminate the continuum,

$$b_{E,\mathbf{n}}(t) = (i/\hbar) \int_{-\infty}^{t} dt' b_1(t') \mu_{\mathbf{n},1}(E) \mathcal{E}_1(t') e^{i\delta_E t'} \ ,$$

where $\delta_E \equiv \omega_{E,1} - \omega_1$.

We obtain

$$\frac{d}{dt} b_1(t) = (\mathcal{E}_1^*(t)/\hbar) \int_{-\infty}^{t} dt' b_1(t') F_1(t-t') \mathcal{E}_1(t') \ ,$$

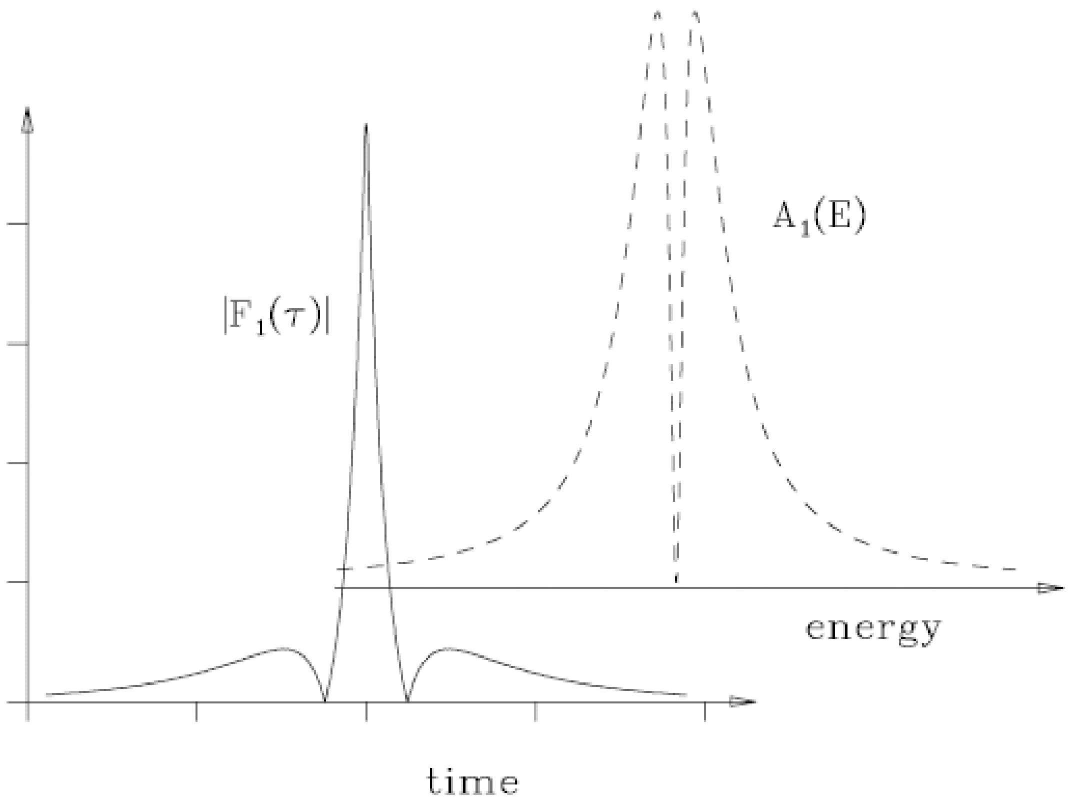
where $F_1(\tau)$, the *spectral auto-correlation* function

$$F_1(\tau) \equiv -\int d\delta_E \sum_{\mathbf{n}} |\mu_{1,\mathbf{n}}(E)|^2 \, e^{-i\delta_E \tau} \ ,$$

is just the Fourier transform of $A_1(E)$, the total absorption probability from state $|\, E_1 \,\rangle$,

$$A_1(E) \equiv \sum_{\mathbf{n}} |\mu_{1,\mathbf{n}}(E)|^2 \ .$$

The lineshape and its spectral autocorrelation function



$$\langle\, E_0 \,|\Psi(t)\rangle = -\frac{ie^{-i(\omega_1+E_1/\hbar)t}}{\mu_{1,0}} \int_{-\infty}^{t} dt' \mathcal{E}_1(t') b_1(t') F_1(t-t') \ .$$

