Lecture 2

The physics of Bose Einstein condensation

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Goal of this lecture:

understand the physics of Bose Einstein condensation for trapped dilute gases, and its properties.

Outline

1. De Broglie wavelength
2. Bose Einstein condensation: historical perspective
3. Evaporative cooling
4. BEC characterization
5. Hydrodynamic formalism and consequences
6. Superfluidity
7. Atom lasers

Two length scales in a perfect gas

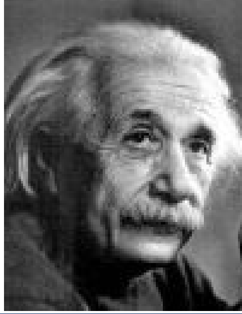
- Distance between particles *d*

- The « mean size » of an individual wave packet is given by the de Broglie wavelength

$$\lambda_{dB} = \frac{h}{\sqrt{2\pi m k_B T}}$$

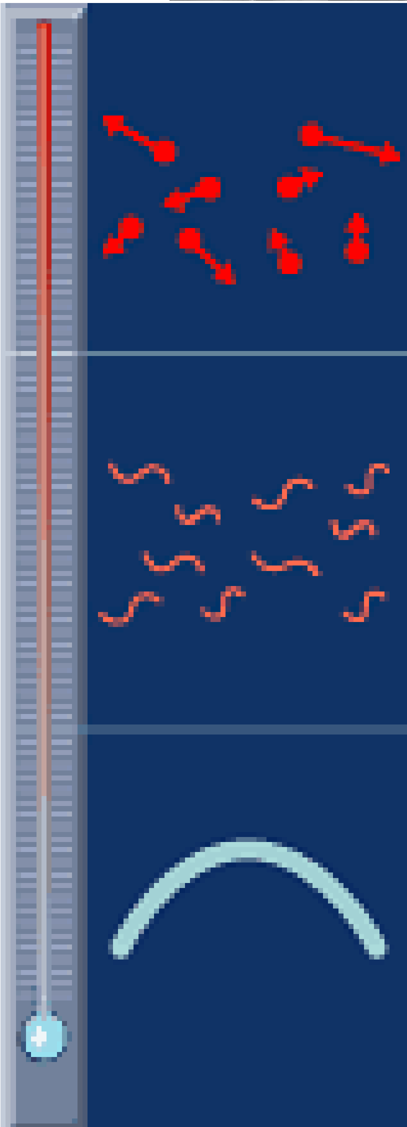
At room temperature : λ **<<** *d* i.e. particle-like behavior.

New phase at low temperature



$$\lambda_{dB} = \frac{h}{\sqrt{2\pi m k_B T}}$$

If T decreases,
λ increases



Einstein (1924) predicts a phase transition:

condensation de Bose-Einstein when $\lambda \sim d$

$\rho = n\lambda^3$ phase space density

$\rho \ll 1$ Classical behavior

$\rho \geq 1$ Collective quantum behavior

Bose-Einstein condensation (BEC)

1938: London relates the superfluidity of ^{4}He to BEC (Tc et Cv)

… in the course of time the degeneracy of Bose-Einstein gas has rather got the reputation of having only a purely imaginary existence …it seems difficult not to imagine a connexion with the condensation phenomenon of Bose-Einstein statistics...

But it is a liquid, and not an ideal gas: condensed fraction < 10 %

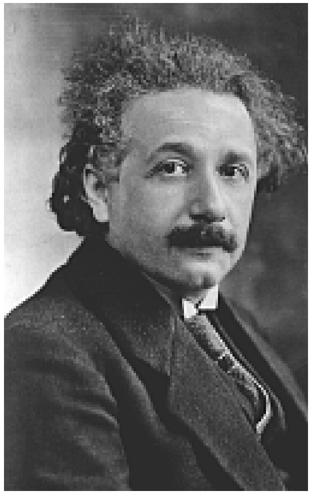
BEC plays a crucial role in a wide variety of physics domains:

Supraconductivity: linked to the bosonic properties of Cooper pairs

Exciton gases in certain types of semi-conductors

Nuclear physics, astrophysics *(neutron stars)* …

Why this is a so fascinating field ?



- Realization of the idea of one of the « father » of modern physics

- Collective quantum behavior at a mesoscopic or even macroscopic scale !

Neutron stars: 10^{39} neutrons/cm^3

Liquid helium: 10^{22} atomes/cm^3

Diluted alkali gases: 10^{14} atoms/cm^3

- Towards the realization of continuous atom laser

Renewal of interest for the realization of BEC with atomic vapors in the 1970's

W. Stwalley and L. Nosanow (P.R.L. **36**, 910, 1976) suggested that polarized hydrogen must remain a gas at any temperature and that the threshold for Bose-Einstein condensation could be reached by cryogenic methods.

Scientific interest of such a system

- The atom-atom interactions are weak in a gas.
  They are well known, especially for H, and their effect can be accurately described.
- The superfluidity was not yet observed in a gas.
- Possible applications (Hydrogen maser)

This paper stimulated several theoretical and experimental studies on polarized Hydrogen.
Groups of D. Kleppner and T. Greytak at MIT, of I. Silvera and J. Walraven in Amsterdan, of Y. Kagan in Moscow…

Laser cooling ➡ Evaporative cooling

Magneto-optical trap: From room temperature to 100 μK

Molasses 100 μK ⟶ 10 μK $n\lambda^3 = 10^{-7}$

Intrisically limited because of the dissipative character of the MOT

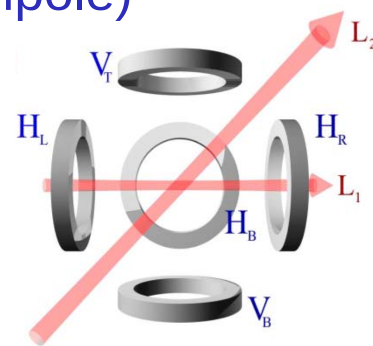
Transfer atoms in a non-dissipative trap

Magnetic trap
(permanent magnetic dipole for polarized atoms)

$$W = -\vec{\mu}.\vec{B}$$

Dipole trap: Off-resonance laser
(induced electric dipole)

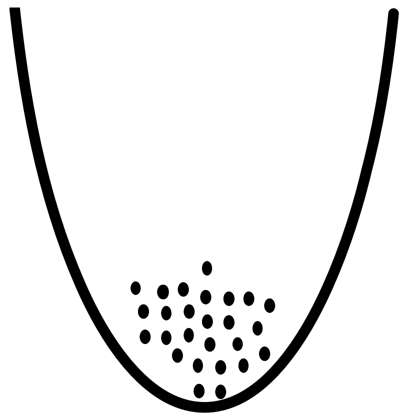
$$W = -\frac{1}{2}\vec{p}.\vec{E}$$



Evaporative cooling: get rid of the most energetical atoms that results from the elastic collisions between atoms. The phase space density of the remaining atoms (open system) can increase.

Evaporation: simplified model (1)

1) Infinite depth



harmonic confinement

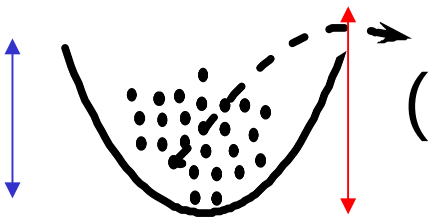
N

$$E = N\left(\frac{3}{2}+\frac{3}{2}\right)k_B T$$

2) Finite depth

$\eta \sim 6$

$\eta k_B T$



$(\eta+\kappa)k_B T$

$\kappa \sim 1$

3) Infinite depth

$N - dN$

$$(N-dN)\left(\frac{3}{2}+\frac{3}{2}\right)k_B T - dE = (N-dN)\left(\frac{3}{2}+\frac{3}{2}\right)k_B (T-dT)$$

$$dE = dN\left[\eta + \kappa - \left(\frac{3}{2}+\frac{3}{2}\right)\right]k_B T$$

Evaporation: simplified model (2)

We deduce a power law dependence

$$\frac{dT}{T} = \alpha \frac{dN}{N}$$

with

$$\alpha = \frac{\eta + \kappa}{3} - 1 > 0$$

The phase space density changes according to

$$\rho \sim n\lambda^3 \sim \frac{N}{(\Delta r)^3} \frac{1}{T^{3/2}} \sim \frac{N}{T^3} \sim N^{1-3\alpha}$$

since

$$\lambda = \frac{h}{\sqrt{2\pi m k_B T}}$$

and

$$\frac{1}{2} m\omega^2 (\Delta r)^2 = \frac{1}{2} k_B T$$

Increase of phase space density when N decreases if

$$1 - 3\alpha < 0$$

Typical numbers

N: 10^9 → 10^6

T: 100 μK → 100 nK

$n\lambda^3$ **x 10^6**

Evaporation kinetics

We have presented a discrete model which does not contain kinetics information.

To evaluate the kinetics, we work out the scalings for the elastic collision rate

$$\gamma \sim n\sigma\bar{v} \sim \frac{N}{(\Delta r)^3} T^{1/2} \sim \frac{N}{T} \sim N^{1-\alpha}$$

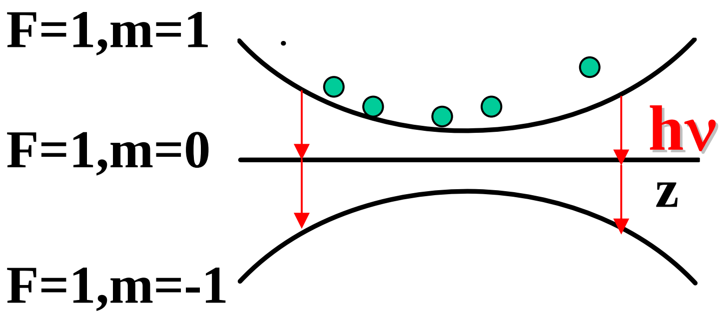
Increase of the collision rate when N decreases if $1-\alpha < 0$

If this condition is fulfilled, the kinetics accelerate during the evaporation
Thi regime is referred to as the RUNAWAY regime of forced evaporation.

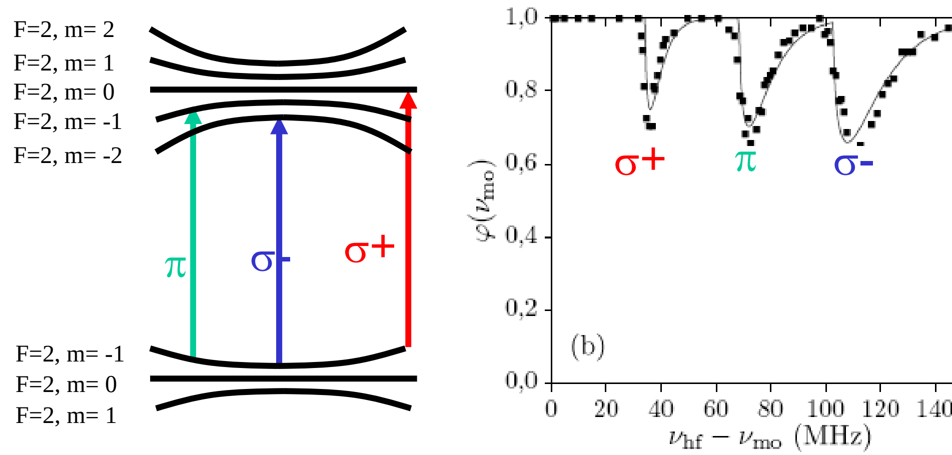
The exponents derive here depends on the dimensionality of the problem
A rigourous treatment can be performed through the Boltzmann equation
or numerically.

Different implementations of evaporative cooling

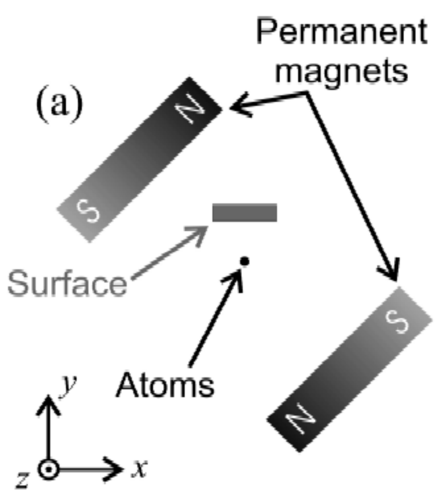
Radio-frequency evaporation



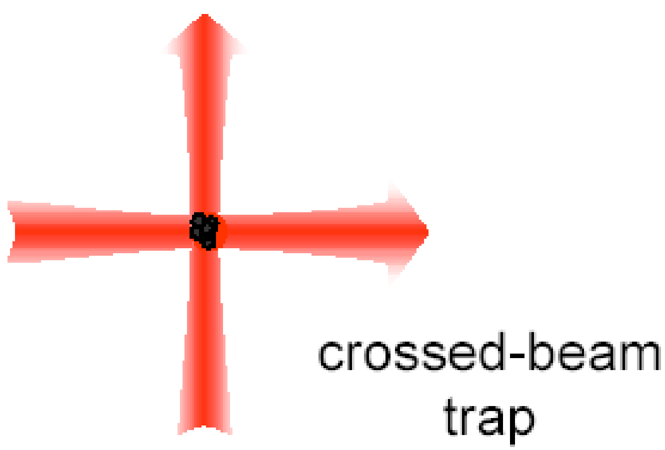
Microwave evaporation



Surface evaporation



Lowering beam intensities

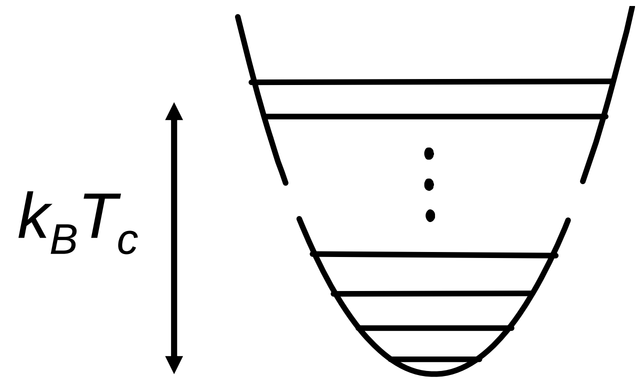


BEC in a harmonic trap

The number of atoms *N'* in the excited states is bounded:

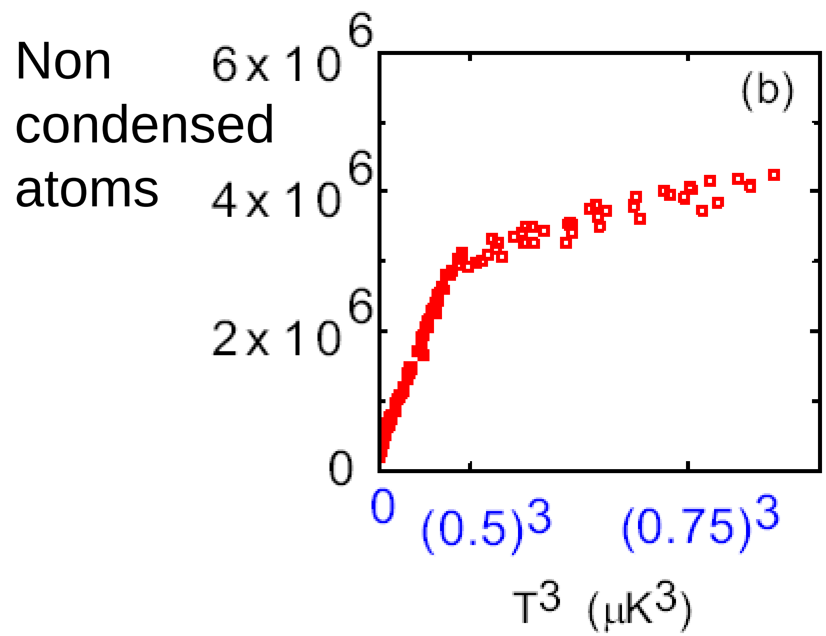
Ideal gas:

$$T_c^0 = \frac{\hbar\omega}{k_B}\left(\frac{N}{1.202}\right)^{1/3} \gg \frac{\hbar\omega}{k_B}$$



$$T < T_c \quad : \quad N = N_0 + N'$$

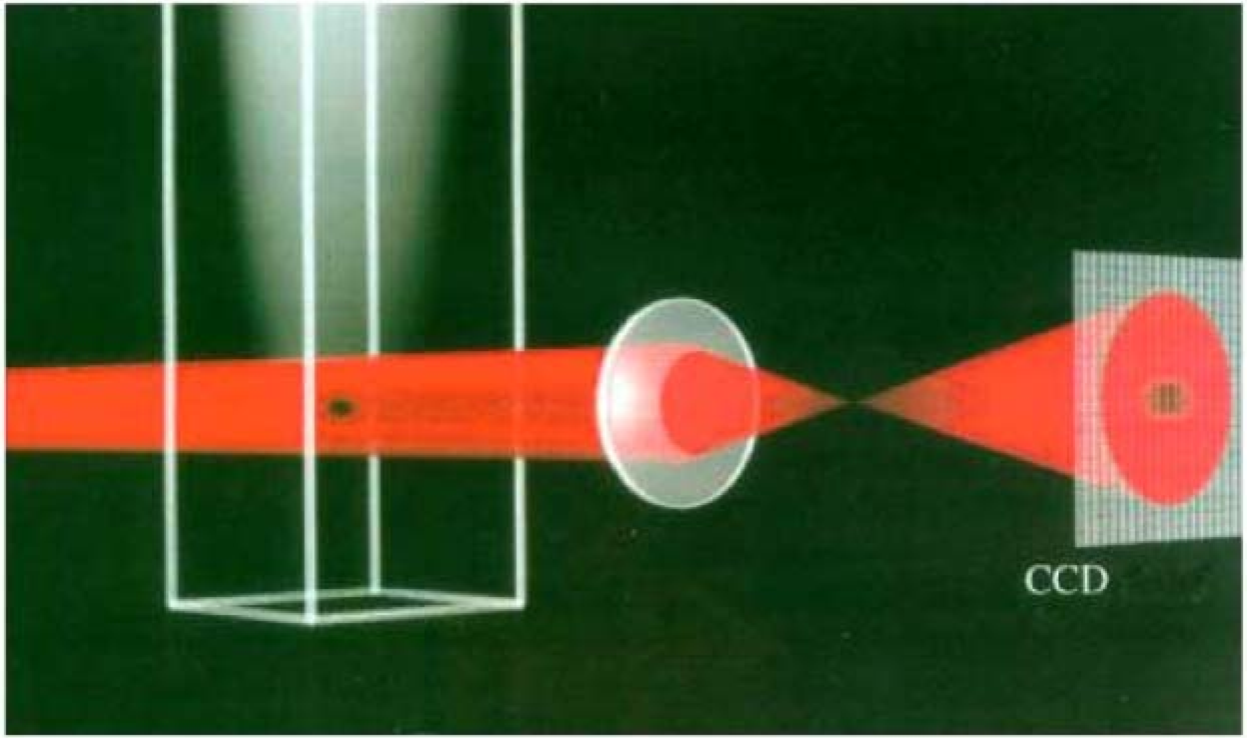
$$N' \leq 1.202\left(\frac{k_B T}{\hbar\omega}\right)^3$$



