Lecture 1

Physics of light forces and laser cooling

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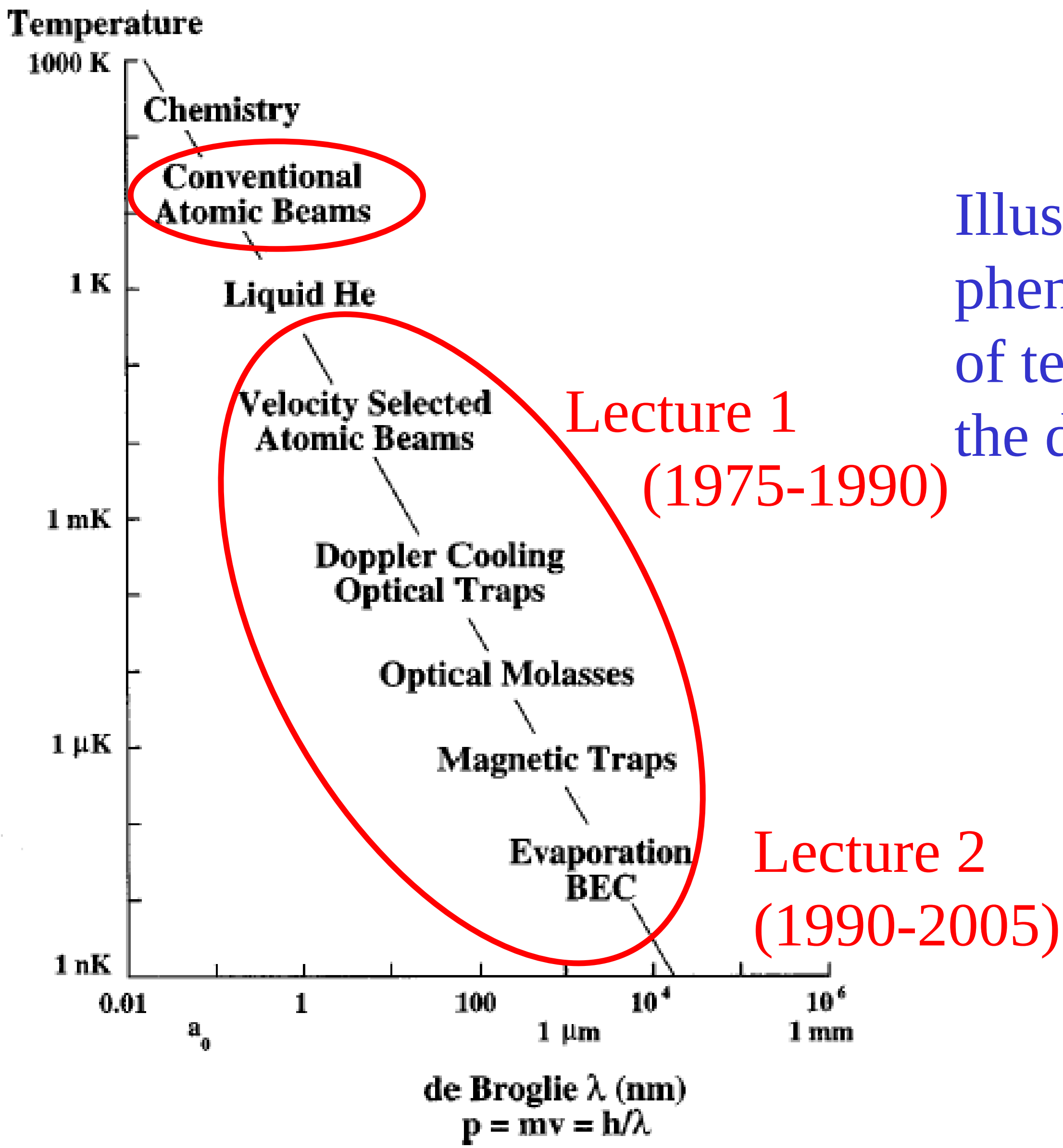
Outline

$$\lambda_{dB} = \frac{h}{\sqrt{2\pi m k_B T}}$$



Illustrative plot of various phenomena along a scale of temperature plotted against the de Broglie Wavelength