$$\langle E_0 | \Psi(t) \rangle = -\frac{ie^{-i(\omega_1 + E_1/\hbar)t}}{\mu_{1,0}} \int_{-\infty}^{t} dt' \mathcal{E}_1(t') b_1(t') F_1(t - t') \ .$$

$$\langle E_2 | \Psi(t) \rangle = -\frac{ie^{-i(\omega_1 + E_1/\hbar)t}}{\mu_{2,0}} \int_{-\infty}^{t} dt' \mathcal{E}_1(t') b_1(t') F_{0,1}(t - t') \ ,$$

$$\langle E_0 | \Psi(t) \rangle = -\frac{ie^{-i(\omega_1 + E_1/\hbar)t}}{\mu_{1,0}} \int_{-\infty}^{t} dt' \mathcal{E}_1(t') b_1(t') F_1(t - t') \ .$$

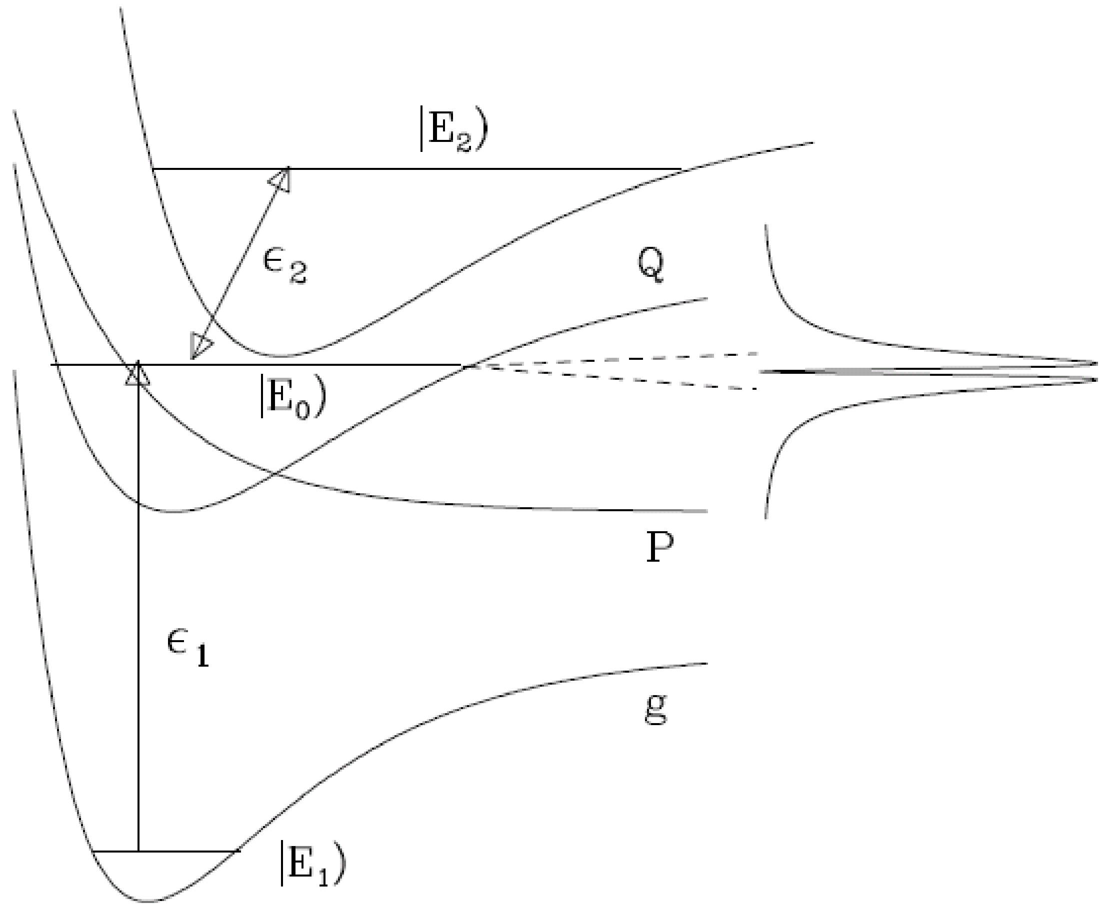
$$\langle E_2 | \Psi(t) \rangle = -\frac{ie^{-i(\omega_1 + E_1/\hbar)t}}{\mu_{2,0}} \int_{-\infty}^{t} dt' \mathcal{E}_1(t') b_1(t') F_{0,1}(t - t') \ ,$$