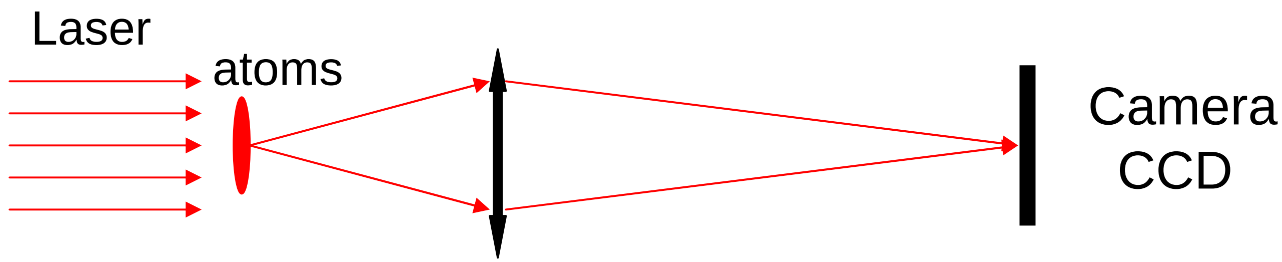
Observation of the atomic cloud

Absorption imaging (destructive)

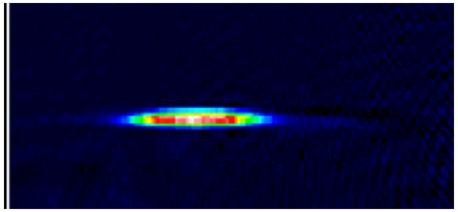
The image can be taken in situ (position measurement) or after time-of-flight (velocity measurement)



Time of flight expansion



3 10[6] atoms in an anisotropic magnetic trap

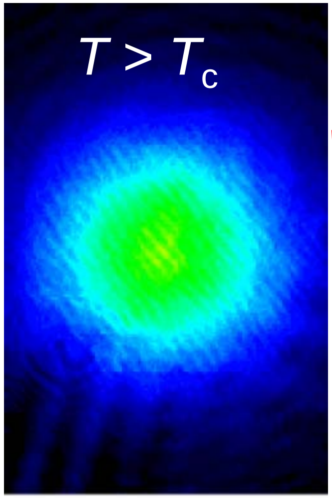


Time of flight

100 μm * 5 μm
0,5 to 1 mK

Boltzmann gas

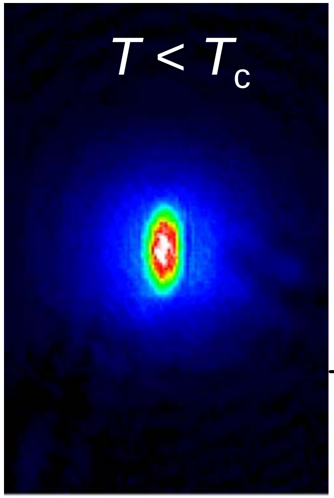
$$\frac{1}{2}mv_z^2 = \frac{1}{2}k_B T$$



isotropic expansion

condensate

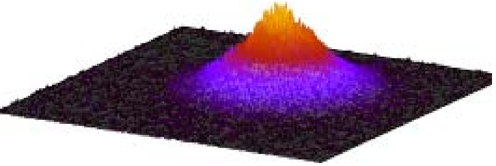
$$\frac{1}{2}mv_i^2 = \frac{1}{4}\hbar\omega_i$$

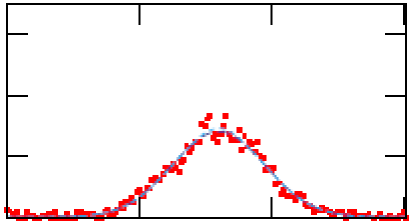


anisotropic expansion

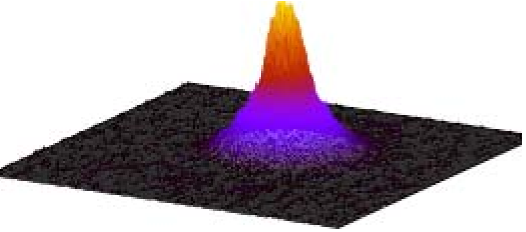
Bimodal structure of the spatial distribution

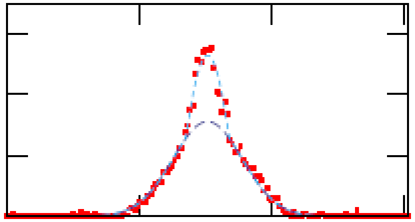
320 nK



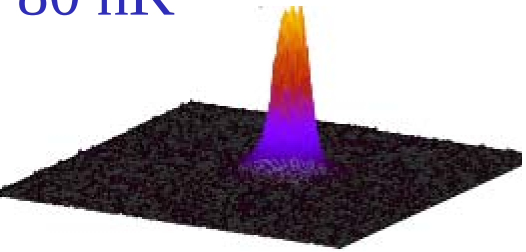


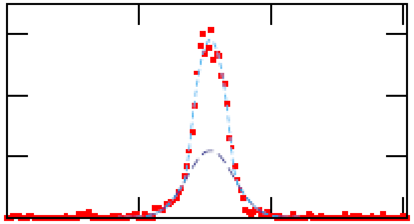
180 nK





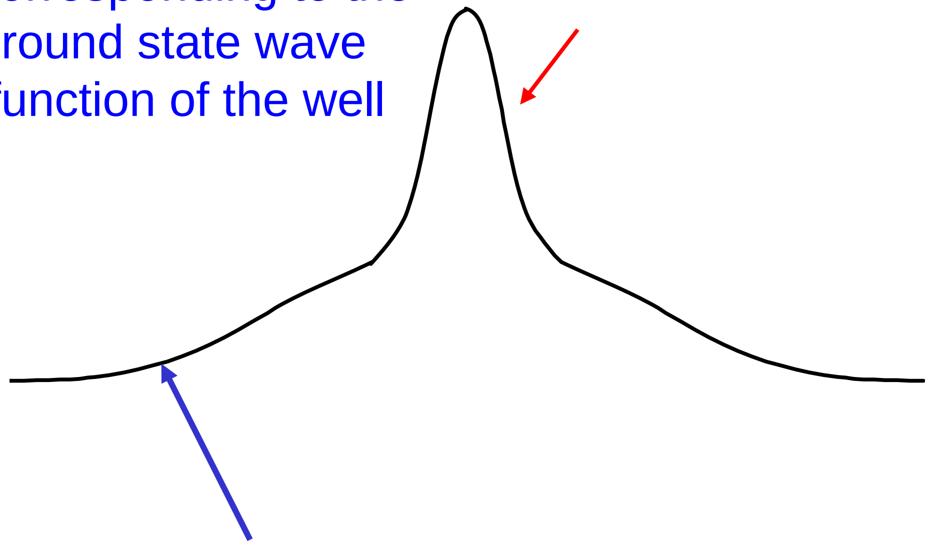
80 nK





Narrow peak with a width corresponding to the ground state wave function of the well

Contribution of the condensed atoms



Contribution of the non condensed atoms

Broad pedestal coming from atoms occupying excited states of the well described by wave functions with a larger width

Bose Einstein condensation

2001 Physics Nobel Prize

E. Cornell, W. Ketterle and C. Wieman

"for the achievement of Bose-Einstein condensation **in dilute gases of alkali atoms**, and for early fundamental studies of the properties of the condensates"

The different elements that have been Bose condensed

[1] ^{87}Rb: M. H. Anderson *et al.*, Science **269**, 198 (1995)

[2] ^{23}Na: K. B. Davis *et al.*, Phys. Rev. Lett. **75**, 3969 (1995)

[3] ^{7}Li: C. C. Bradley *et al.*, Phys. Rev. Lett. **78**, 985 (1997)

[4] ^{1}H: D. G. Fried *et al.*, Phys. Rev. Lett. **81**, 3811 (1998)

[5] ^{85}Rb: S. L. Cornish *et al.*, Phys. Rev. Lett. **85**, 1795 (2000)

[6] ^{4}He: A. Robert *et al.*, Science **292**, 461 (2001), F. Pereira Dos Santos, *et al.*, Phys. Rev. Lett. **86**, 3459 (2001)

[7] ^{41}K: G. Modugno *et al.*, Science **294**, 1320 (2001)

[8] ^{133}Cs: T. Weber, *et al.*, Science **299**, 232 (2002)

[9] ^{174}Yb: Y. Takasu, *et al.*, Phys. Rev.Lett. **91**, 040404 (2003)

Books:

- C. Pethick, H. Smith, Bose Einstein condensation in dilute Bose gases, Cambridge University Press, 2002.

-L. Pitaevskii, S. Stringari, Bose Einstein condensation, Clarendon Press, Oxford, 2003.

-Bose-Einstein Condensation in Atomic Gases: Proceedings of the International School of Physics (Enrico Fermi) Course Cxl : Varenna on Lake Como Villa Monastero July 1998. (M. Inguscio, S. Stringari, C. E. Wieman), Ed Ios Pr Inc.

Theoretical description of an ideal BEC

The Hamiltonian: $H = \sum_{i=1}^{N} \frac{p_i^2}{2m} + V(\vec{r}_i)$ with

$$V(\vec{r}_i) = \frac{1}{2} m(\omega_x^2 r_{i,x}^2 + \omega_y^2 r_{i,y}^2 + \omega_z^2 r_{i,z}^2)$$

➡ At zero temperature, all atoms are in the ground state

$$\left|\Psi(1,2,\ldots,N)\right\rangle = \left|\varphi_0(1)\right\rangle \otimes \left|\varphi_0(2)\right\rangle \otimes \ldots \otimes \left|\varphi_0(N)\right\rangle$$

$$\Psi(\vec{r}_1,\vec{r}_2,\ldots,\vec{r}_N) = \varphi_0(\vec{r}_1)\varphi_0(\vec{r}_2)\ldots\varphi_0(\vec{r}_N)$$

with $\left\langle \vec{r} \middle| \varphi_0 \right\rangle = \varphi_0(\vec{r}) = \left(\frac{1}{\pi\sigma^2}\right)^{1/4} e^{-r^2/2\sigma^2}$ and $\sigma = \sqrt{\frac{\hbar}{m\omega}}$

Oscillator length

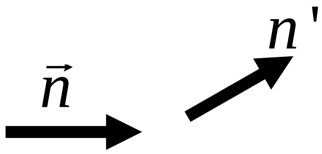
Interactions between cold atoms

Two-body problem:

$$\frac{p_1^2}{2m}+\frac{p_2^2}{2m}+W\left(\vec{r}_1-\vec{r}_2\right) \quad \Longrightarrow \quad H=\frac{p^2}{2\mu}+W\left(\vec{r}\right)$$

Scattering state (eigenstate of *H* with a positive energy)

$$\psi_{\vec{k}}(\vec{r}) \sim e^{i\vec{k}\cdot\vec{r}} + f(k,\vec{n},\vec{n}')\frac{e^{ikr}}{r}$$



scattering amplitude

At low energy, and if *W* decreases faster than r^{-3} at infinity:

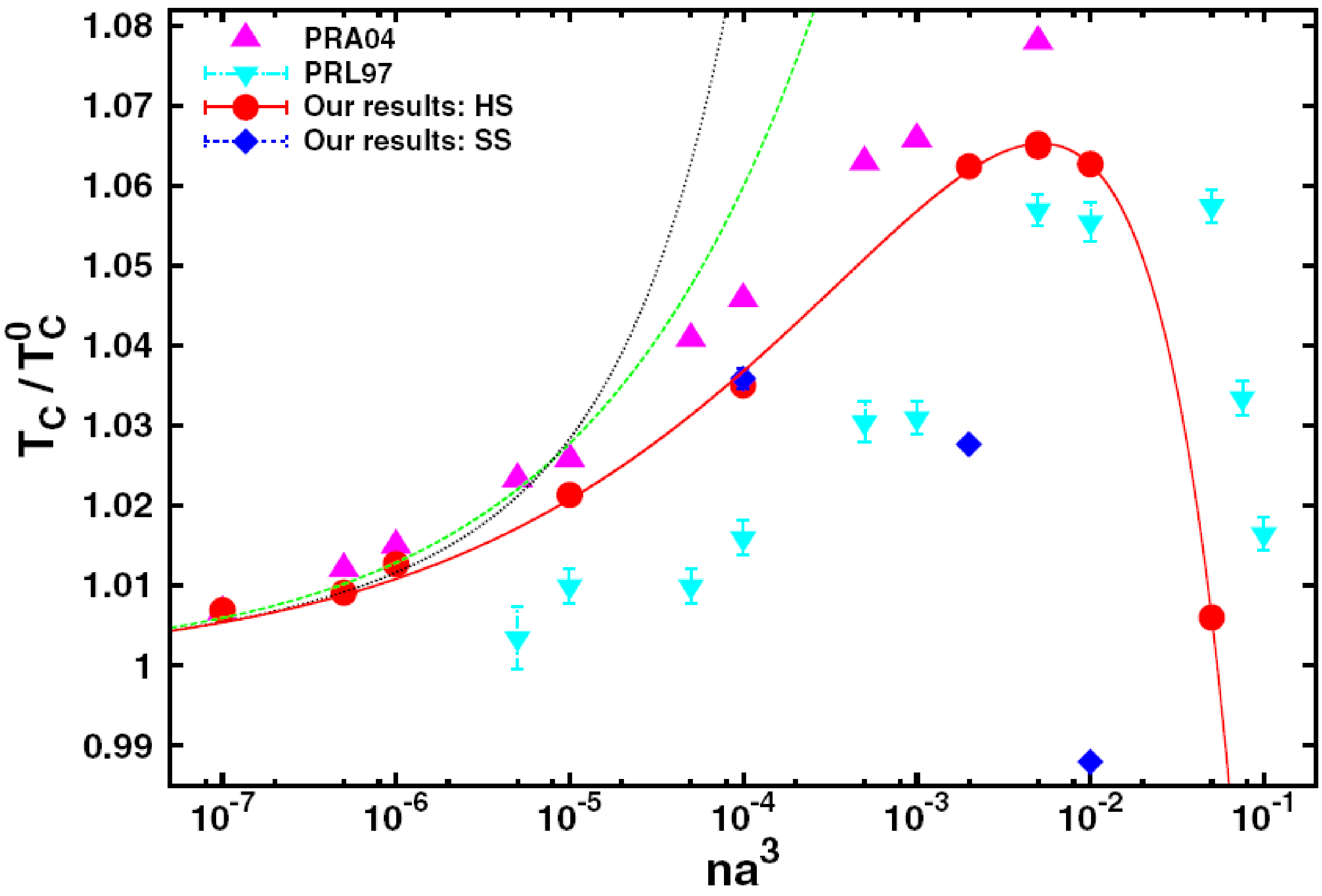
$$f(k,\vec{n},\vec{n}') \xrightarrow{k\to 0} -a$$

scattering length

Two interaction potentials with the same scattering length lead to the same properties at sufficiently low temperature

a = 5 nm (!) for ^{87}Rb

Interactions: minor role on the critical temperature



S. Pilati, S. Giorgini and N. Prokof'ev, PRL **100**, 140405 (2008)

Theoretical description of the condensate

The Hamiltonian: $$H=\sum_{i=1}^{N}\frac{p_i^2}{2m}+V(\vec{r}_i)+\sum_{i<j}W(\vec{r}_i-\vec{r}_j)$$

Confining potential ($V(\vec{r}_i)$)

Interactions between atoms ($W(\vec{r}_i-\vec{r}_j)$)

➡ At low temperature, we can replace the real potential $W(\vec{r}_i-\vec{r}_j)$ by :

$$W(\vec{r}_i-\vec{r}_j)\longrightarrow g\,\delta(\vec{r}_i-\vec{r}_j) \qquad g=\frac{4\pi\hbar^2 a}{m}$$, a : scattering legnth

➡ Hartree approximation: $$\Psi(\vec{r}_1,\vec{r}_2,\ldots,\vec{r}_N)\approx\psi(\vec{r}_1)\psi(\vec{r}_2)\ldots\ \psi(\vec{r}_N)$$

Treatment valid in the dilute regime: $na^3\ll 1$

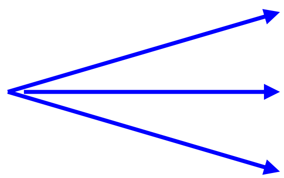
Gross-Pitaevski equation (or non-linear Schrödinger's equation) :

$$\left(-\frac{\hbar^2}{2m}\Delta+V(\vec{r})+Ng\,\left|\psi(\vec{r})\right|^2\right)\psi(\vec{r})=\mu\,\psi(\vec{r})$$

Different regime of interactions

The scattering length can be modified:

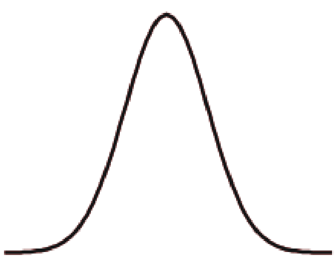
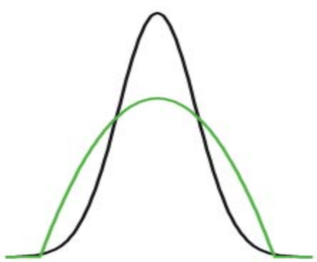
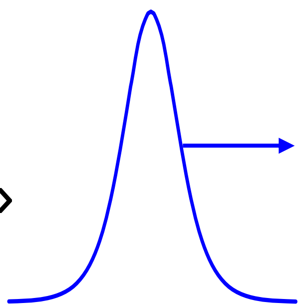
a (B) Feshbach's resonances