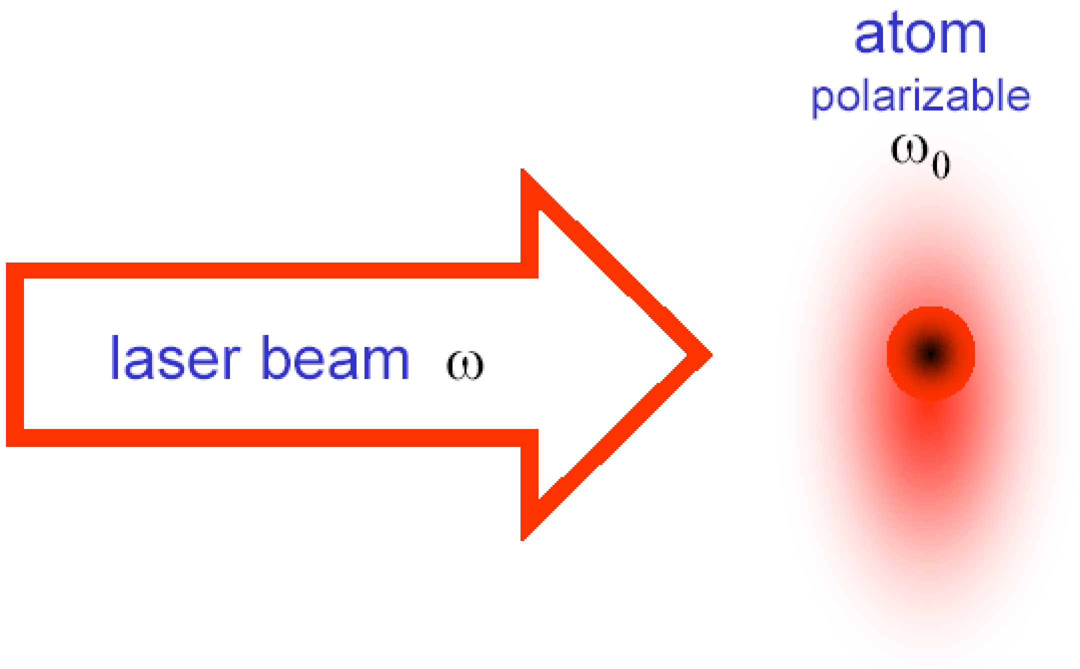
Goal of this lecture:

understand how to use light to manipulate the external degrees of freedom of atoms, with an application to laser cooling

Outline

1. “Classical” picture
2. Elementary processes: Spontaneous and stimulated emissions
3. Forces on an atom at rest
4. The radiation pressure force and the dipole force
5. Doppler cooling
6. The magneto-optical trap
7. Sisyphus cooling

Simple Classical Picture (1)



Atom =nucleus + electron

The electron is harmonically bounded to the nucleus

and subjected to the classical radiation field of the laser

Simple Classical Picture (2)

Driven oscillations of the electron obey the Newton equation:

$$\ddot{x} + \underbrace{\Gamma_\omega \, \dot{x}} + \omega_0^2 x = -\frac{eE(t)}{m_e}$$

Dissipative term according to Larmor well known formula for the power radiated by an accelerated charge:

$$\Gamma_\omega = \frac{e^2 \omega^2}{6\pi\varepsilon_0 m_e c^3}$$

Simple Classical Picture (3)

Physical interpretation = interaction between the laser field and the the atomic dipole that it induces

$$p = (-e)x = \alpha E$$

we deduce the atomic polarizability

$$\alpha = \frac{e^2}{m} \frac{1}{\omega_0^2 - \omega^2 + i\omega\Gamma_\omega}$$

with Γ the on resonance damping rate

$$\alpha = 6\pi\epsilon_0 c^3 \frac{\Gamma/\omega_0^2}{\omega_0^2 - \omega^2 - i(\omega^3/\omega_0^2)\Gamma}$$