where $F_{0,1}(\tau)$ is the *spectral cross-correlation* function

$$F_{0,1}(\tau) \equiv -\int d\delta_E A_{0,1}(E) e^{-i\delta_E \tau} \ ,$$

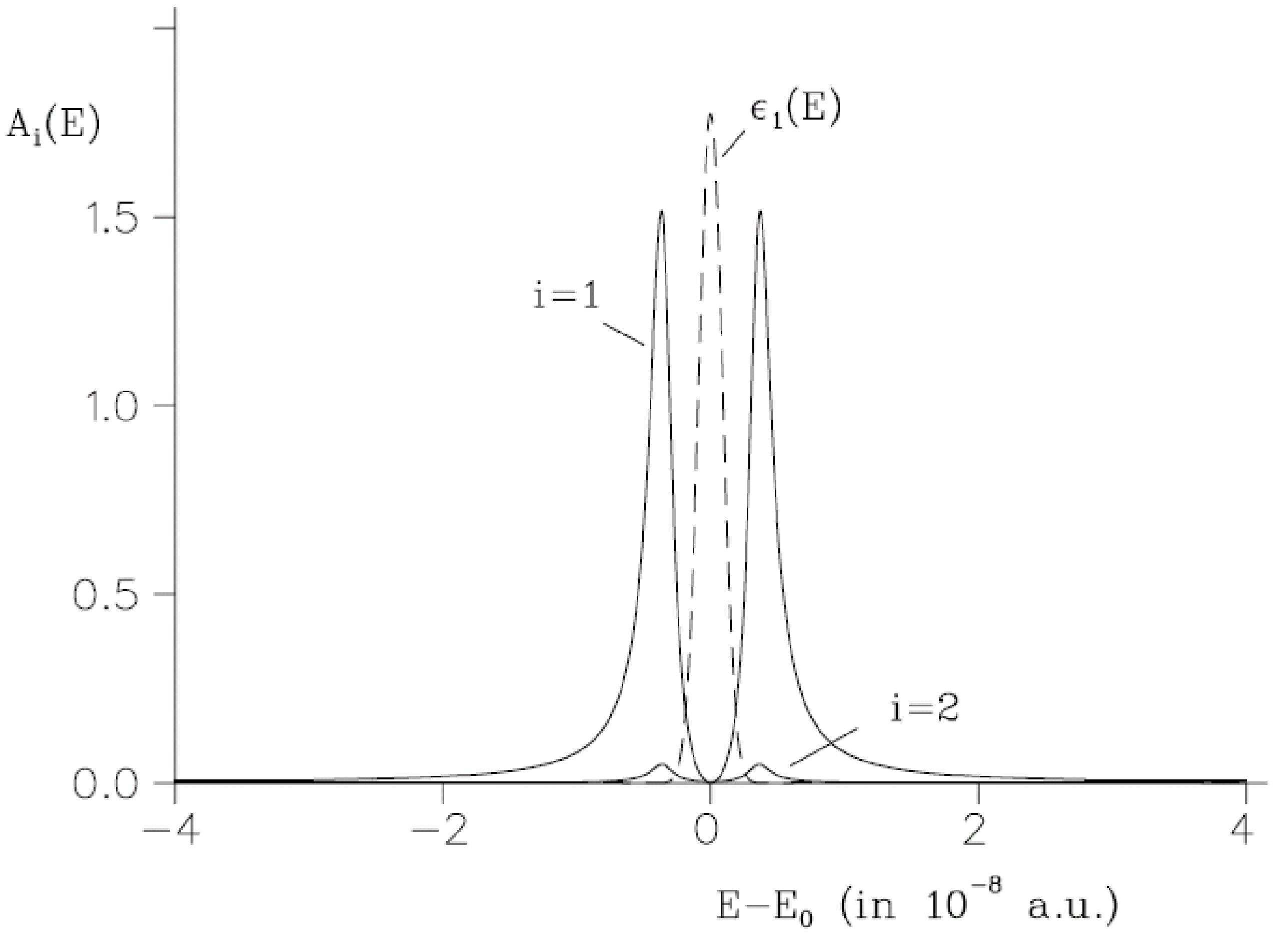
where

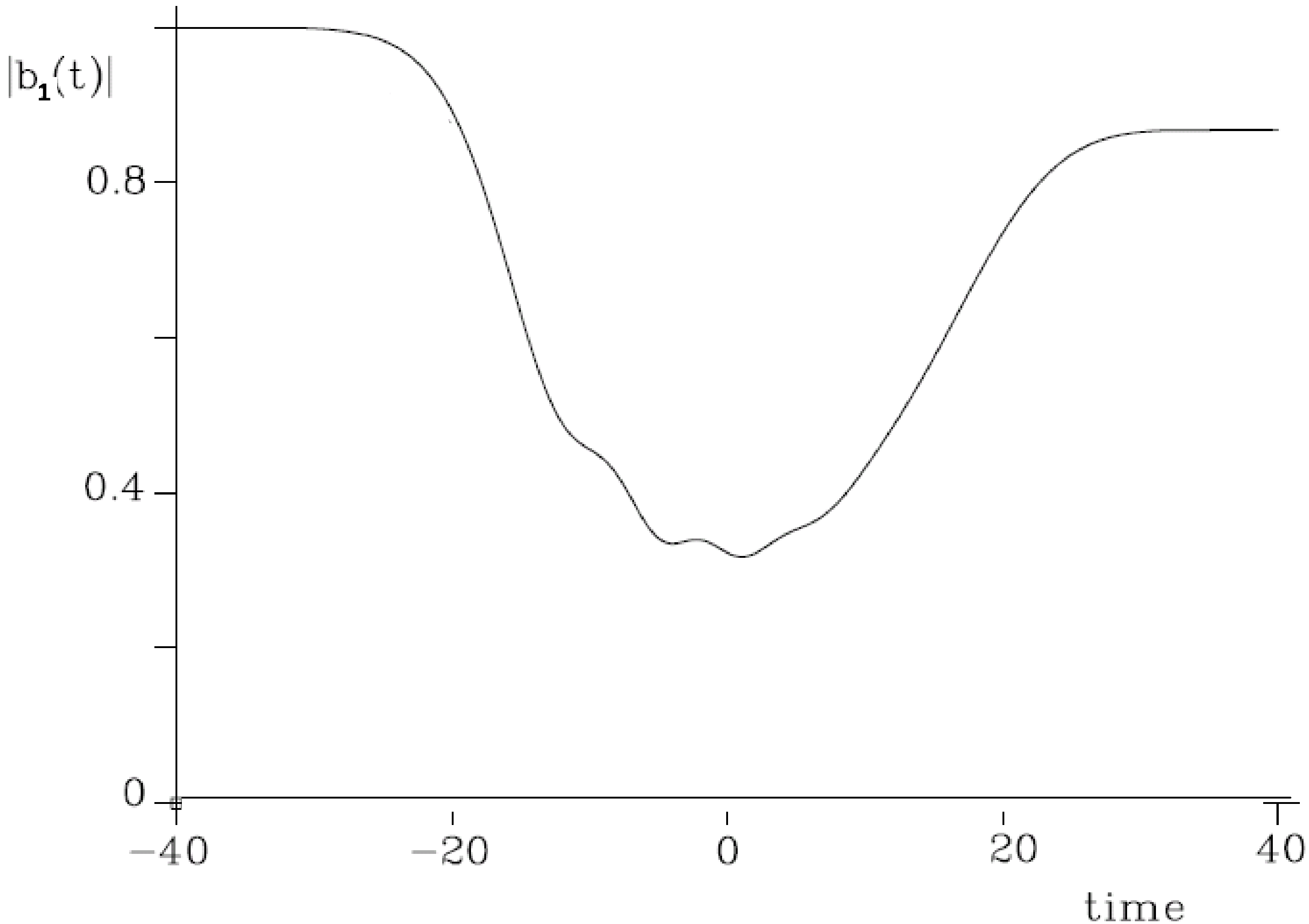
$$A_{0,1}(E) \equiv \sum_{\mathbf{n}} \mu_{0,\mathbf{n}}(E) \mu_{\mathbf{n},1}(E).$$