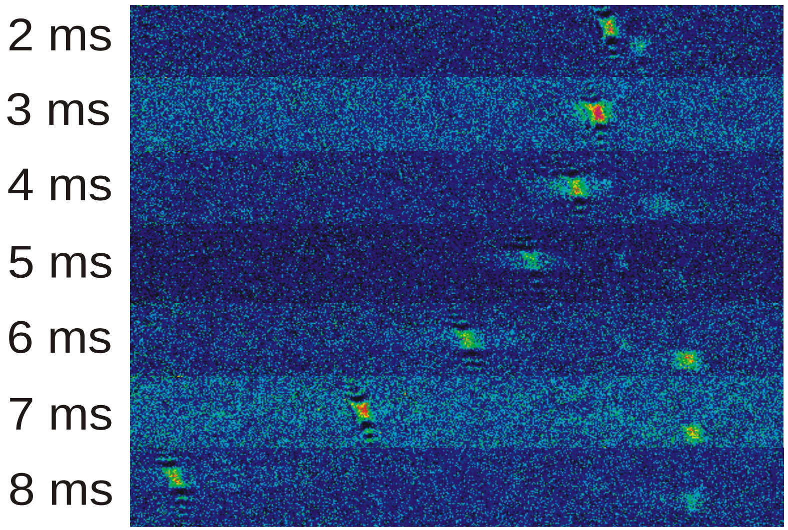
$a > 0$: Repulsive interactions

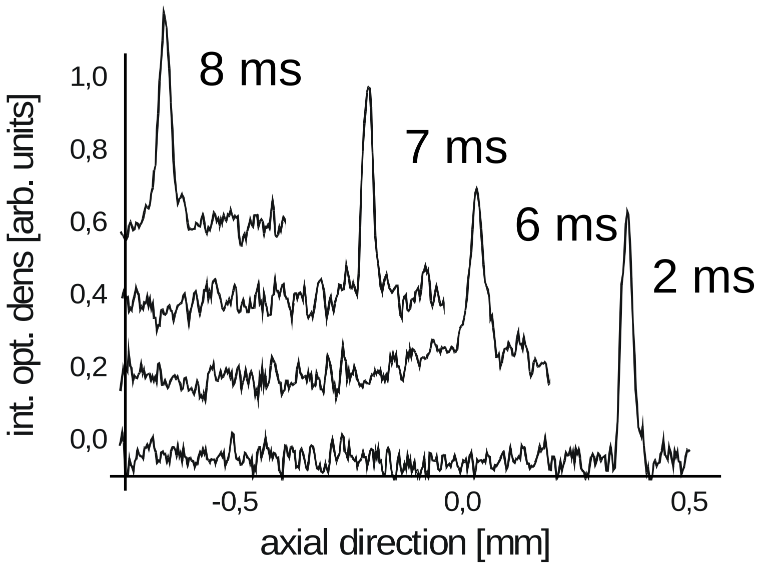
$a = 0$: Ideal gas

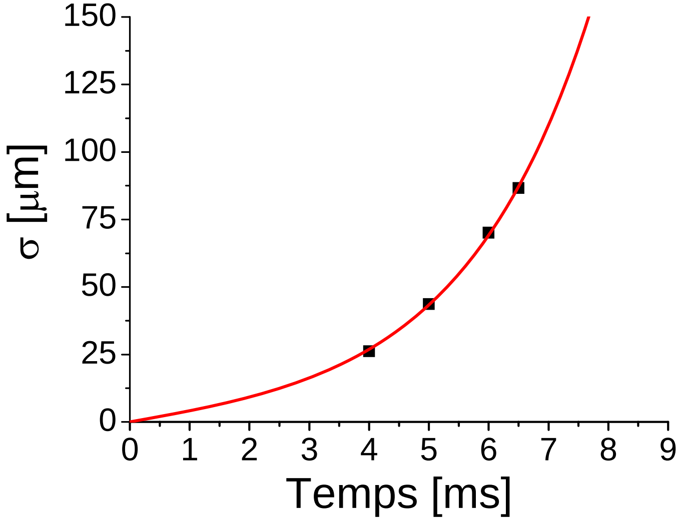
$a < 0$: Attractive interaction

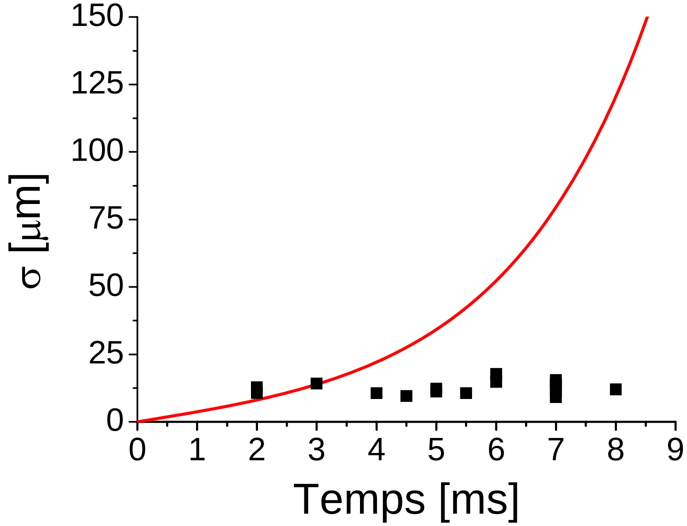
$a = 0$	$a > 0$	$a < 0$, 3D	$a < 0$, 1D
		$N < N_c$ « Collapse »	
Gaussian $\varphi_0(\vec{r})$	Parabolic		Soliton

Experimental evidence for matter wave soliton









Science **296**, 1290 (2002)

Time dependent Gross Pitaevskii equation

$$i\hbar\frac{\partial\varphi(\boldsymbol{r},t)}{\partial t}=-\frac{\hbar^2}{2m}\Delta\varphi(\boldsymbol{r},t)+V(\boldsymbol{r})\varphi(\boldsymbol{r},t)+\frac{4\pi\hbar^2 a}{m}\left|\varphi(\boldsymbol{r},t)\right|^2\varphi(\boldsymbol{r},t)$$

with the normalization $\int d^3r|\varphi(\boldsymbol{r},t)|^2=N$

The time-dependent behaviour of Bose–Einstein condensed clouds is an important source of information about the physical nature of condensate.

For instance, it enables the study of collective modes and the expansion of a BEC when released from a trap.

The spectrum of elementary excitations of the condensate is an essential ingredient in calculations of thermodynamic properties.

From this equation one may derive equations very similar to those of classical hydrodynamics, which we shall use to calculate properties of collective modes.

Hydrodynamic formalism

$$i\hbar \frac{\partial \varphi(\boldsymbol{r},t)}{\partial t} = -\frac{\hbar^2}{2m}\Delta\varphi(\boldsymbol{r},t) + V(\boldsymbol{r})\varphi(\boldsymbol{r},t) + \frac{4\pi\hbar^2 a}{m}\left|\varphi(\boldsymbol{r},t)\right|^2 \varphi(\boldsymbol{r},t)$$

with the normalization $\int d^3r|\varphi(\boldsymbol{r},t)|^2 = N$

Phase-modulus formulation

$$\varphi(\boldsymbol{r},t) = \sqrt{\rho(\boldsymbol{r},t)}\; e^{iS(\boldsymbol{r},t)} \qquad \boldsymbol{v}(\boldsymbol{r},t) = \frac{\hbar}{m}\,\boldsymbol{\nabla} S(\boldsymbol{r},t)$$

evolve according to a set of hydrodynamic equations **(<u>exact):</u>**

$$\frac{\partial \rho}{\partial t} + \boldsymbol{\nabla}\cdot(\rho\boldsymbol{v}) = 0 \quad \text{continuity}$$

$$m\left(\frac{\partial \boldsymbol{v}}{\partial t} + \frac{1}{2}\boldsymbol{\nabla} v^2\right) = \boldsymbol{\nabla}\left(\frac{\hbar^2}{2m}\frac{\Delta\left(\sqrt{\rho}\right)}{\sqrt{\rho}} - V(\boldsymbol{r}) - \frac{4\pi\hbar^2 a}{m}\rho\right) \quad \text{Euler}$$

Thomas Fermi regime (1)

Kinetic energy

$$E_c \sim \frac{\hbar^2}{2mR^2}$$

Potential energy

$$E_p \sim \frac{1}{2}m\omega^2 R^2$$

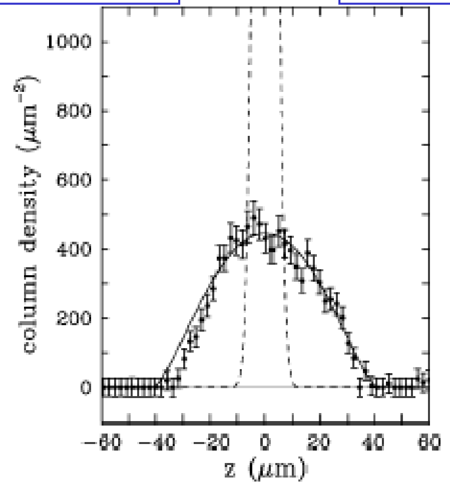
Interaction energy

$$E_{int} \sim \frac{4\pi\hbar^2 a}{m}\frac{N}{R^3}$$

$$\frac{E_c}{E_p} = \left(\frac{\sigma}{R}\right)^4 \ll 1$$

$$\frac{E_c}{E_{int}} = \frac{R}{Na} \ll 1$$

$$\sigma = \sqrt{\frac{\hbar}{m\omega}}$$



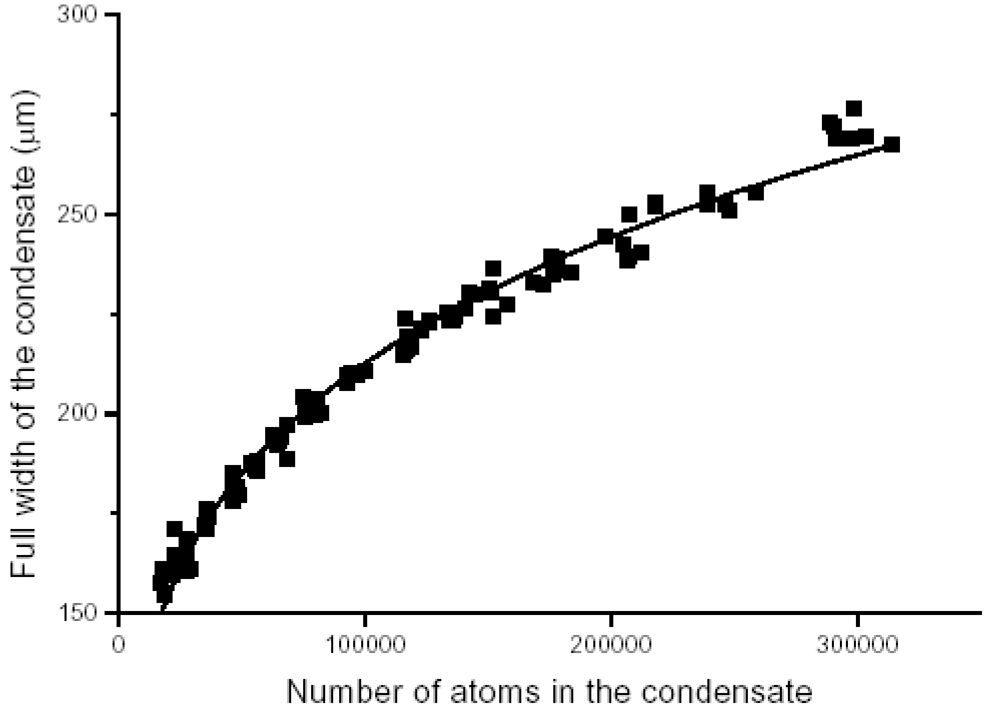
^{87}Rb : a = 5 nm
N = 10^5
R = 1 μm

Thomas Fermi regime (2) $Na^3 \gg \sigma$

$$\boldsymbol{\nabla}\left[\frac{\hbar^2}{2m}\frac{1}{\sqrt{\rho_0}}\Delta\sqrt{\rho_0} - V_{\mathrm{ext}} - \frac{4\pi\hbar^2 a}{m}\rho_0\right] = \mathbf{0} \quad\Rightarrow\quad V(\boldsymbol{r}) + \frac{4\pi\hbar^2 a}{m}\rho_0(\boldsymbol{r}) = \mu$$

(the term $\frac{\hbar^2}{2m}\frac{1}{\sqrt{\rho_0}}\Delta\sqrt{\rho_0}$ is crossed out)

$$V(\boldsymbol{r}) = \frac{1}{2}m\omega^2 r^2 \quad\Rightarrow\quad N = \int \rho_0(\boldsymbol{r})d^3r \quad\Rightarrow\quad \boxed{R \propto N^{1/5}}$$



Appl. Phys. B **69**, 257 (1999)

Scaling like solutions of GPE

Scaling ansatz

$$\rho(\boldsymbol{r},t) = \frac{1}{\prod_j b_j}\rho_0\left(\frac{r_i}{b_i}\right)$$

Scaling parameters
Time dependent

Normalization

yields exact solutions in 2D for harmonic confinement (hc)

exact solution in 3D in the Thomas Fermi limit for hc :

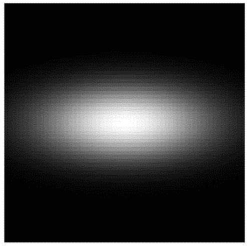
Equation of continuity

$$v_i = \frac{\dot{b}_i}{b_i} r_i$$

Euler equation

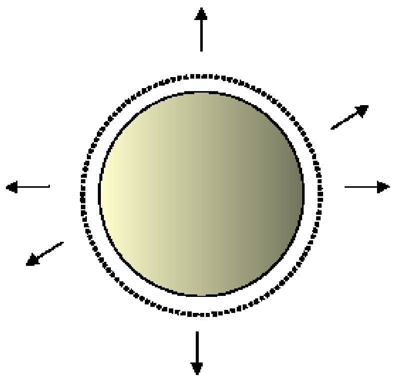
$$\ddot{b}_i + \omega_i^2 b_i = \frac{\omega_i^2}{\prod_j b_j}\frac{1}{b_i}$$

Scaling solutions of GPE:
low lying collective modes

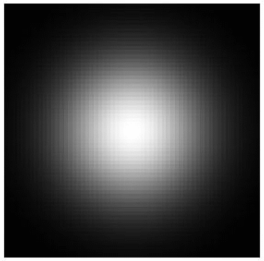


Isotropic harmonic confinement

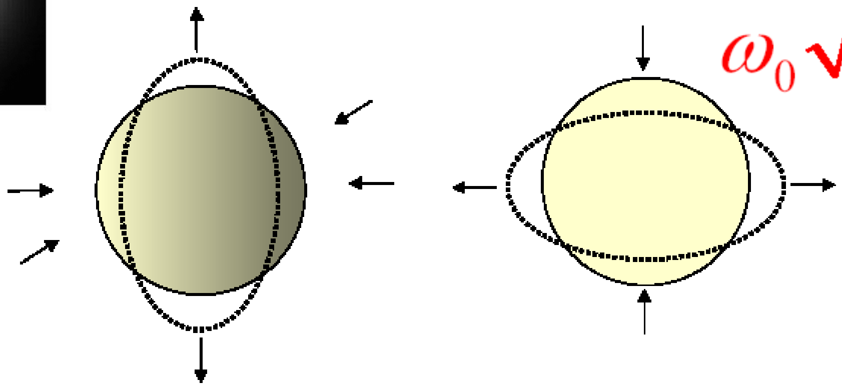
Monopole mode



$\omega_0\sqrt{5}$



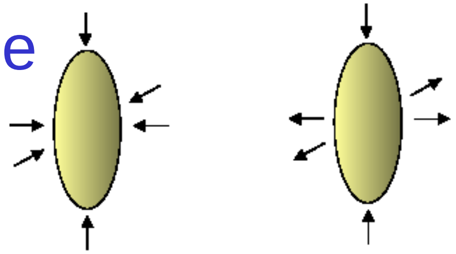
Quadrupole mode



$\omega_0\sqrt{2}$

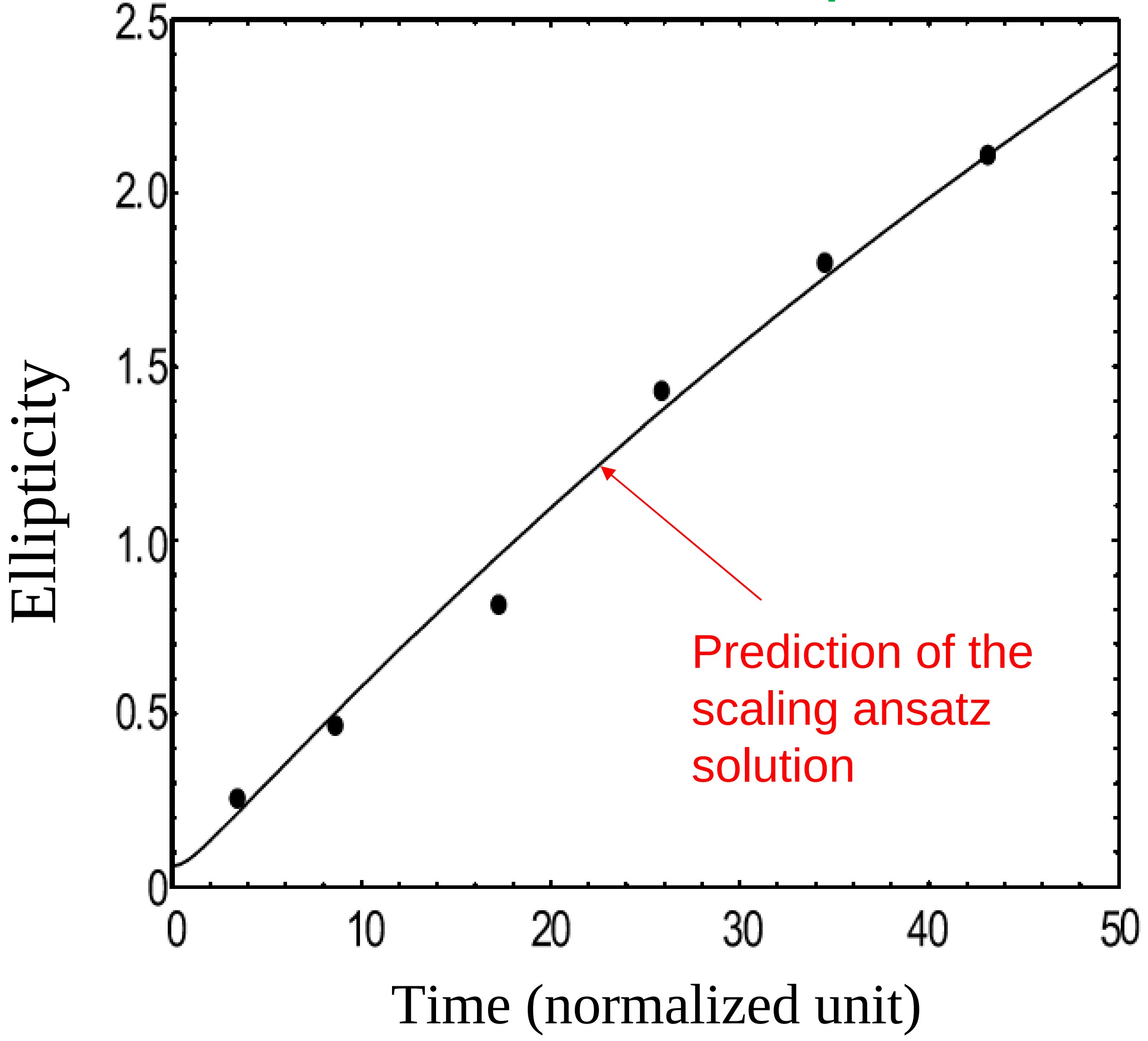
Harmonic confinement with cylindrical symmetr $\lambda = \omega_z/\omega_\perp$

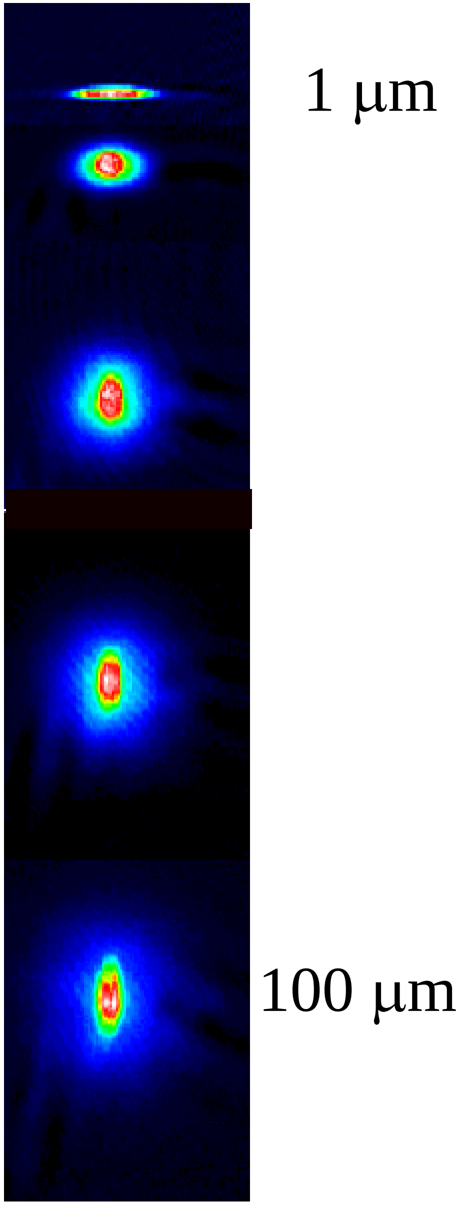
Coupling between monopole and quadrupole mode in anisotropic harmonic traps



$$\omega^2/\omega_\perp^2 = 2 + 3\lambda^2/2 \pm (\sqrt{16 - 16\lambda^2 + 9\lambda^4})/2$$

Anisotropic expansion: «microscope effect»



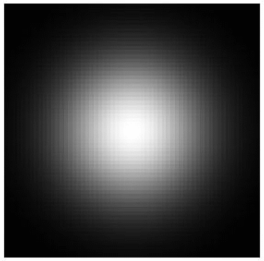


Collective modes versus collective oscillations
Are unambigously probing BEC properties ?