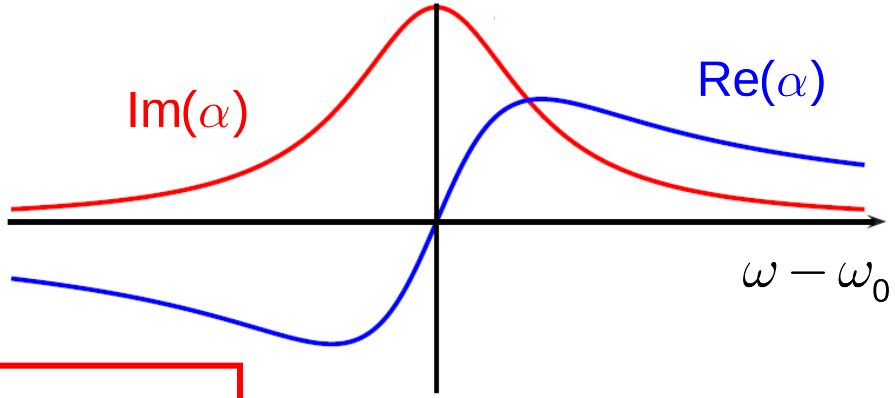
$$\Gamma \equiv \Gamma_{\omega_0} = \left(\frac{\omega_0}{\omega}\right)^2 \Gamma_\omega$$

Simple Classical Picture (4)

Conservative component (Dispersive shape)

$$U_{\mathrm{dip}} = -\frac{1}{2}\langle \mathbf{p}.\mathbf{E} \rangle = -\frac{1}{2\epsilon_0 c}\mathrm{Re}(\alpha) I$$

$$\Rightarrow \quad \mathbf{F}_{\mathrm{dip}} = -\boldsymbol{\nabla} U_{\mathrm{dip}} = \frac{1}{2\epsilon_0 c}\mathrm{Re}(\alpha)\boldsymbol{\nabla} I(\mathbf{r}).$$



Dissipative component (Lorentzian shape)

$$P_{\mathrm{abs}} = \langle \dot{\mathbf{p}}.\mathbf{E} \rangle = 2\omega \mathrm{Im}(p.E^*) = \frac{\omega}{\epsilon_0 c}\mathrm{Im}(\alpha) I$$

$$\Rightarrow \quad \Gamma_{\mathrm{sc}} = \frac{P_{\mathrm{abs}}}{\hbar\omega} = \frac{1}{\hbar\omega} = \frac{1}{\hbar\epsilon_0 c}\mathrm{Im}(\alpha) I(\mathbf{r})$$

Semiclassical approach in quantum mechanics

In a semiclassical approach, the atomic polarizability can be calculated by consideringq a **two-level atom** as a two level quantum system interacting with a classical radiation field.

One finds that, **when saturation effects can be neglected**, the semiclassical calculation yields exactly the same result as the classical calculation with only one modification:

The damping rate can no longer be calculated from Larmor's Formula, but it is determined by the dipole matrix element:

$$\Gamma = \frac{\omega^3}{3\pi\varepsilon_0\hbar c^3}\left|\langle e|\hat{D}|g\rangle\right|^2$$

Two-level atom and elementary processes

Emission

The atom goes from a state e to a lower state g
by emitting a photon h ν

Absorption

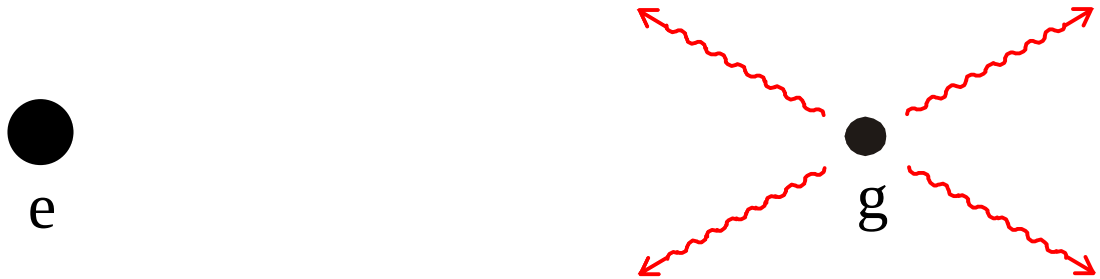
The atom goes from g to e by absorbing a photon h ν

In his attempts to derive Planck's law for blackbody radiation from an analysis of the energy exchanges between a 2-level atom and a radiation in thermal equilibrium, Einstein was led in 1917 to introduce **2 types of emission**

Spontaneous emission of a photon (dissipative)

An atom does not remain indefinitely in the excited state e. After a finite time τ_R, it falls down to the ground state g by spontaneously emitting a photon in all possible directions.