Electromagnetically Induced Transparency

Overlap of the probe pulse with the EIT absorption curve





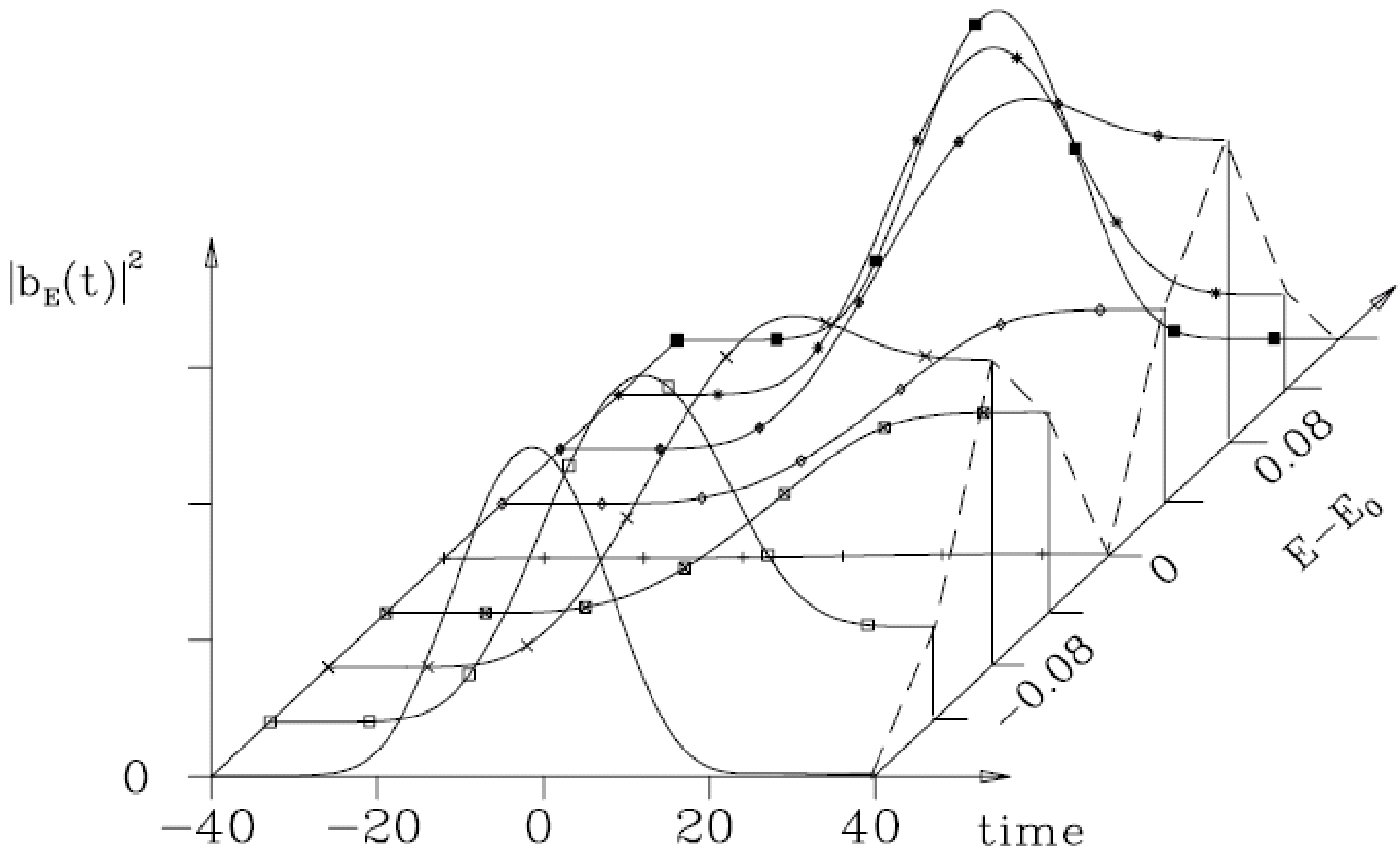
|b_1(t)|
0.8
0.4
0
−40
−20
0
20
40
time

$$b_{E,\mathbf{n}}(t) = (i/\hbar) \int_{-\infty}^{t} dt' b_1(t') \mu_{\mathbf{n},1}(E) \mathcal{E}_1(t') e^{i\delta_E t'} \ ,$$

where $\delta_E \equiv \omega_{E,1} - \omega_1$.