Low-lying collective modes of a BEC has a classical counterpart : collective oscillations for a classical gas

Classical gases are described by the Boltzmann equation (BE)

First example: monopole mode in an isotropic trap



$$\frac{d\langle \mathbf{r}^2\rangle}{dt} = 2\langle \mathbf{r}\cdot\mathbf{v}\rangle ,$$

$$\frac{d\langle \mathbf{r}\cdot\mathbf{v}\rangle}{dt} = \langle \mathbf{v}^2\rangle - \omega_0^2\langle \mathbf{r}^2\rangle$$

$$\frac{d\langle \mathbf{v}^2\rangle}{dt} = -2\omega_0^2\langle \mathbf{r}\cdot\mathbf{v}\rangle$$

$$U_{trap}(\boldsymbol{r}) = \frac{1}{2}m\omega_0^2 r^2$$

$$\omega = 2\omega_0$$

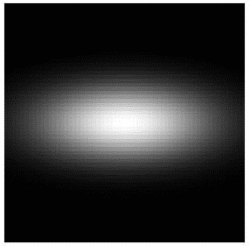
Response Classical gas

$\neq$

For BEC $\omega_0\sqrt{5}$

BEC

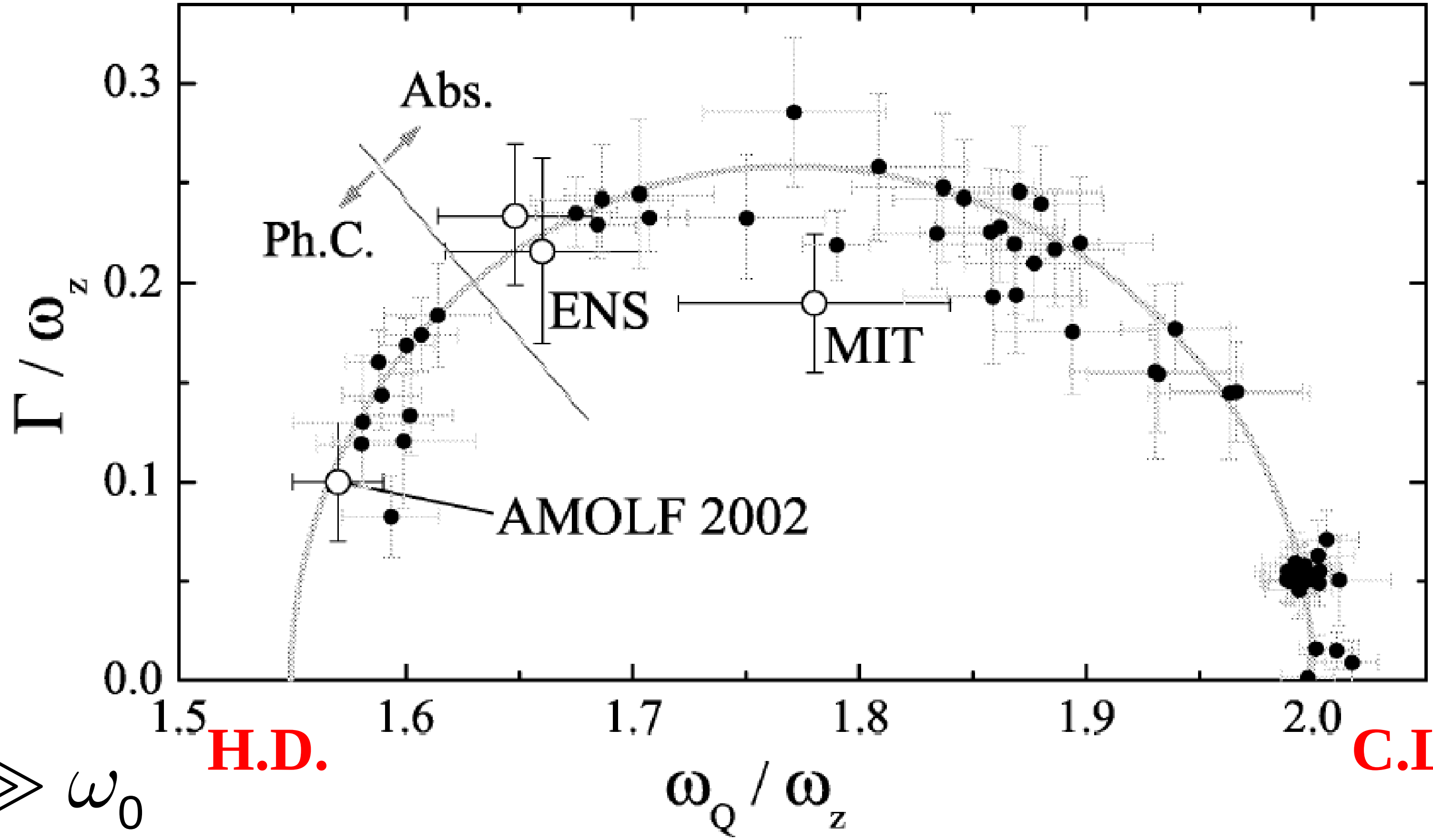
Collective modes versus collective oscillations



Second example: quadrupole mode

γ Collision rate

$$\omega = \omega_Q + i\Gamma$$

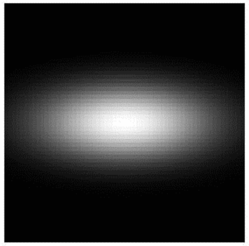


H.D. $\gamma \gg \omega_0$

C.L. $\gamma \ll \omega_0$

Collective modes versus collective oscillations

Quadrupole mode in isotropic trap



$\gamma \gg \omega_0$ Hydrodynamic limit $\omega_0\sqrt{2}$

For BEC $\omega_0\sqrt{2}$

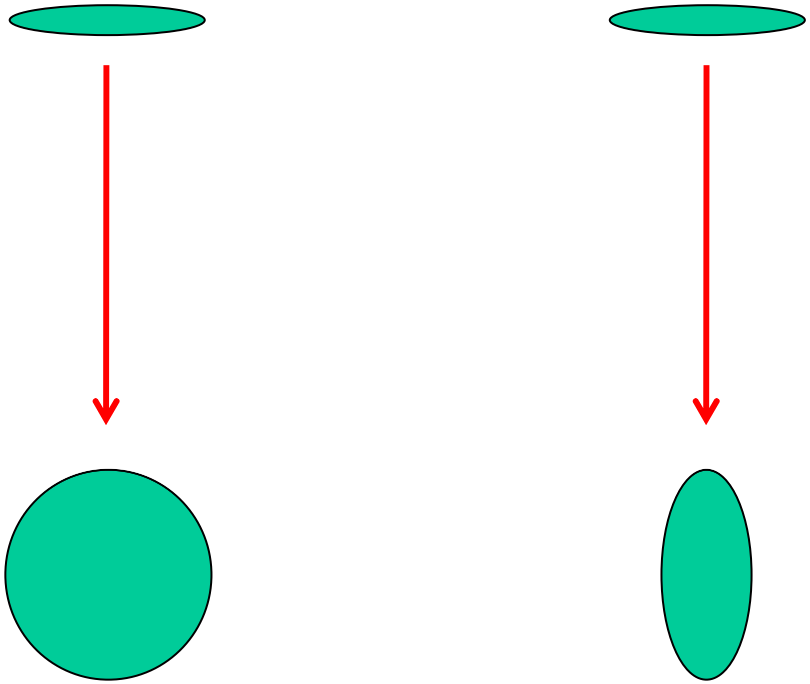
Response Classical gas = BEC !!!

For harmonic and isotropic confinement, all surface modes (quadrupole, octopole, …) give the same results as the ones for a classical gas in the hydrodynamic regime !

What about the expansion ?

Collisionless gas

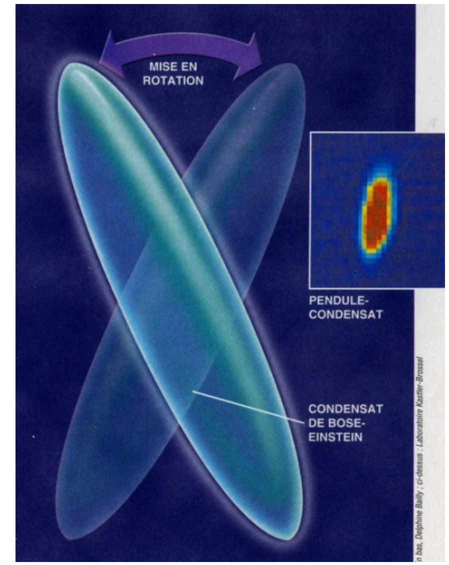
Hydrodynamic gas



$$\vec{\nabla} P \propto \vec{\nabla} n \propto \begin{pmatrix} \omega_x^2 x \\ \omega_y^2 y \\ \omega_z^2 z \end{pmatrix}$$

Used for thermometry

Scissors mode



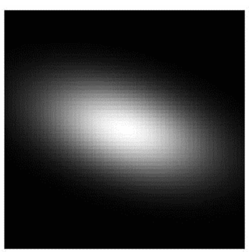
Moment of inertia

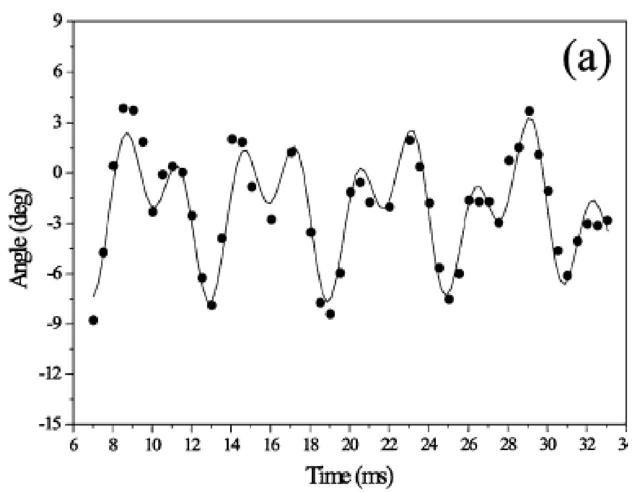
$$I = \begin{cases} I_{\mathrm{rig}} = Nm\langle x^2 + y^2 \rangle \\ I_{\mathrm{s}} \sim \epsilon^2 I_{\mathrm{rig}} \end{cases}$$

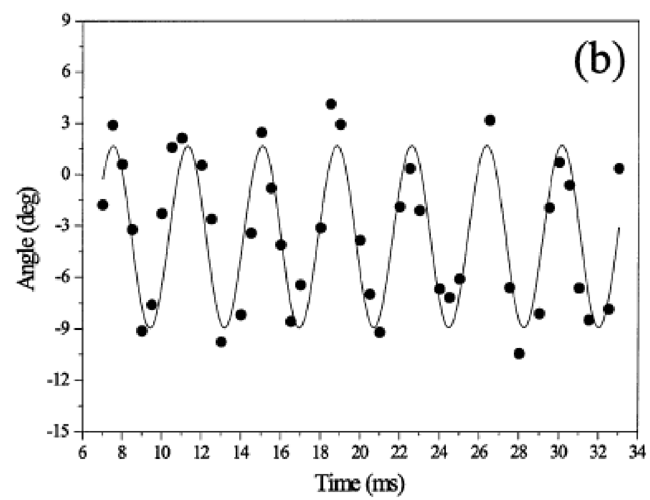
Classical gas

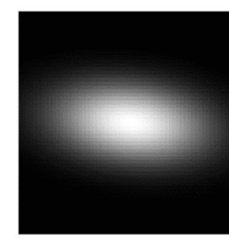
BEC in an anisotropic trap

Pendulum oscillation in an anisotropic trap









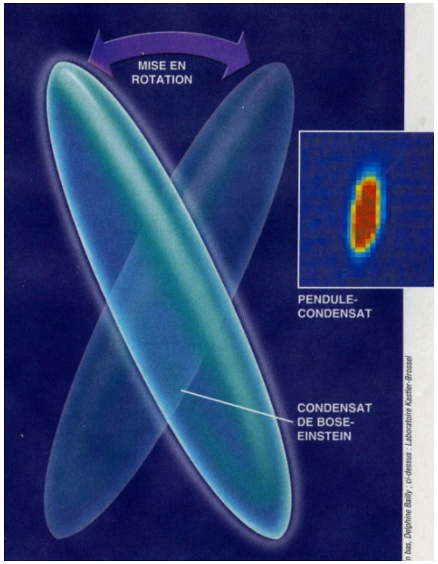
DGO and S. Stringari PRL **83** 4452 (1999)

Marago *et al.* PRL **84** 2056 (2000)

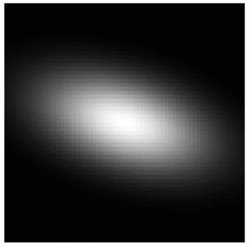
Scissors mode

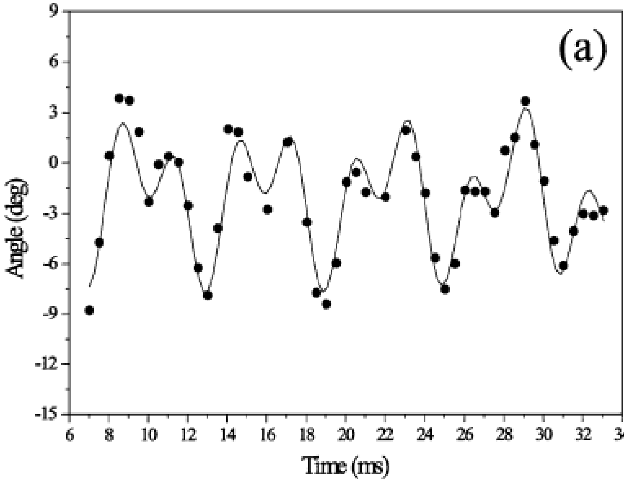
Moment of inertia

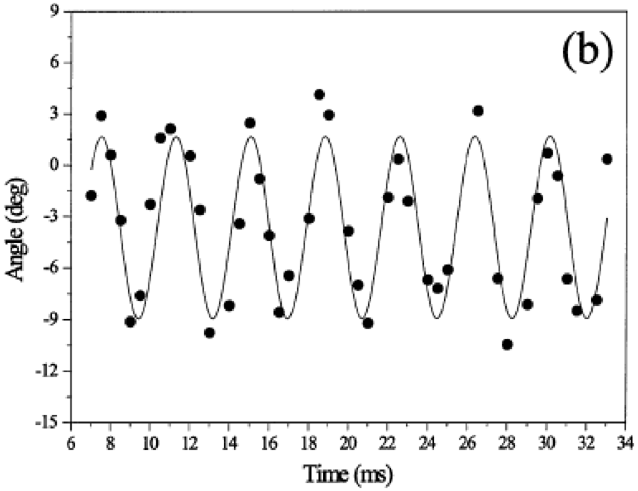
$$I = \begin{cases} I_{\rm rig} = Nm\langle x^2 + y^2 \rangle & \text{Classical gas} \\ I_{\rm s} \sim \epsilon^2 I_{\rm rig} & \text{BEC in an anisotropic trap} \end{cases}$$

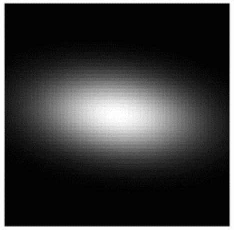


Pendulum oscillation in an anisotropic trap









DGO and S. Stringari PRL **83** 4452 (1999)

Marago *et al*. PRL **84** 2056 (2000)

Intrinsically due to the two types of solutions of HD equations: **rotational and irrotational solutions**

Bogolubov spectrum and speed of sound

Equilibrium state **in a box**

$$\begin{cases} \rho(\boldsymbol{r},t) = \rho_0 \\ \boldsymbol{v}(\boldsymbol{r},t) = \boldsymbol{0} \end{cases}$$

uniform

Linearization of the hydrodynamic equations

$$\begin{cases} \rho(\boldsymbol{r},t) = \rho_0 + \delta\rho(\boldsymbol{r},t) \\ \boldsymbol{v}(\boldsymbol{r},t) = \boldsymbol{0} + \delta\boldsymbol{v}(\boldsymbol{r},t) \end{cases}$$

We obtain

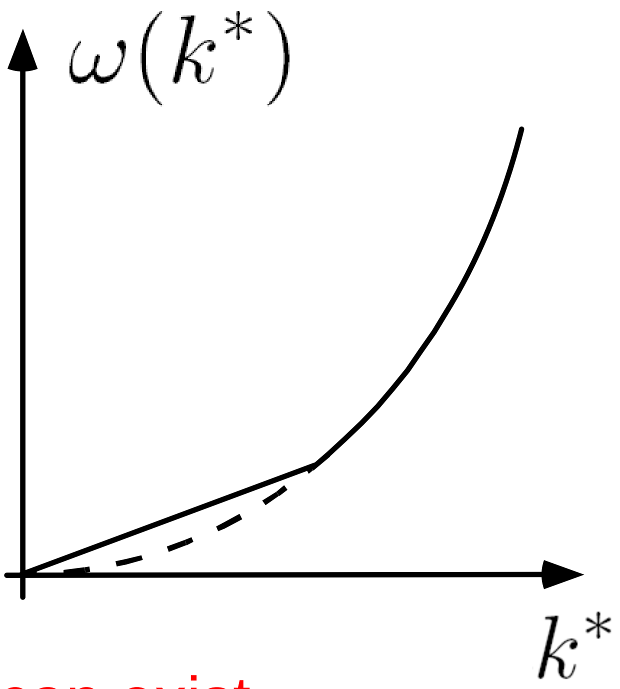
$$\frac{\partial^2 \delta\rho}{\partial t^2} + \frac{\hbar^2}{4m^2}\Delta(\Delta(\delta\rho)) - \frac{4\pi\hbar^2 a\rho_0}{m^2}\Delta(\delta\rho) = 0$$

$$\delta\rho(\boldsymbol{r},t) = \delta\rho_0 e^{i(kx-\omega t)} \Rightarrow \begin{cases} \omega^2 = c^2k^2\left(1 + \frac{\hbar^2k^2}{2m}\frac{1}{2mc^2}\right) \\ c = \sqrt{\frac{4\pi\hbar^2 a\rho_0}{m^2}} \end{cases}$$

Speed of sound
Dictated by the interactions

Landau argument for superfluidity

At low momentum, the **collective** excitations have a linear dispersion relation: $\omega(k^*) \simeq ck^*$



Microscopic probe-particle:

k before collision and k' after collision

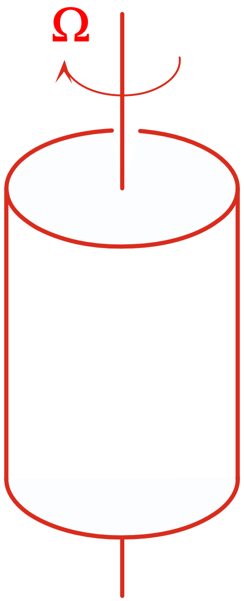
$$\begin{cases} \dfrac{\hbar^2 k^2}{2m} = \dfrac{\hbar^2 k'^2}{2m} + ck^* \\ k = k' + k^* \end{cases}$$

A solution can exist if and only if $\hbar k > mc$

Conclusion : For $\hbar k < mc$ the probe cannot deposit energy in the fluid. Superfluidity is a consequence of interactions.