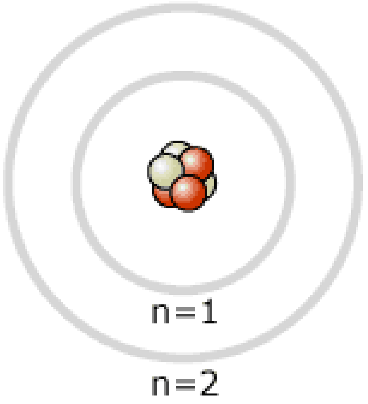
τ_R : Radiative lifetime of e, on the order of 10^{-8} s



Stimulated emission of a photon (conservative)

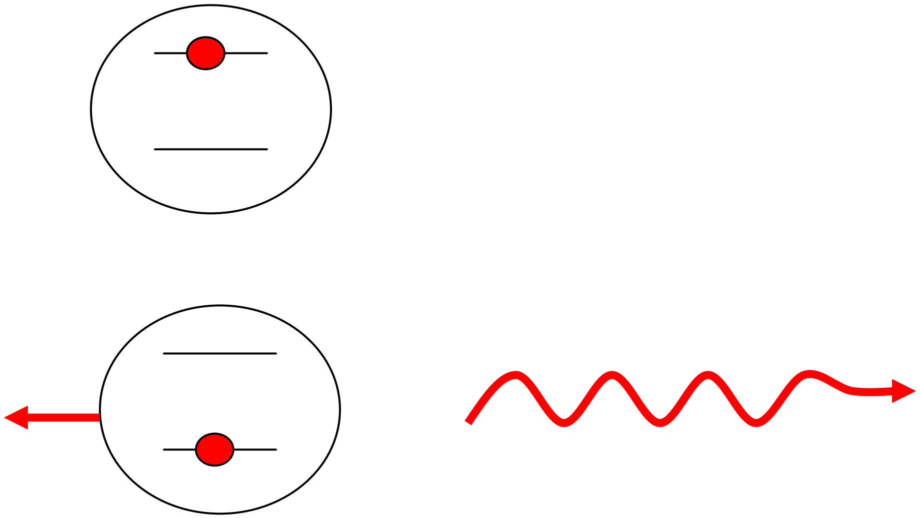
A photon with energy $h\,\nu = E_e - E_g$, impinging on an atom in the excited state e stimulates (or induces) this atom to emit a photon exactly identical to the impinging photon (same energy, same direction of propagation, same polarization)

Light forces: momentum exchange



The absorption and emission are accompanied with a momentum exchange.

Example with emission



The recoil velocity of the atom, assumed initially at rest, is $v_R = \dfrac{\hbar k}{m}$

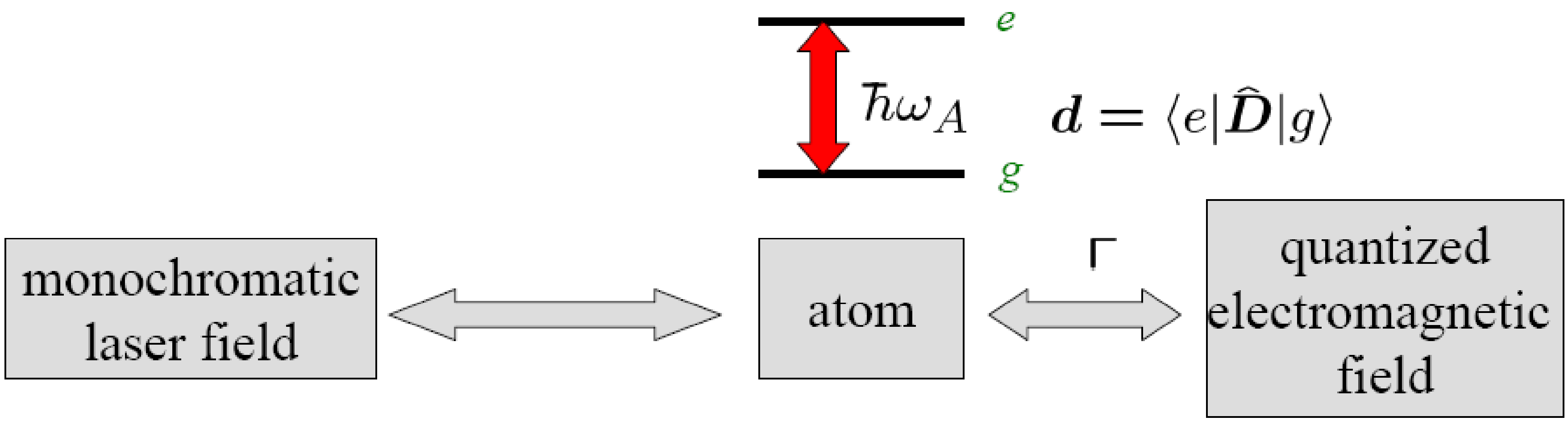
Example: $v_R = 6\ \text{mm.s}^{-1}$ for rubidium atoms

How should we proceed to describe quantum mechanically the force exerted by quasi-resonnant light on atoms without any restriction on saturation ?