$$b_{E,\mathbf{n}}(t) = (i/\hbar) \int_{-\infty}^{t} dt' b_1(t') \mu_{\mathbf{n},1}(E) \mathcal{E}_1(t') e^{i\delta_E t'} \ ,$$

where $\delta_E \equiv \omega_{E,1} - \omega_1$.



$$\langle\, E_0 \,|\Psi(t)\rangle = -\frac{i e^{-i(\omega_1 + E_1/\hbar)t}}{\mu_{1,0}} \int_{-\infty}^{t} dt' \mathcal{E}_1(t') b_1(t') F_1(t-t')\ .$$

$$\langle E_0 | \Psi(t) \rangle = -\frac{i e^{-i(\omega_1 + E_1/\hbar)t}}{\mu_{1,0}} \int_{-\infty}^{t} dt' \mathcal{E}_1(t') b_1(t') F_1(t - t') \ .$$

$$\langle E_2 | \Psi(t) \rangle = -\frac{i e^{-i(\omega_1 + E_1/\hbar)t}}{\mu_{2,0}} \int_{-\infty}^{t} dt' \mathcal{E}_1(t') b_1(t') F_{0,1}(t - t') \ ,$$

$$\langle E_0 | \Psi(t) \rangle = -\frac{ie^{-i(\omega_1 + E_1/\hbar)t}}{\mu_{1,0}} \int_{-\infty}^{t} dt' \mathcal{E}_1(t') b_1(t') F_1(t - t') \; .$$

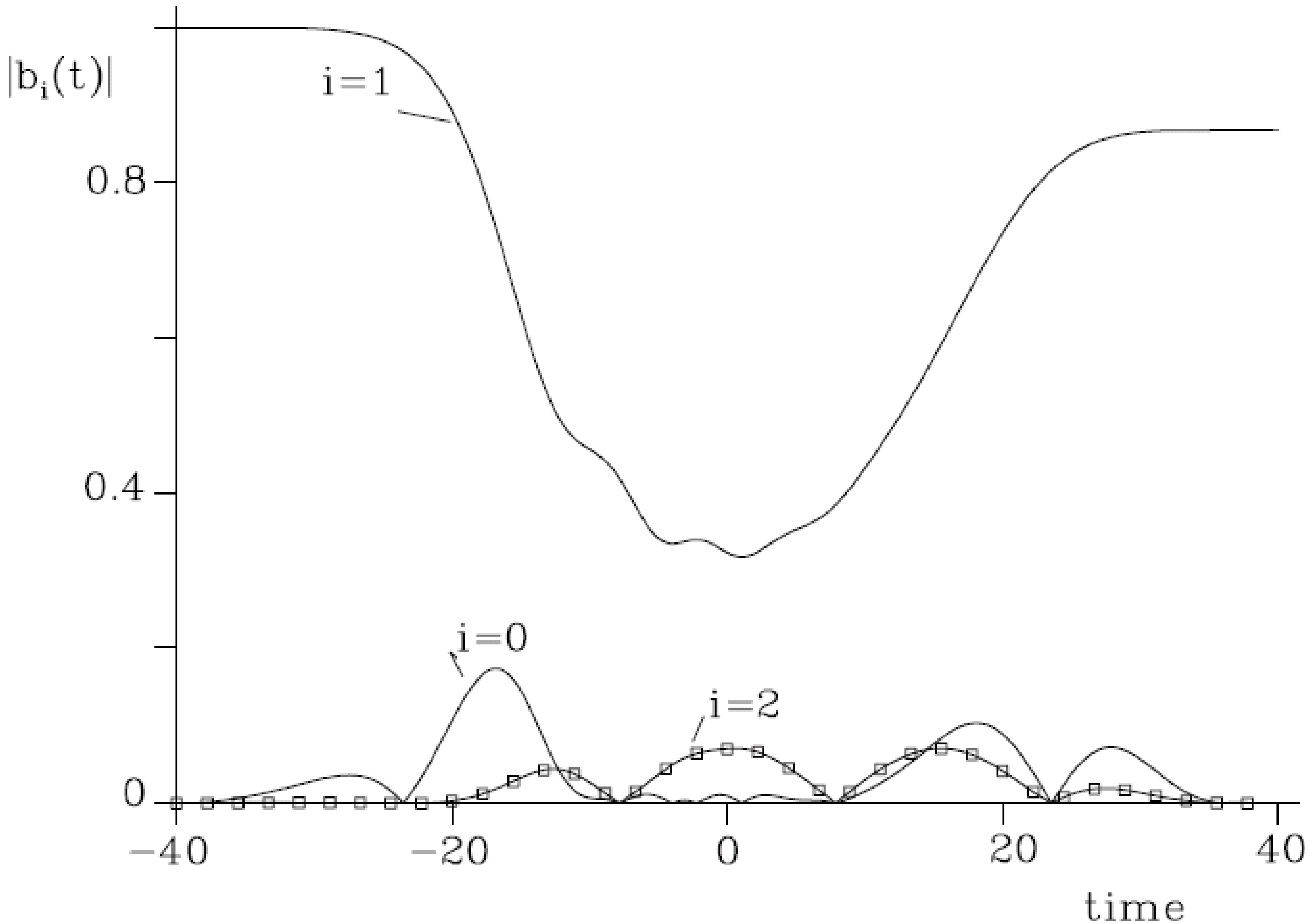
$$\langle E_2 | \Psi(t) \rangle = -\frac{ie^{-i(\omega_1 + E_1/\hbar)t}}{\mu_{2,0}} \int_{-\infty}^{t} dt' \mathcal{E}_1(t') b_1(t') F_{0,1}(t - t') \; ,$$