For a macroscopic probe: it also exists a threshold velocity, PRL **91**, 090407 (2003)

Vortices in a rotating quantum fluid



Liquid superfluid helium

In a condensate $\psi(\vec{r}) = \sqrt{\rho(\vec{r})}\; e^{iS(\vec{r})}$

the velocity $\vec{v} = \dfrac{\hbar}{m}\vec{\nabla}S$ is such that $\oint \vec{v}\,.\,d\vec{r} = \dfrac{nh}{m}$

incompatible with rigid body rotation $\vec{v} = \vec{\Omega}\times\vec{r}$

Below a critical rotation Ω_c, no motion at all

Above Ω_c, apparition of singular lines on which the density is zero and around which the circulation of the velocity is quantized

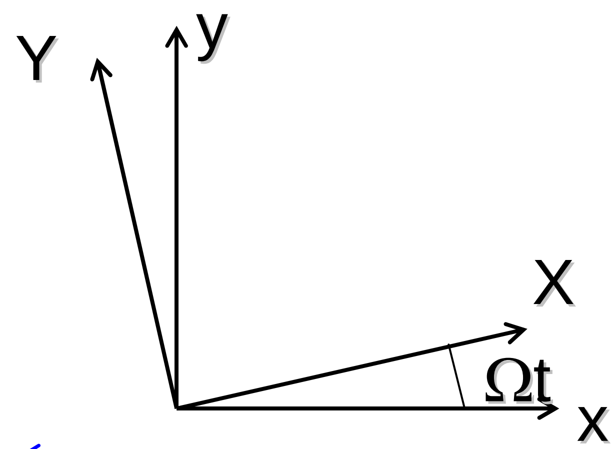
Onsager - Feynman

Ground state in the rotating frame

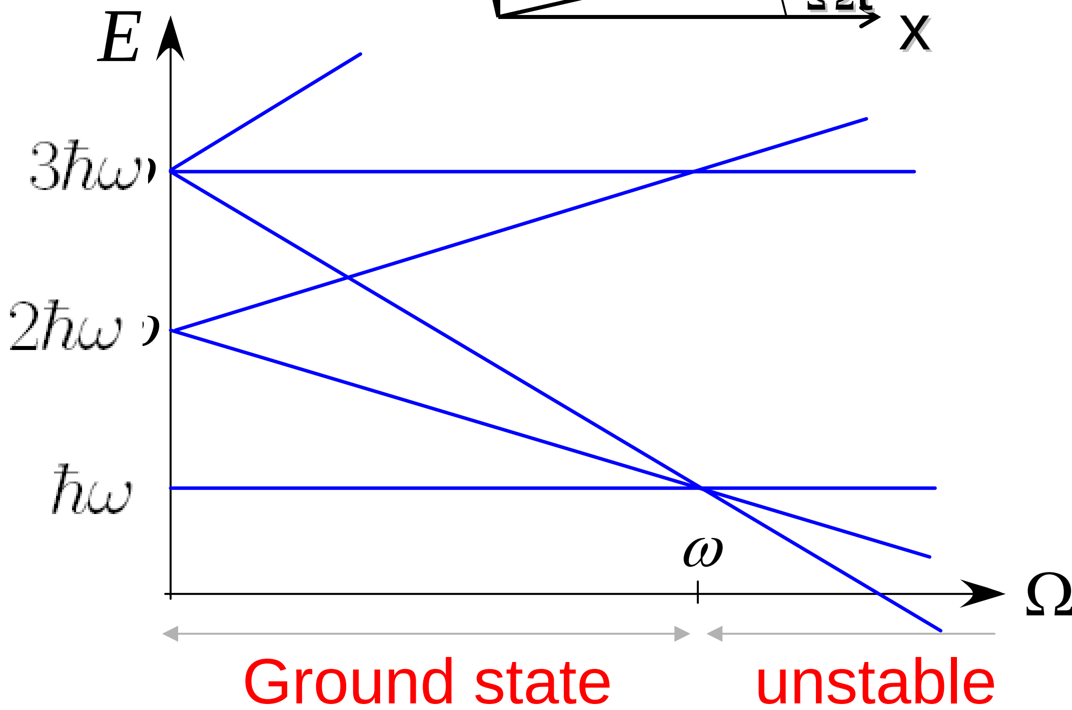
osc. 2D $\quad V(\boldsymbol{r}) = \frac{1}{2} m\omega^2 (x^2 + y^2)$

$$H = \frac{\boldsymbol{p}^2}{2m} + V(\boldsymbol{r})$$

$$\rightarrow H = \frac{\boldsymbol{p}^2}{2m} + V(\boldsymbol{r}) - \Omega L_z$$



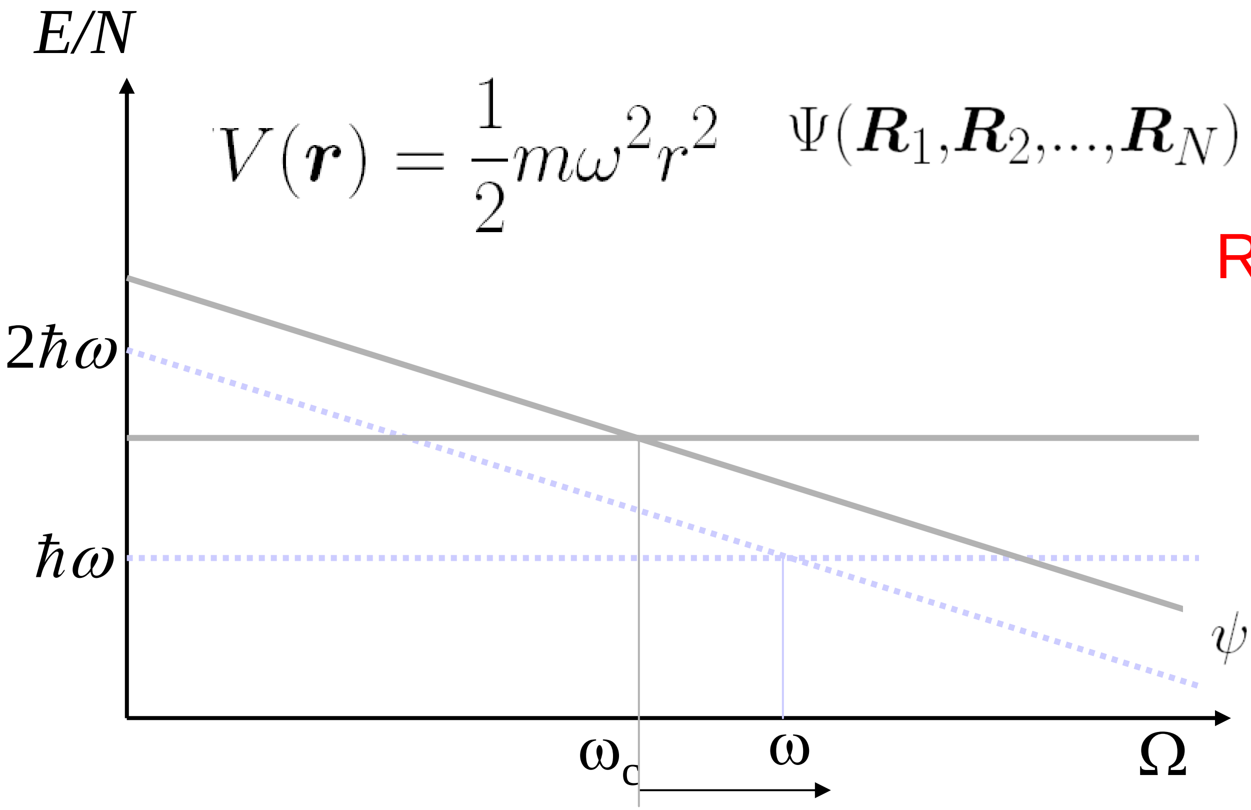
An anisotropy, even very small, to favorize the rotating frame at the angular freqency Ω.



$$\delta V(\boldsymbol{r}) = \frac{\epsilon}{2} m\omega^2 (X^2 - Y^2)$$

$$\left| n_x = 0, n_y = 0 \right\rangle$$

BEC in the weakly interacting regime



$$V(\boldsymbol{r}) = \frac{1}{2} m \omega^2 r^2$$

$$\Psi(\boldsymbol{R}_1, \boldsymbol{R}_2, \ldots, \boldsymbol{R}_N) = \psi(\boldsymbol{R}_1)\psi(\boldsymbol{R}_2)..\psi(\boldsymbol{R}_N)$$

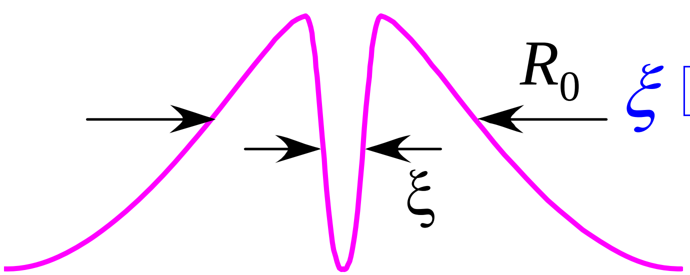
Repulsive interactions

$$\psi(\boldsymbol{R}) = e^{-(X^2+Y^2)}$$

$$\psi(\boldsymbol{R}) = (X + iY)e^{-(X^2+Y^2)}$$

Ground state in presence of a quantum vortex

Increasing the interactions:



ξ □ $\dfrac{\hbar}{\sqrt{m\, g\, n_0}} = \dfrac{1}{\sqrt{4\pi\, a\, n_0}}$

Preparation of a condensate with vortices

1. Preparation of a quasi-pure condensate (20 seconds)

Laser+evaporative cooling of ^{87}Rb atoms in a magnetic trap

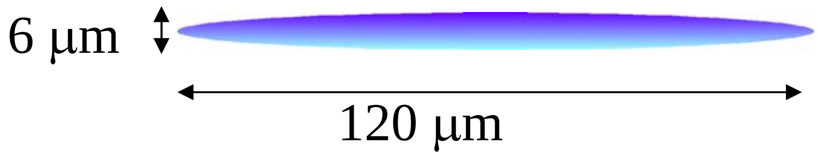
$$\frac{1}{2} m\omega_\perp^2 \left(x^2 + y^2\right) + \frac{1}{2} m\omega_z^2 z^2$$

$$\omega_\perp / 2\pi = 200 \ \text{Hz}$$

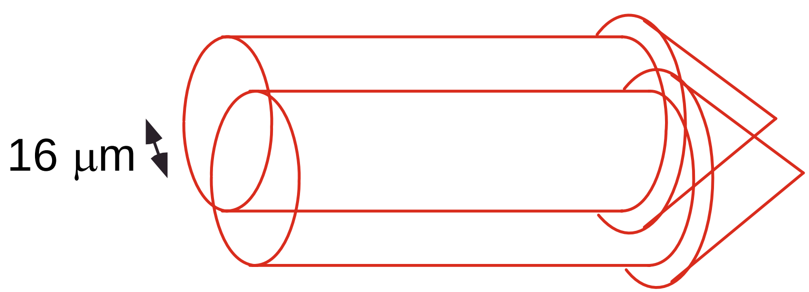
$$\omega_z / 2\pi = 10 \ \text{Hz}$$

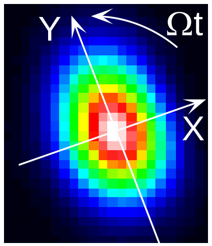
10^5 to 4×10^5 atoms

$T < 100$ nK



2. Stirring using a laser beam (0.5 seconds)





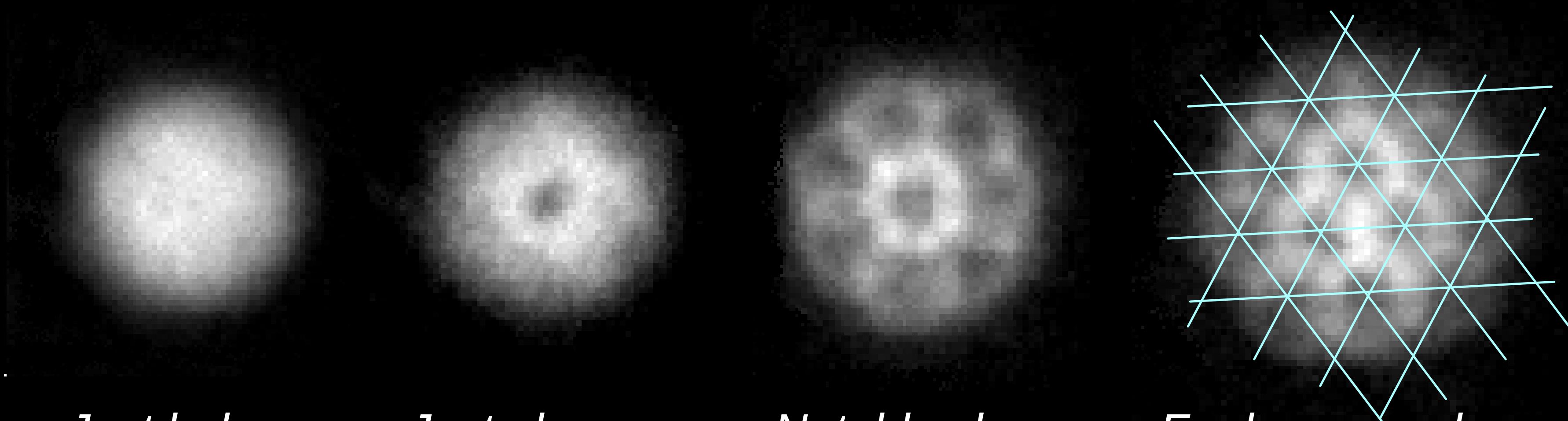
controlled with acousto-optic modulators

$$\delta \mathrm{U}(\vec{r}) = \frac{1}{2} m\omega_\perp^2 \left(\varepsilon_X X^2 + \varepsilon_Y Y^2\right)$$

$\varepsilon_X = 0.03$, $\varepsilon_Y = 0.09$

From single to multiple vortices

PRL **84**, 806 (2000)



Just below the critical frequency	*Just above the critical frequency*	*Notably above the critical frequency*	*For large numbers of atoms: Abrikosov lattice*

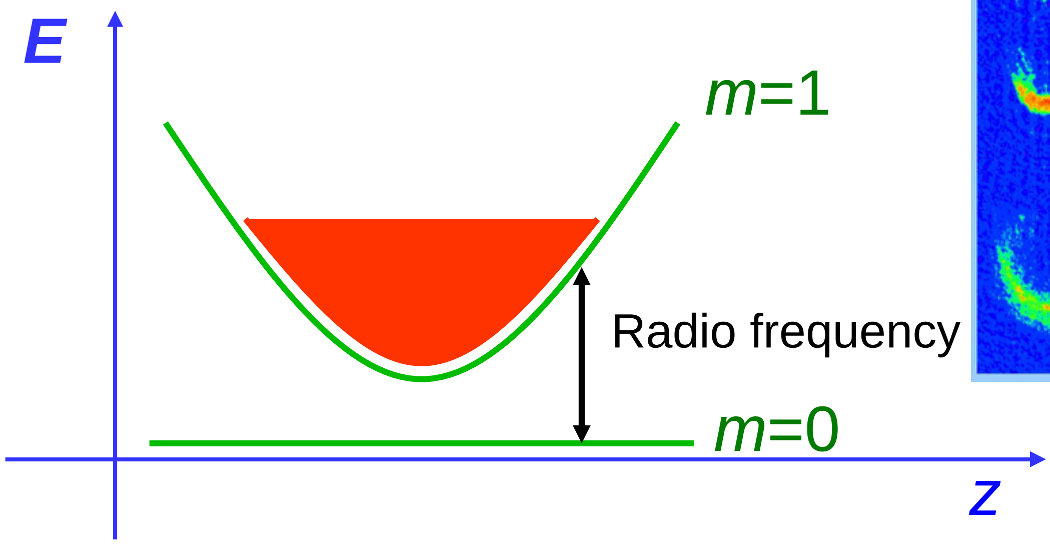
It is a real quantum vortex: angular momentum $\hbar$