1 – internal degrees of freedom (*g* and *e*)

2 – external degrees of freedom ($\hat{R}$ and $\hat{P}$)

The systems in interaction



$$\boldsymbol{d} = \langle e|\widehat{\boldsymbol{D}}|g\rangle$$

Laser field: $\boldsymbol{E}_L(\boldsymbol{r},t) = \frac{1}{2}\mathcal{E}_L(\boldsymbol{r})\left(\boldsymbol{\epsilon}_L(\boldsymbol{r})\, e^{-i\omega_L t - i\phi(\boldsymbol{r})} + \text{c.c.}\right)$

Electric dipole coupling between the atom and the laser field

Rabi frequency: $\hbar\Omega_1(\boldsymbol{r}) = -\left(\boldsymbol{d}\cdot\boldsymbol{\epsilon}_L(\boldsymbol{r})\right)\,\mathcal{E}_L(\boldsymbol{r})$

One often measures the laser intensity I in term of the saturation intensity I_s :

$$\frac{I}{I_s} = \frac{2\Omega_1^2}{\Gamma^2}$$

typical I_s: a few mW/cm^2

The hamiltonian of the problem

Total hamiltonian: $\hat{H} = \hat{H}_A + \hat{V}_{AL} + \hat{H}_R + \hat{V}_{AR}$

$\Longrightarrow$ $\hat{H}_A = \dfrac{\hat{\boldsymbol{P}}^2}{2m} + \hbar\omega_A |e\rangle\langle e|$

atomic center-of-mass kinetic energy + internal energy

$\Longrightarrow$ $V_{AL}(t) = -\hat{\boldsymbol{D}}\cdot\boldsymbol{E}_L(\hat{\boldsymbol{R}},t) \simeq \dfrac{\hbar\Omega_1(\hat{\boldsymbol{R}})}{2}\left(|e\rangle\langle g|\, e^{-i\omega_L t - i\phi(\hat{\boldsymbol{R}})} + h.c.\right)$

rotating wave approximation

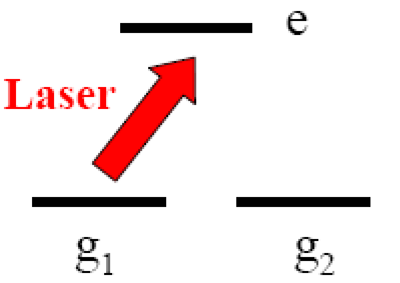
$\Longrightarrow$ $\hat{H}_R$ *: collection of harmonic oscillators*

$\Longrightarrow$ $\hat{V}_{AR}$ *: responsible for spontaneous emission and Lamb shift, no need to write it explicitly.*

The timescales of the problem

- Internal atomic variables:
for a two-level atom, the steady state is reached in a time $T_{\text{int}} \sim \Gamma^{-1}$
irrespective of the intensity and detuning of the laser.

This would not be true for a more complex atomic transition where long optical pumping times may appear :



- External atomic variables: how long does it take to get the atom out of resonance with the laser beam?

$$k\,\Delta v \sim \Gamma$$
$$\delta v|_{\text{per photon}} = \hbar k/m \quad \Longrightarrow \quad \frac{m\Gamma}{\hbar k^2} \text{ photons needed}$$

time Γ^{-1} per photon $\Longrightarrow$ $T_{\text{ext}} \sim \dfrac{m}{\hbar k^2}$

The broadline condition

For most atomic lines: $\frac{m\Gamma}{\hbar k^2} \gg 1$ (broad line condition)

$m\Gamma/(\hbar k^2) \sim 800$ for the resonance line of rubidium atoms

This means that one needs the absorption and the emission of several photons to get the atom out of resonance because of Doppler effect

$$\Longrightarrow \quad T_{\text{int}} \ll T_{\text{ext}}$$

Concept of mean force and Heisenberg inequalities

To define the concept of mean force $\mathcal{F}$(x) in a given point x, we need to atomic wave packets sufficiently well localized in position and velocity:

$$\Delta x \ll 1/k_L \qquad k_L \Delta \mathrm{v} \ll \Gamma \qquad \Rightarrow \qquad \Delta x . \Delta \mathrm{v} \ll \Gamma / k_L^2$$