where $F_{0,1}(\tau)$ is the *spectral cross-correlation* function

$$F_{0,1}(\tau) \equiv -\int d\delta_E A_{0,1}(E) e^{-i\delta_E \tau} \; ,$$

where

$$A_{0,1}(E) \equiv \sum_{\mathbf{n}} \mu_{0,\mathbf{n}}(E) \mu_{\mathbf{n},1}(E).$$



$|b_i(t)|$
i=1
0.8
0.4
0
i=0
i=2
−40
−20
0
20
40
time

Calculating $\langle\, E', \mathbf{m}_1^- \,|\Psi(t)\rangle$

Calculating $\langle E', \mathbf{m}_1^- | \Psi(t) \rangle$

↑

Bare photon states

Calculating $\langle E', \mathbf{m}_1^- | \Psi(t) \rangle$

↑
Bare photon states

$$|\Psi(t)\rangle = \sum_{\mathbf{n}} \int dE b_{E,\mathbf{n}}(t) \quad |E, \mathbf{n}^-\rangle e^{-iEt/\hbar} \; .$$

Calculating $\langle E', \mathbf{m}_1^- | \Psi(t) \rangle$

↑ Bare photon states

Fully dressed photon states ↓

$$|\Psi(t)\rangle = \sum_{\mathbf{n}} \int dE b_{E,\mathbf{n}}(t) \qquad |E, \mathbf{n}^-\rangle e^{-iEt/\hbar} \; .$$

Calculating $\langle E', \mathbf{m}_1^- | \Psi(t) \rangle$

↑ Bare photon states

Fully dressed photon states ↓

$$\langle E', \mathbf{m}_1^- | \Psi(t) \rangle = \sum_{\mathbf{n}} \int dE b_{E,\mathbf{n}}(t) \langle E', \mathbf{m}_1^- | E, \mathbf{n}^- \rangle e^{-iEt/\hbar} \; .$$

Calculating $\langle\, E', \mathbf{m}_1^- \,|\Psi(t)\rangle$

Bare photon states

Fully dressed photon states

$$\langle\, E', \mathbf{m}_1^- \,|\Psi(t)\rangle = \sum_{\mathbf{n}} \int dE b_{E,\mathbf{n}}(t) \langle\, E', \mathbf{m}_1^- \,|E, \mathbf{n}^-\rangle e^{-iEt/\hbar} \;.$$