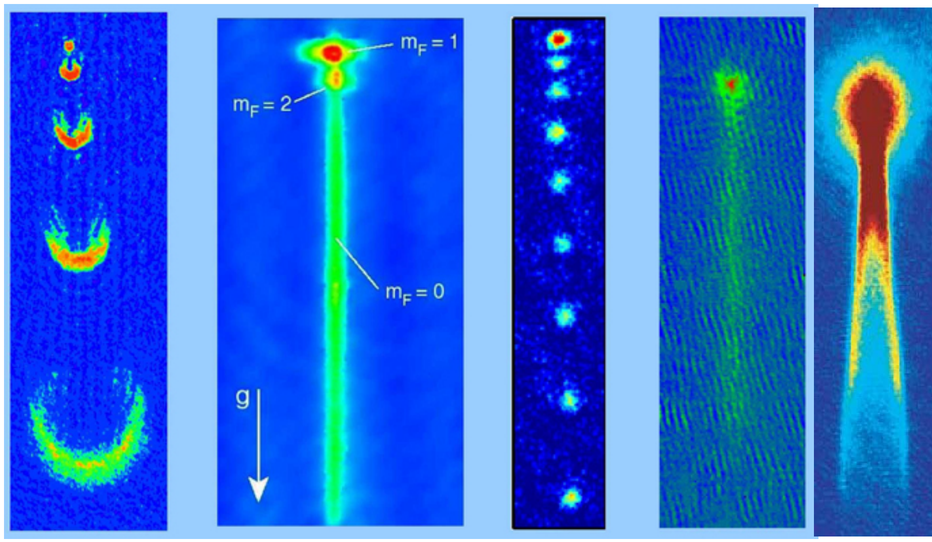
PRL 85, 2223 (2000)

also at MIT, Boulder, Oxford

Free falling coherent source of matter wave

Radio-frequency extraction of matter wave from a magnetic trap

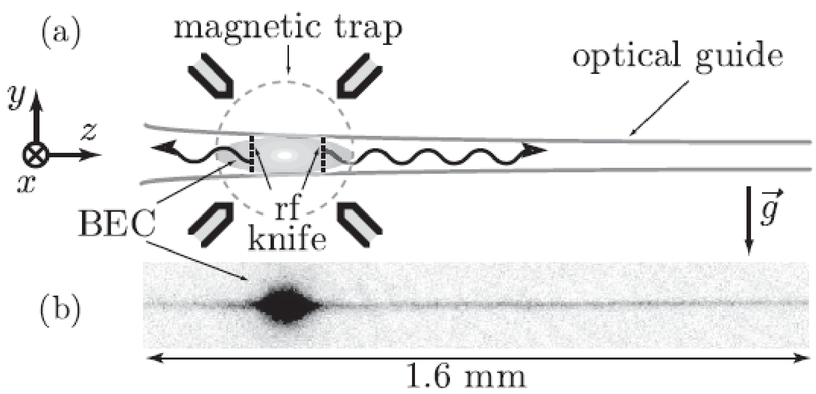




Atom laser experiment: MIT, Munich, Yale, NIST, Orsay, Canberra …

Guided coherent source of matter wave

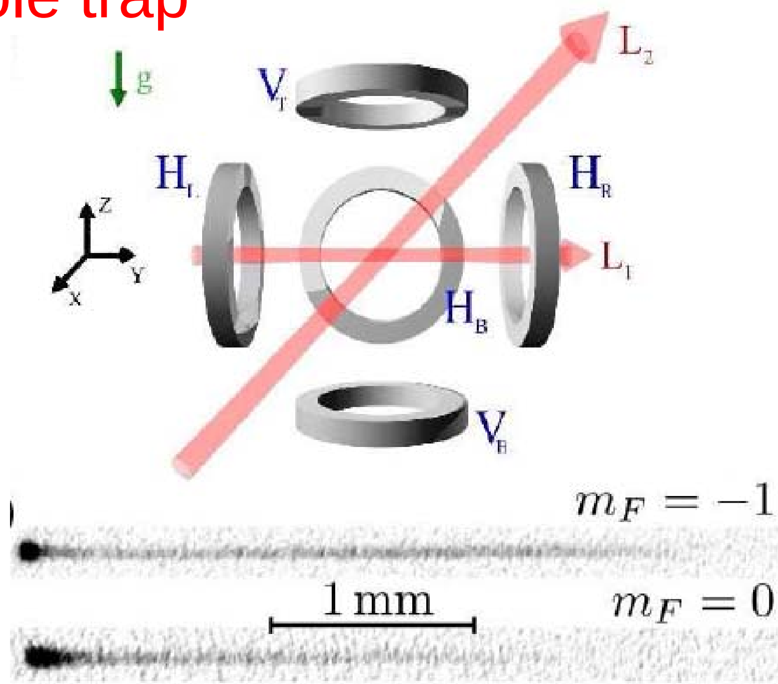
From a magnetic trap



$$\langle n \rangle = 2 \qquad m_F = 0$$

W. Guérin *et al.*
Phys. Rev. Lett. **97**, 200402 (2006)

From a dipole trap

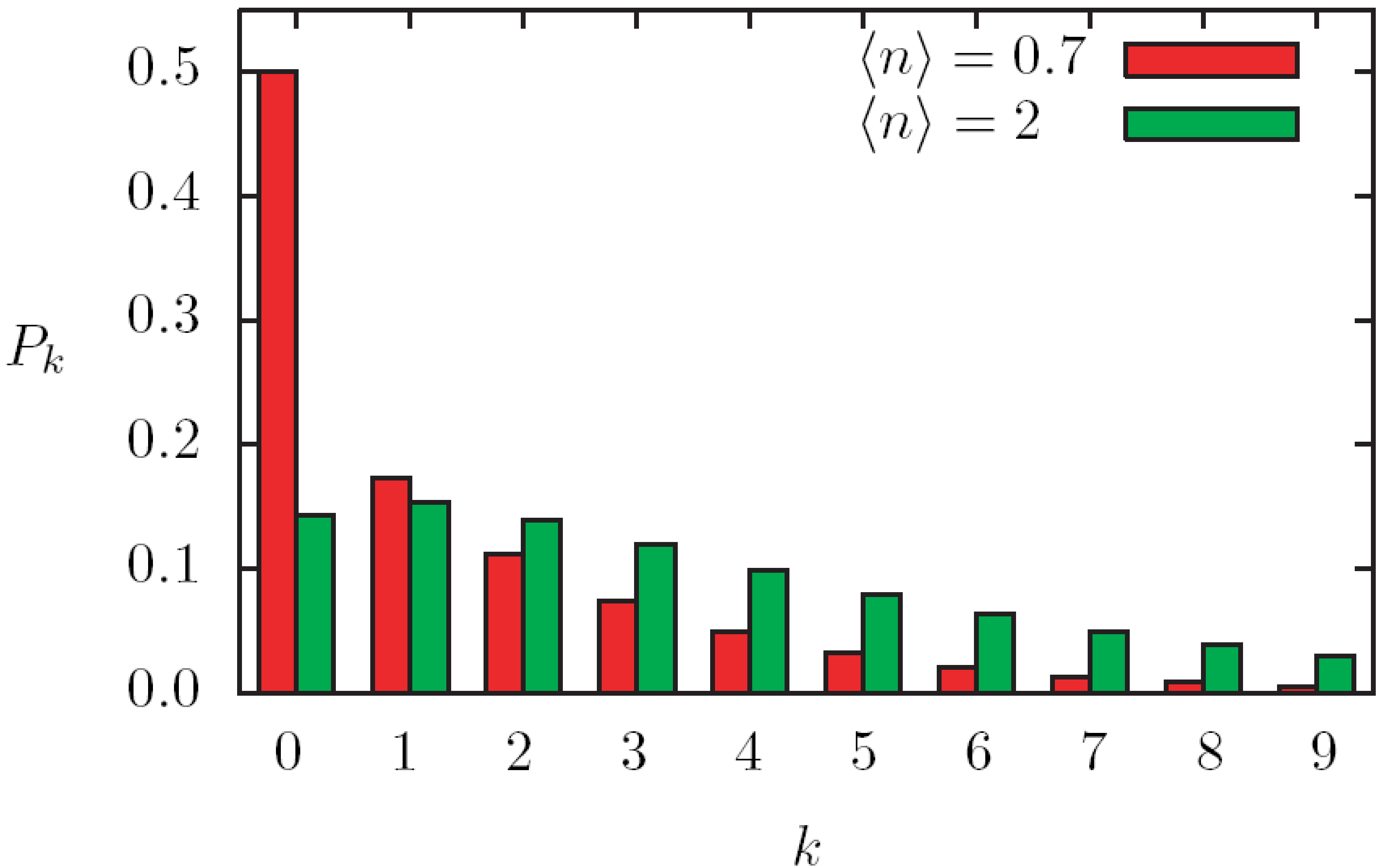


$$\langle n \rangle = 0.7 \qquad m_F = -1, 0, +1$$

A. Couvert *et al.*
Europhys. Lett. **83**, 50001 (2008)

Quasi-monomode guided atom laser

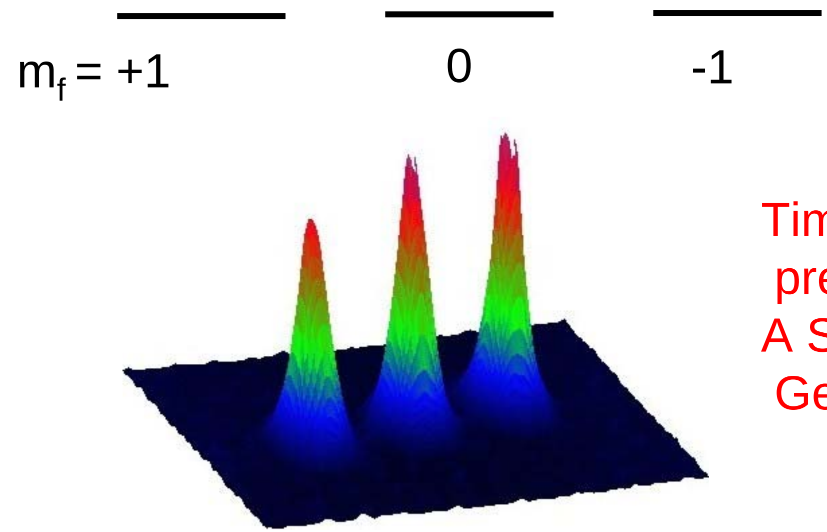
P_k occupation number of the transverse state of energy $\varepsilon_k = k\hbar\omega$



Spin-1 condensate

A vector field

$$\vec{\hat{\Psi}} = \begin{pmatrix} \hat{\Psi}_+ \\ \hat{\Psi}_0 \\ \hat{\Psi}_- \end{pmatrix}$$



Time-of-flight in prensence of A Stern and Gerlach field

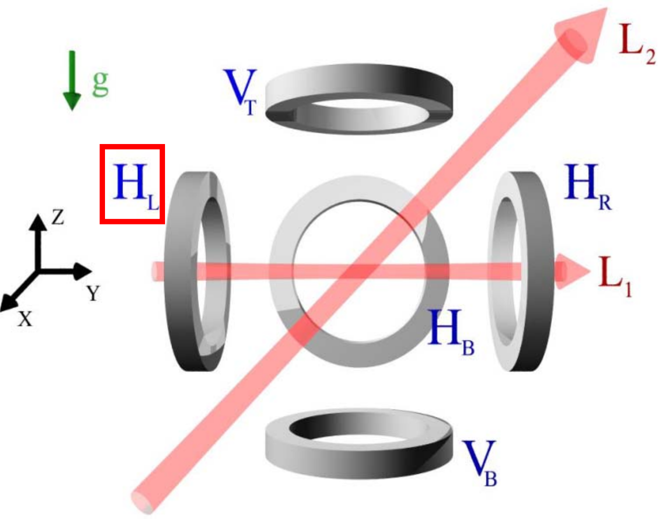
- Two conserved quantities:

(1) $N = \int d\vec{r}\, n(\vec{r}) \to \mu;$

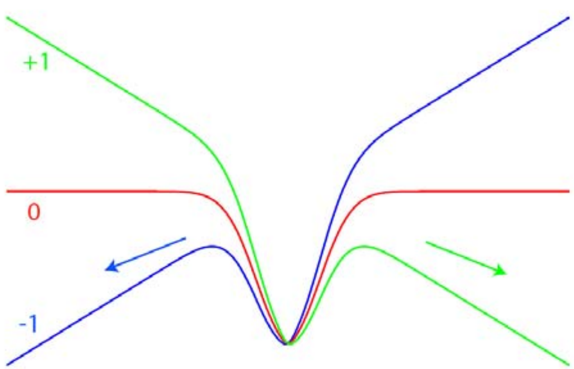
(2) $\mathcal{M} = \int d\vec{r}\, [n_+(\vec{r}) - n_-(\vec{r})] \to \mathcal{B}.$

T.-L Ho, Phys. Rev. Lett. 81, 742 (1998), T. Ohmi and K. Machida, J. Phys. Soc. Jpn 67, 1822 (1998), C. K. Law, H. Pu, and N. P. Bigelow,

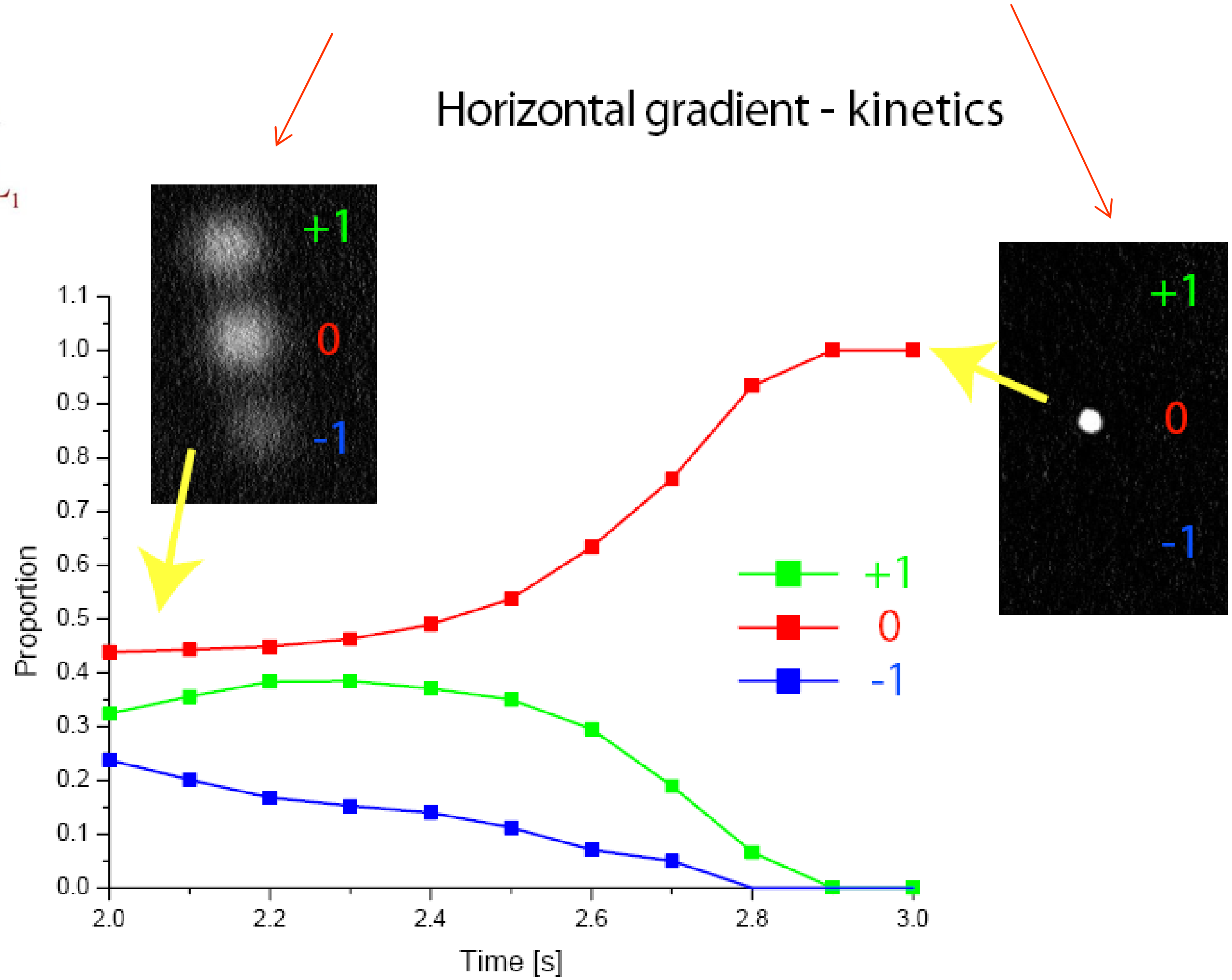
« Horizontal » spin distillation : m_F=0 state



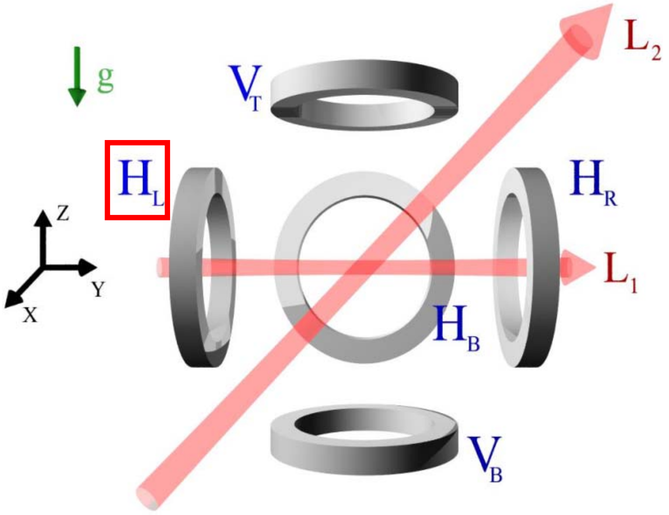
$$W = -\vec{\mu}.\vec{B} = m_F g_F \mu_B \, || \vec{B} ||$$

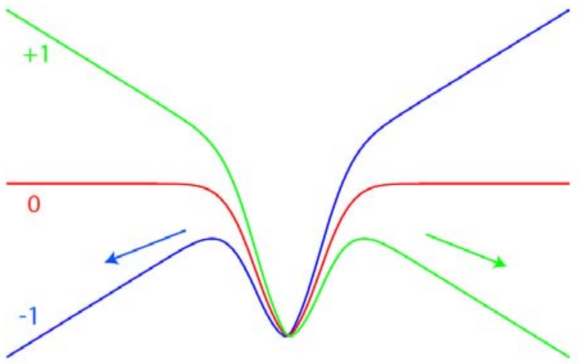


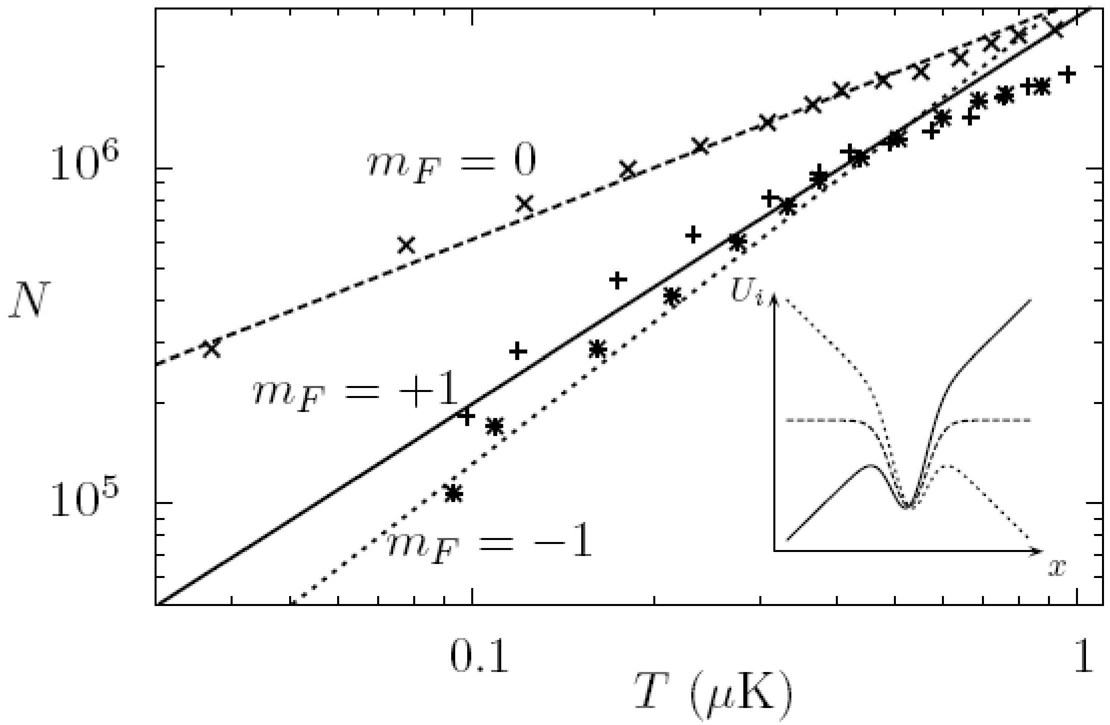
Time of flight (Stern and Gerlach field)



« Horizontal» spin distillation trajectories

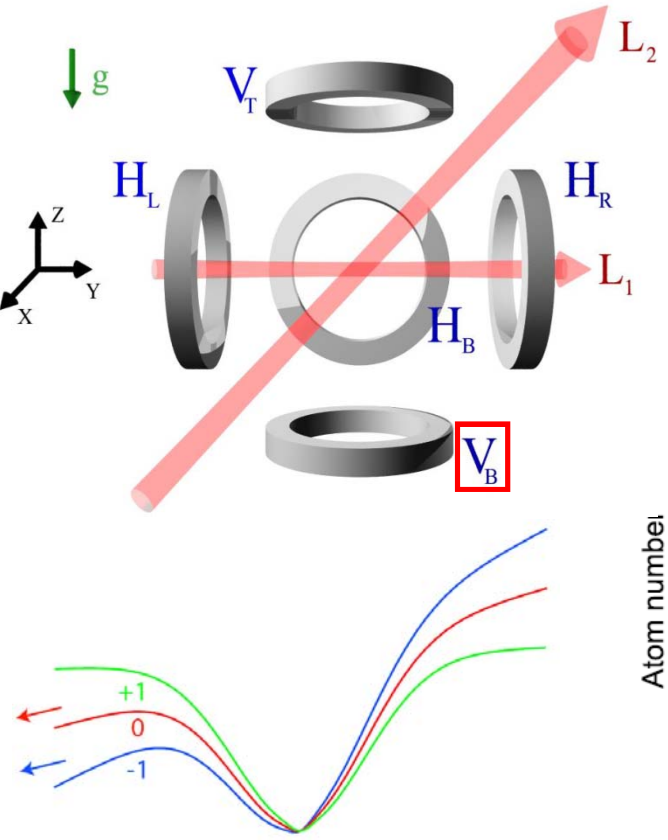


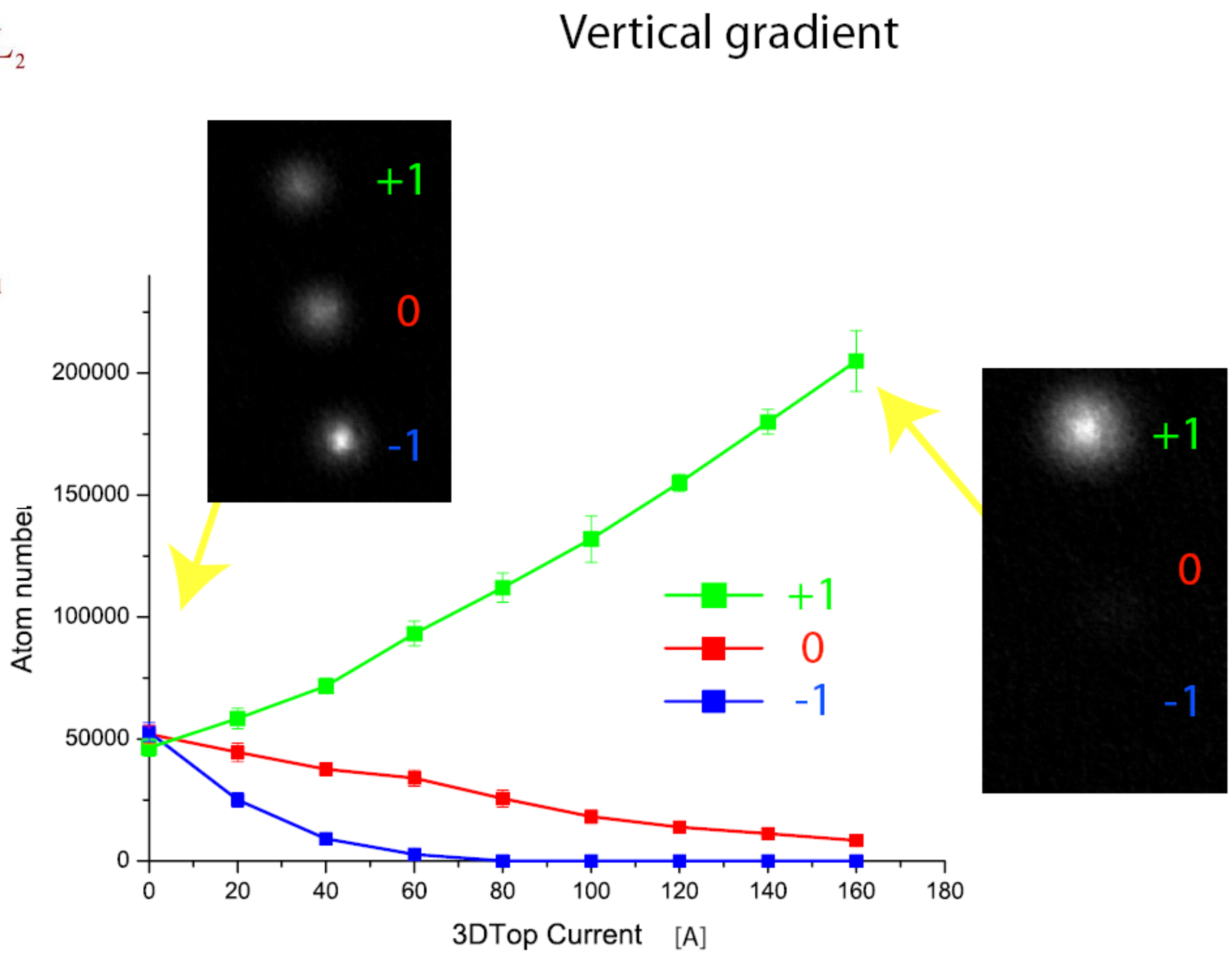




Combine sympathetic and evaporative cooling

«Vertical» distillation : m_F=1 preparation

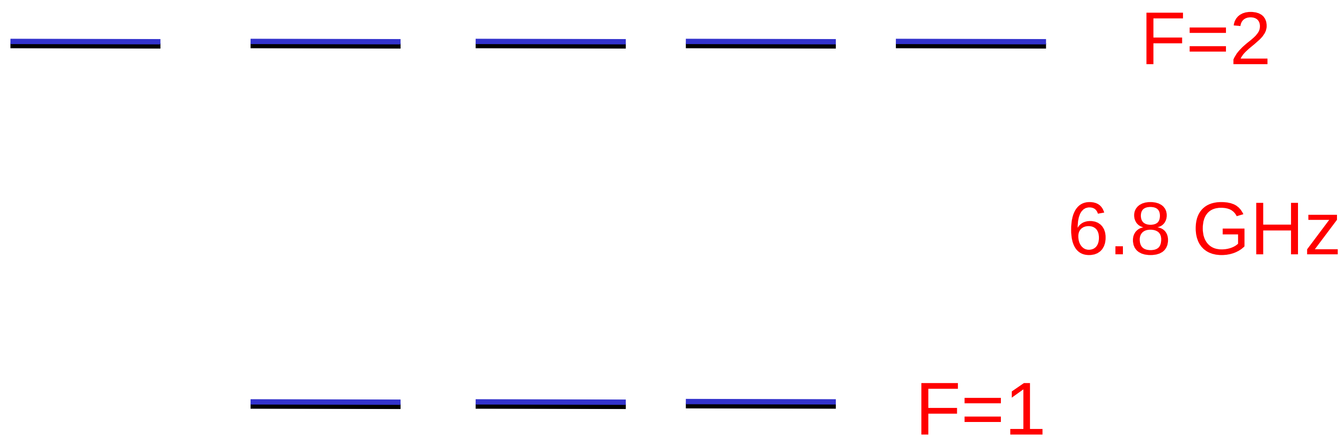




Interesting state for coupling to magnetic structure (trap, guide, …)

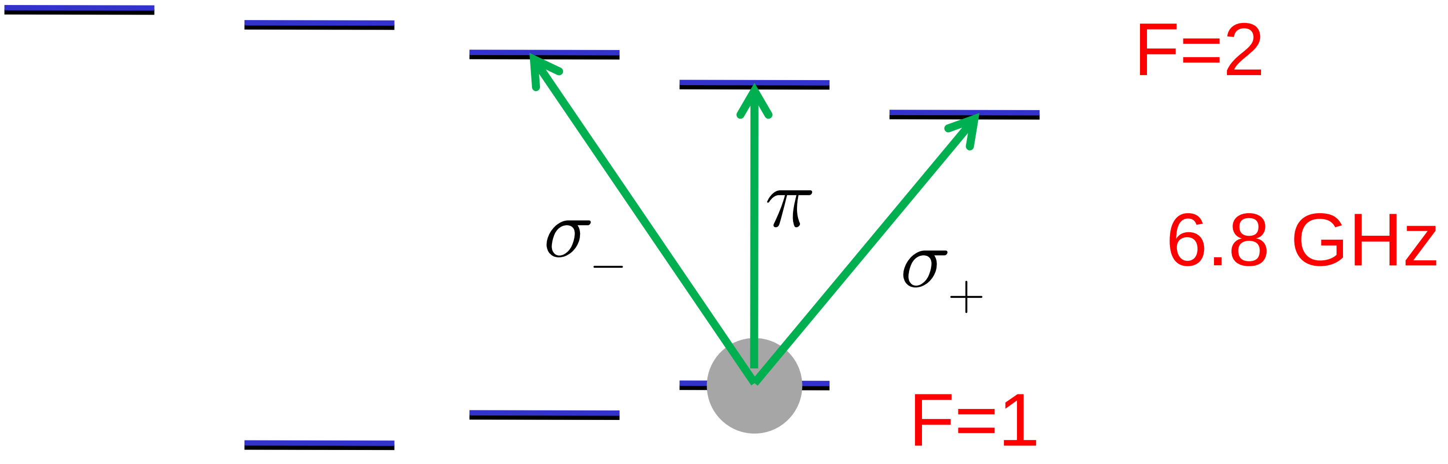
Production of a condensate in any zeeman state of the ground state

Hyperfine levels of the ground state of ^{87}Rb



Production of a condensate in any zeeman state of the ground state

We apply a small magnetic field to lift the degeneracy



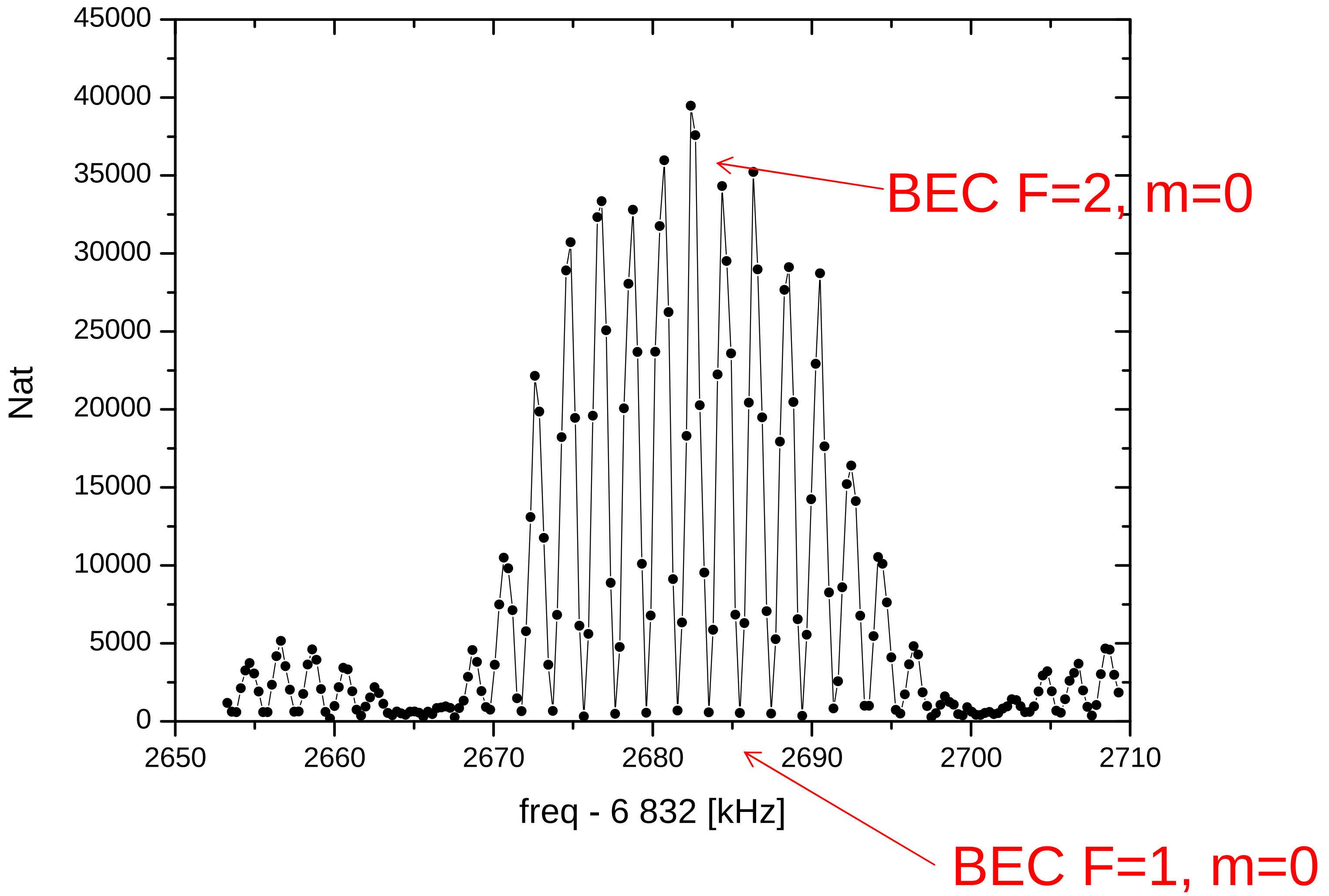
Example: Applying a π-pulse of microwave with a linear polarization along the magnetic field axis

F=1,mF=1 F=2,mF=1

Control of the internal state:

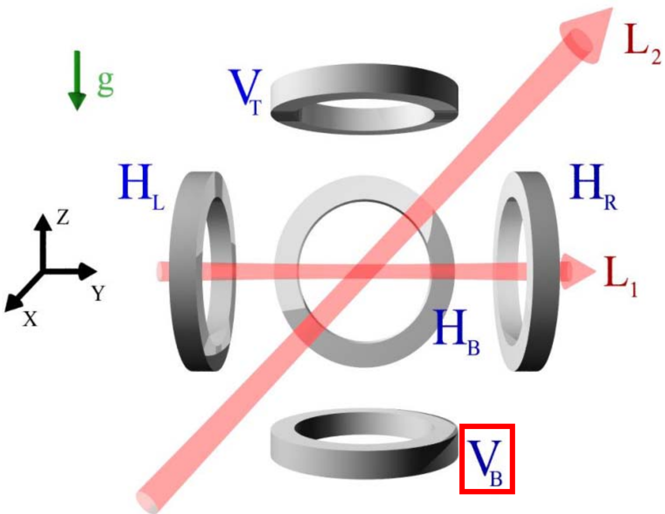
Ramsey fringes with BEC

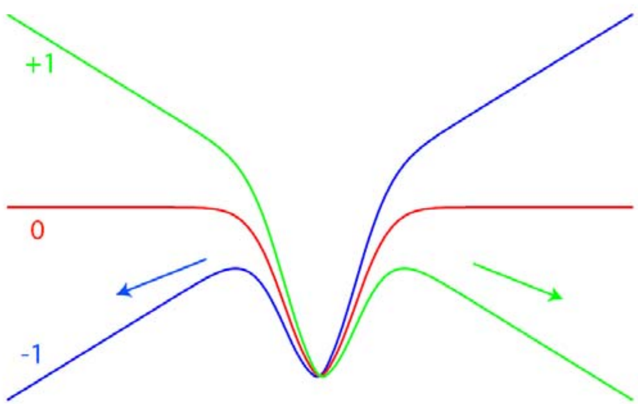
Young's double slit experiment in time domain



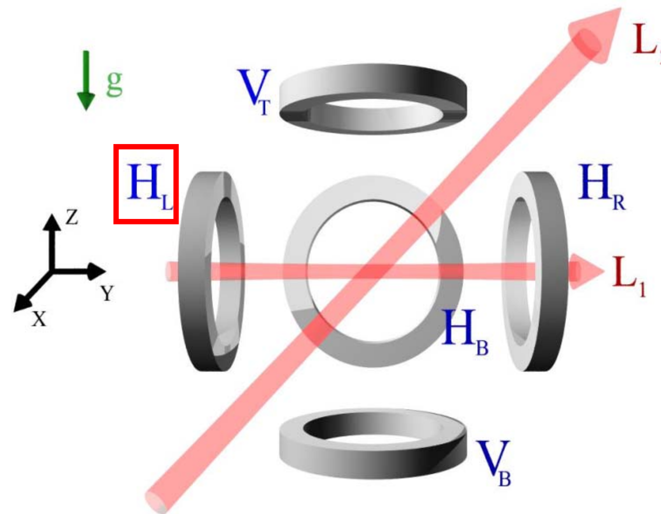
Optically guided atom laser (1)

1) We prepare a BEC in m=+1 or -1





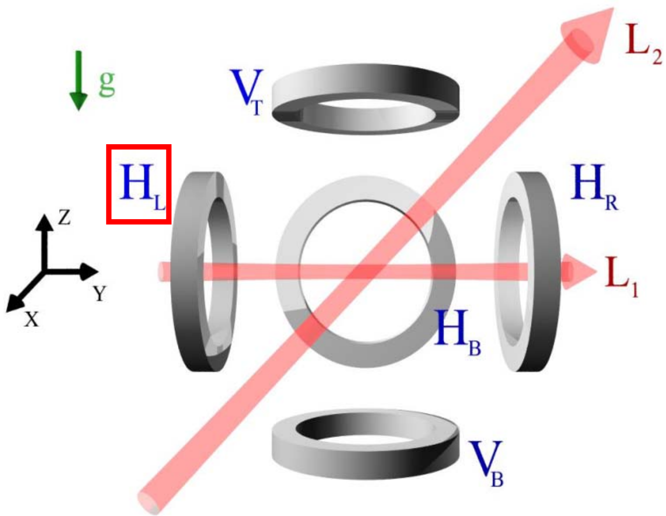
2) We outcouple in the horizontal arm (first order Zeeman effect)