After some manipulation we get that

$$\langle E', \mathbf{m}_1^- | \Psi(t) \rangle = i e^{-i\omega_1 t}.$$

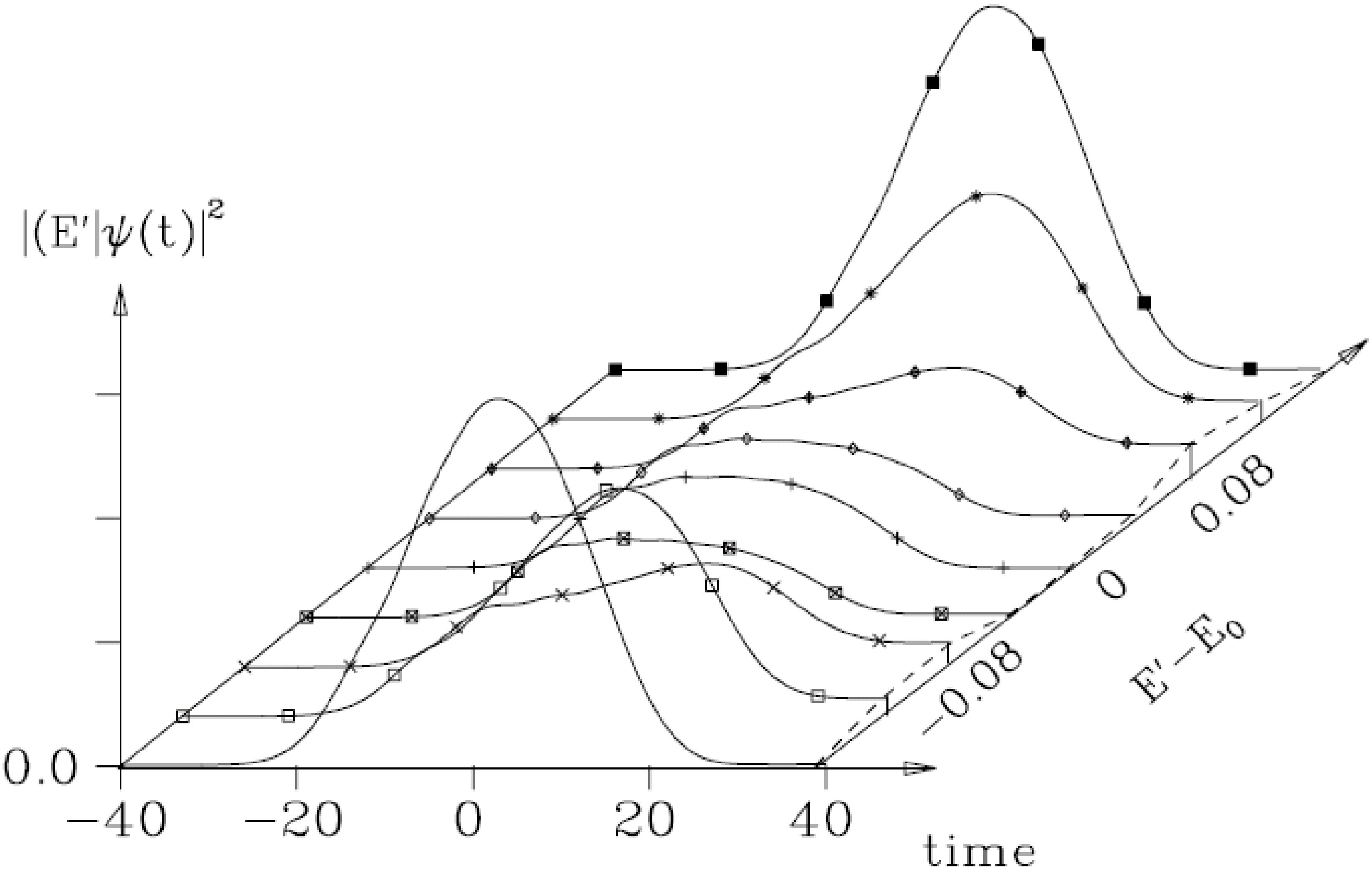
$$\left\{ \frac{V_{\mathbf{m},0}(E')}{\mu_{1,0}} \int_{-\infty}^{t} dt' b_1(t') \mathcal{E}_1(t') G_1(E', t-t') + \right.$$

$$\left. \frac{V_{\mathbf{m},2}(E')}{\mu_{0,2}} \int_{-\infty}^{t} dt' b_1(t') \mathcal{E}_1(t') G_{1,0}(E', t-t') \right\} .$$

where

$$G_1(E', \tau) \equiv \int d\delta_E e^{-i\delta_E \tau} \frac{A_1(E)}{E - i\epsilon - E'}$$

$$G_{0,1}(E', \tau) \equiv \int d\delta_E e^{-i\delta_E \tau} \frac{A_{0,1}(E)}{E - i\epsilon - E'} .$$



Lasing without inversion! (Kochorovskaya, Scully, Harris)