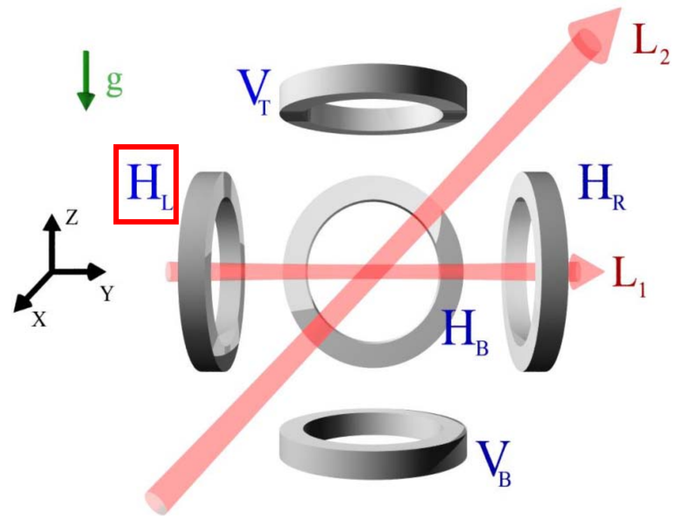
3) A picture is taken after a time of flight

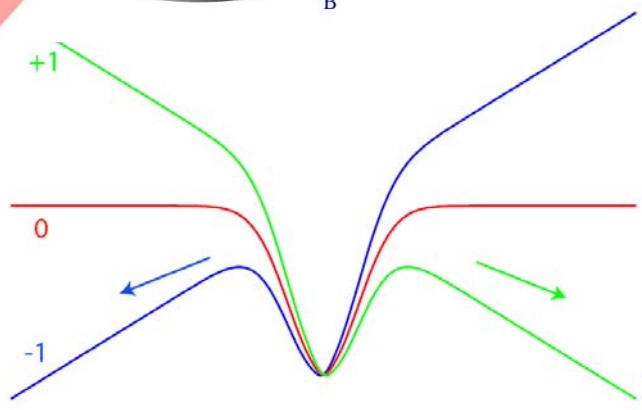
Optically guided atom laser (2)

1) We prepare a BEC in m=0

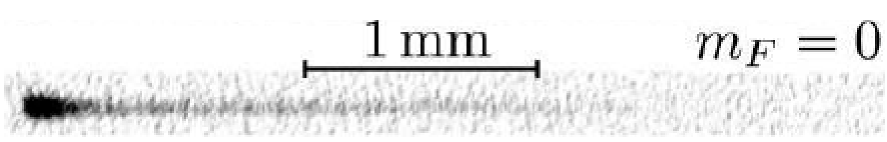


2) We outcouple in the horizontal arm (second order Zeeman effect)

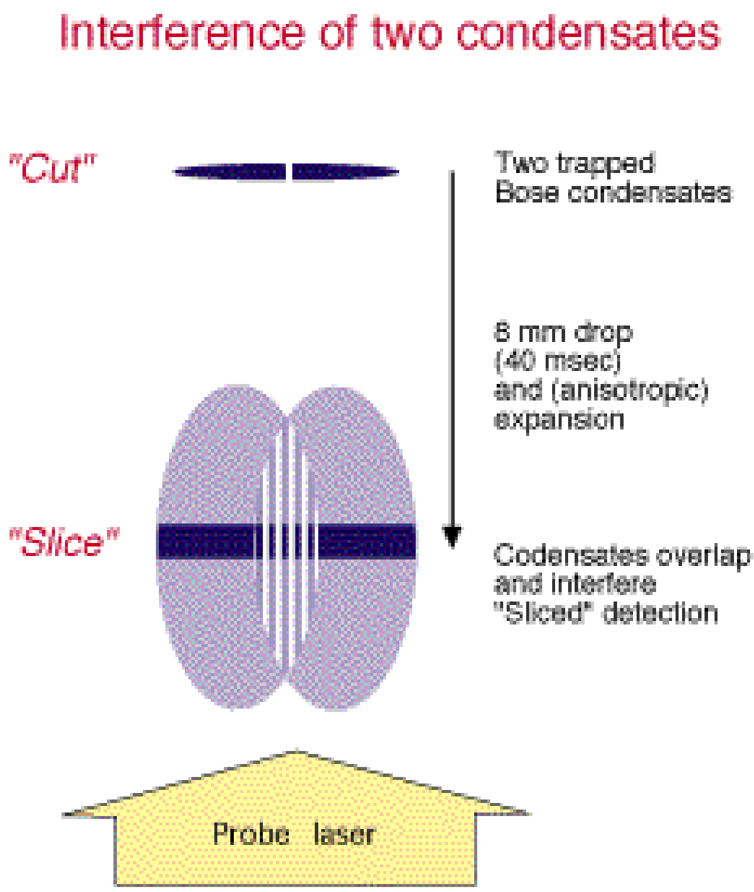


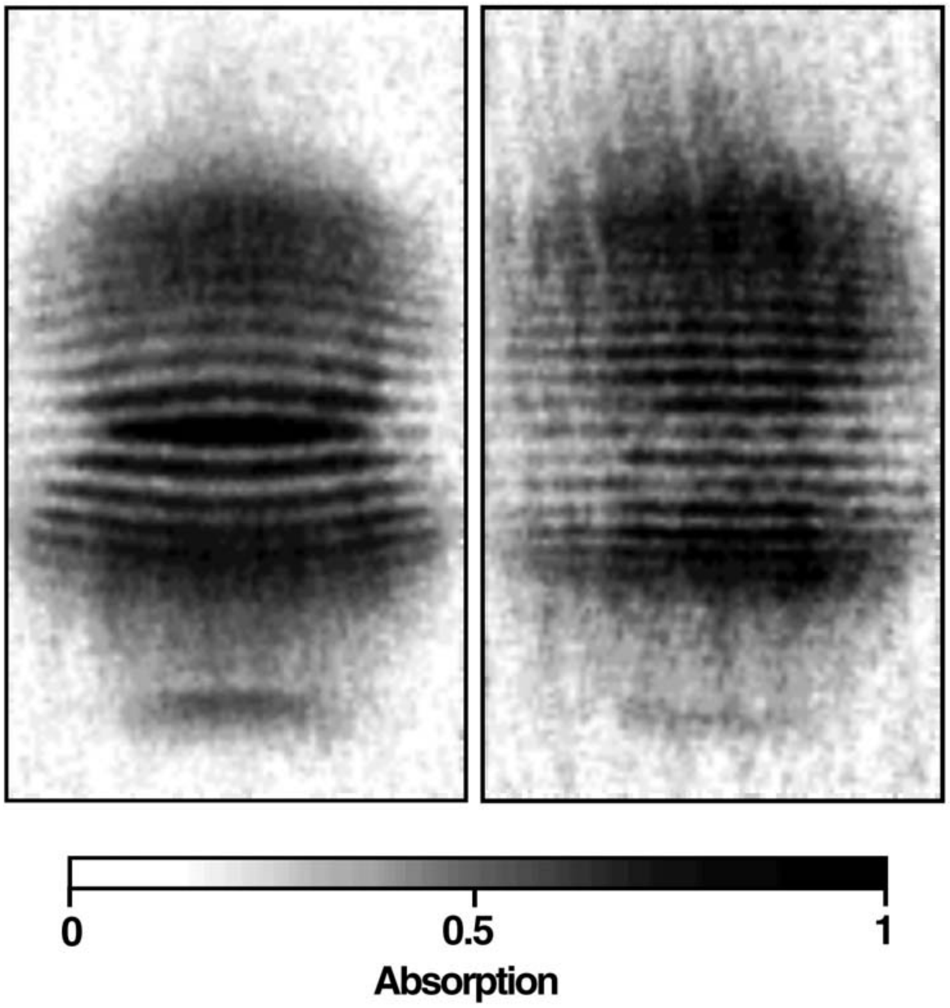


3) A picture is taken after a time of flight



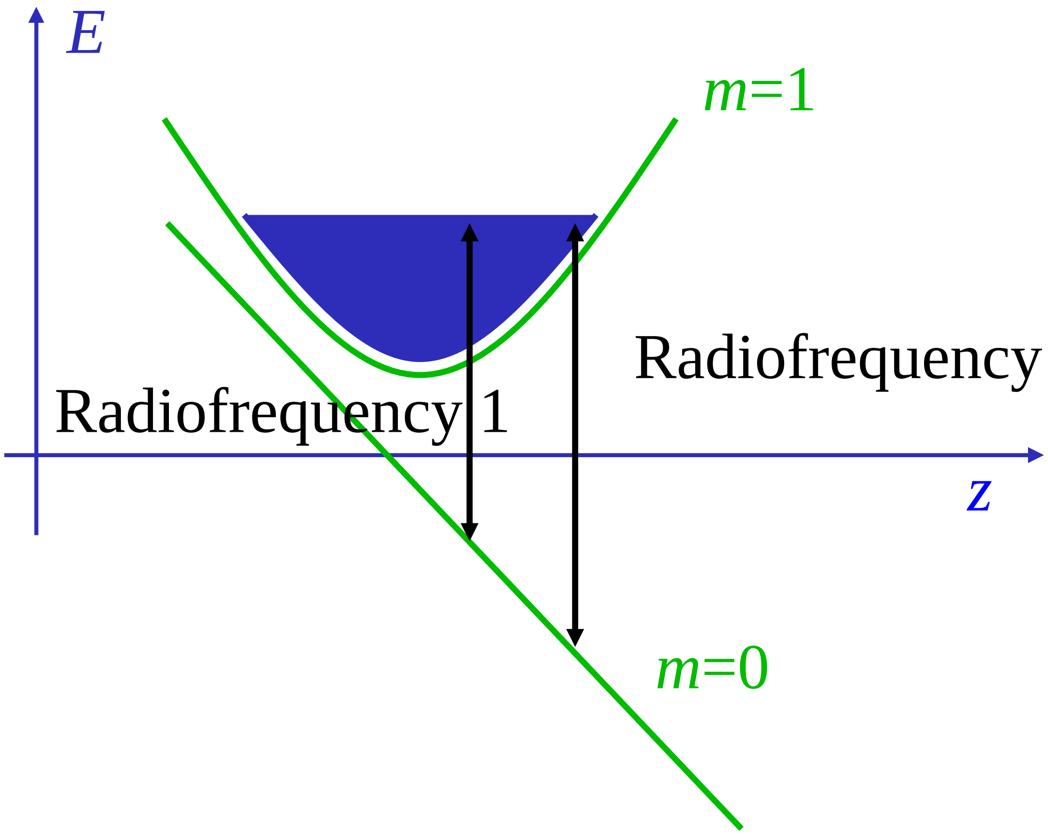
Matter wave interferences at a mesoscopic scale



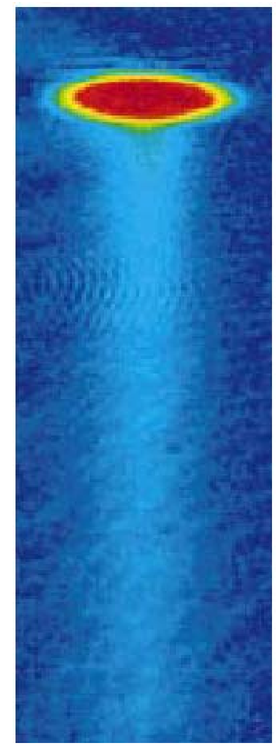


Coherence properties of BEC

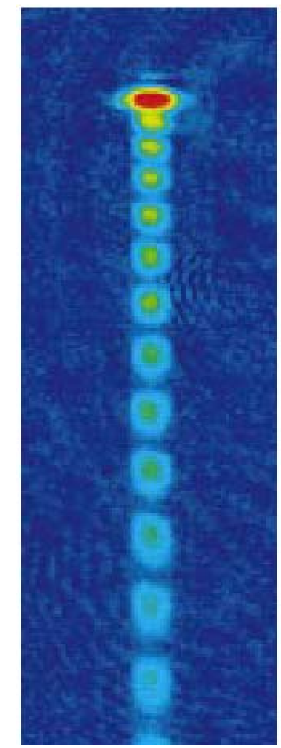
Young's two slit like experiment



$T > T_c$

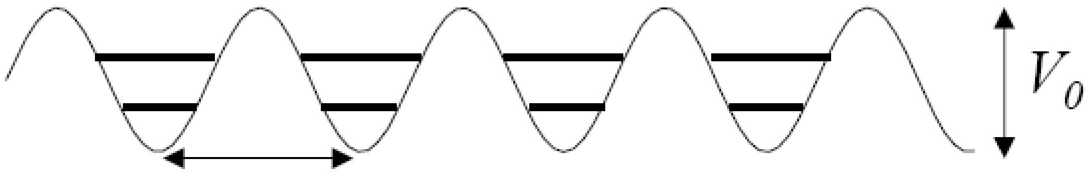


$T < T_c$

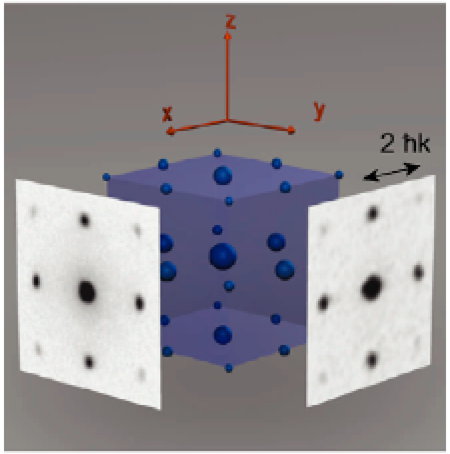


Transition between a BEC state and a Mott insulator

3D periodic potential created by a laser standing wave



$\lambda/2 = \pi/k = 0.4\ \mu\text{m}$

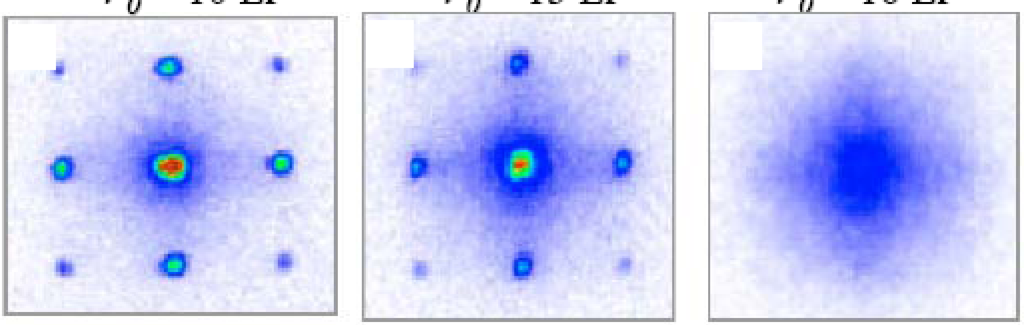


~ one atom per lattice site and 10^5 sites

For small V_0, tunnelling dominates and maintains full coherence over the lattice:

⟶ time of flight with Bragg peaks

V_0 = 10 Er V_0 = 13 Er V_0 = 16 Er



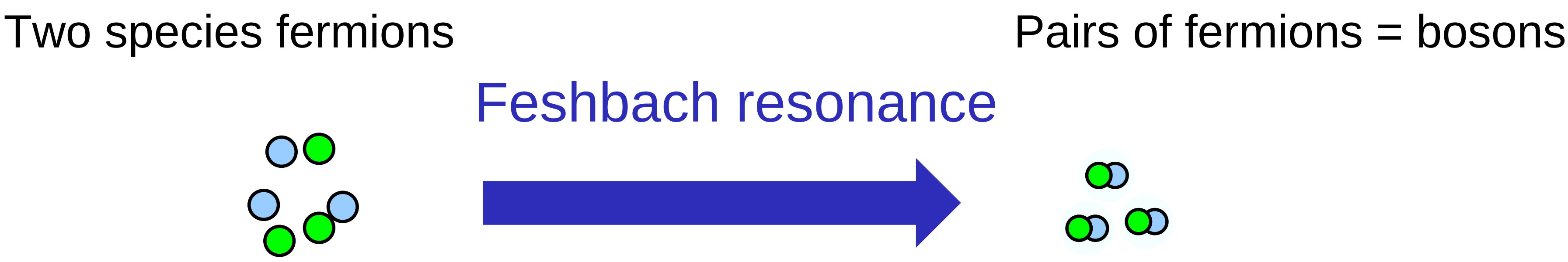
For larger V_0, repulsive interactions dominate over tunnelling:
the system evolves to a state with « exactly » one atom/site

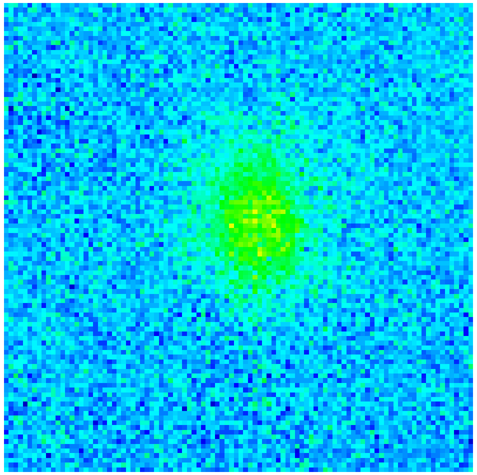
$E_r = \hbar^2 k^2/2m$

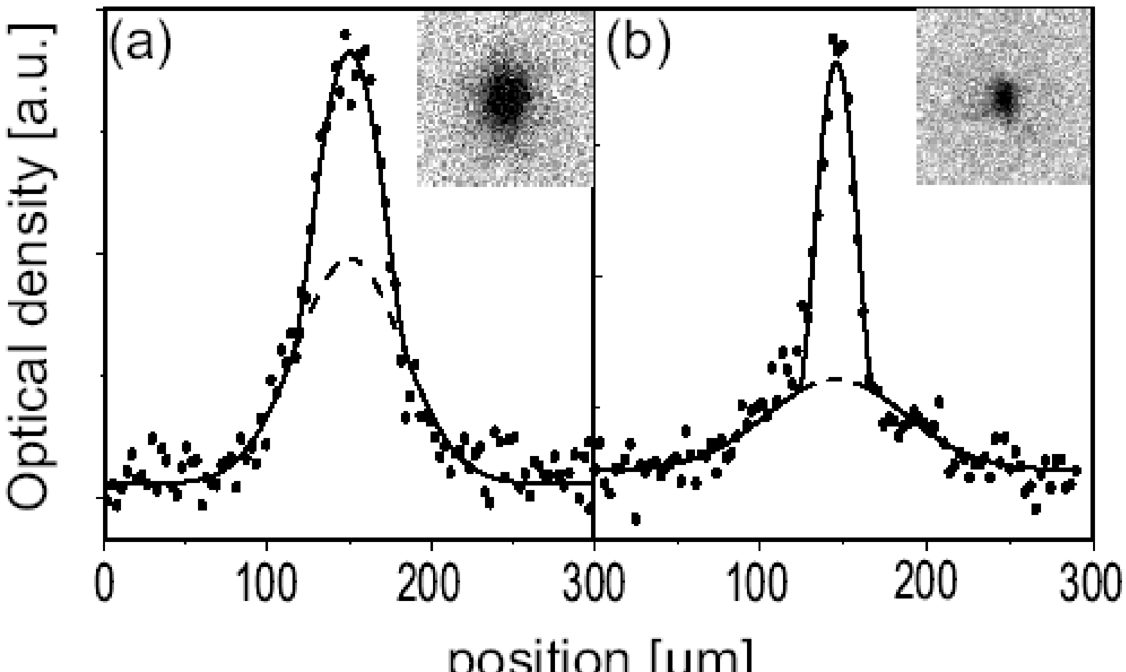
Munich 2002

coherence is lost!

BEC by pairing of two fermions







MIT, ENS, Rice,…