This localization condition is compatible with Heisenberg relation

$$\Delta x . \Delta \mathrm{v} \geq \hbar / M$$

only if : $\Gamma / k_L^2 \gg \hbar / M$

One recovers the condition : $\hbar \Gamma \gg E_{\mathrm{rec}}$

or in other terms $T_{\mathrm{int}} \ll T_{\mathrm{ext}}$

The force acting on an atom

Equations of motion in Heisenberg point of view:

$$\frac{d\hat{\boldsymbol{R}}}{dt} = \frac{1}{i\hbar}[\hat{\boldsymbol{R}}, \hat{H}] = \frac{\hat{\boldsymbol{P}}}{m}$$

$$\frac{d\hat{\boldsymbol{P}}}{dt} = \frac{1}{i\hbar}[\hat{\boldsymbol{P}}, \hat{H}] = -\boldsymbol{\nabla}\hat{V}_{AL}(t) - \boldsymbol{\nabla}\hat{V}_{AR} = \hat{\boldsymbol{F}}$$

We now take the average over the atomic internal state

The average force

➡ The coupling with the vacuum field does not contribute.

➡ Contribution of the coupling with the laser field:

$$\boldsymbol{F} = -\left\langle \boldsymbol{\nabla} \widehat{V}_{AL}(t) \right\rangle = \left\langle \sum_{i=x,y,z} \widehat{D}_i \, \boldsymbol{\nabla} E_{Li}(\widehat{\boldsymbol{R}}, t) \right\rangle$$

$$\simeq \sum_{i=x,y,z} \langle \widehat{D}_i \rangle(t) \, \boldsymbol{\nabla} E_{Li}(\boldsymbol{R}(t), t)$$

The evolution of the atomic dipole is obtained using the optical Bloch equations (OBEs): evolution of the 2x2 atomic density matrix

The OBEs are solved (analytically or numerically) for a given position and a given velocity of the atom

The forces on an atom at rest

The laser field can have an intensity gradient and/or a phase gradient.

Within the framework of the two-level atom model, polarization gradients are meaningless, but may appear for more realistic atomic structures (Sisyphus cooling)

- The ***radiation pressure force*** originates from the phase gradient. Consider for simplicity a plane running wave $\phi(\boldsymbol{R}) = -\boldsymbol{k}_L \cdot \boldsymbol{r}$

$$\boldsymbol{F}_{\mathrm{RP}} = \frac{\hbar \boldsymbol{k}_L \Gamma}{2} \frac{s}{1+s} \qquad s = \frac{\Omega_1^2/2}{\delta^2 + \Gamma^2/4}$$

- The ***dipole force*** originate from the intensity gradient

$$\boldsymbol{F}_{\mathrm{dip}}(\boldsymbol{R}) = -\frac{\hbar\delta}{2} \frac{\boldsymbol{\nabla} s(\boldsymbol{R})}{1+s(\boldsymbol{R})}$$

Potential related to this force: $U_{\mathrm{dip}}(\boldsymbol{R}) = \frac{\hbar\delta}{2} \ln(1+s(\boldsymbol{R}))$

The radiation pressure force

resonant photons

atom

$$\boldsymbol{F}_{\mathrm{RP}} = \frac{\hbar \boldsymbol{k}_L \Gamma}{2} \frac{s}{1+s}$$

- The atom undergoes a succession of absorption – spontaneous emission cycles. Each cycle changes the atom momentum by $\hbar k = m v_{\mathrm{rec}}$

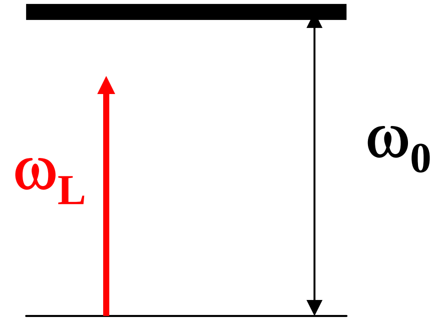
- Fluorescence rate (obtained from EBOs): $\gamma_{\mathrm{fluo}} = \frac{\Gamma}{2} \frac{s}{1+s}$

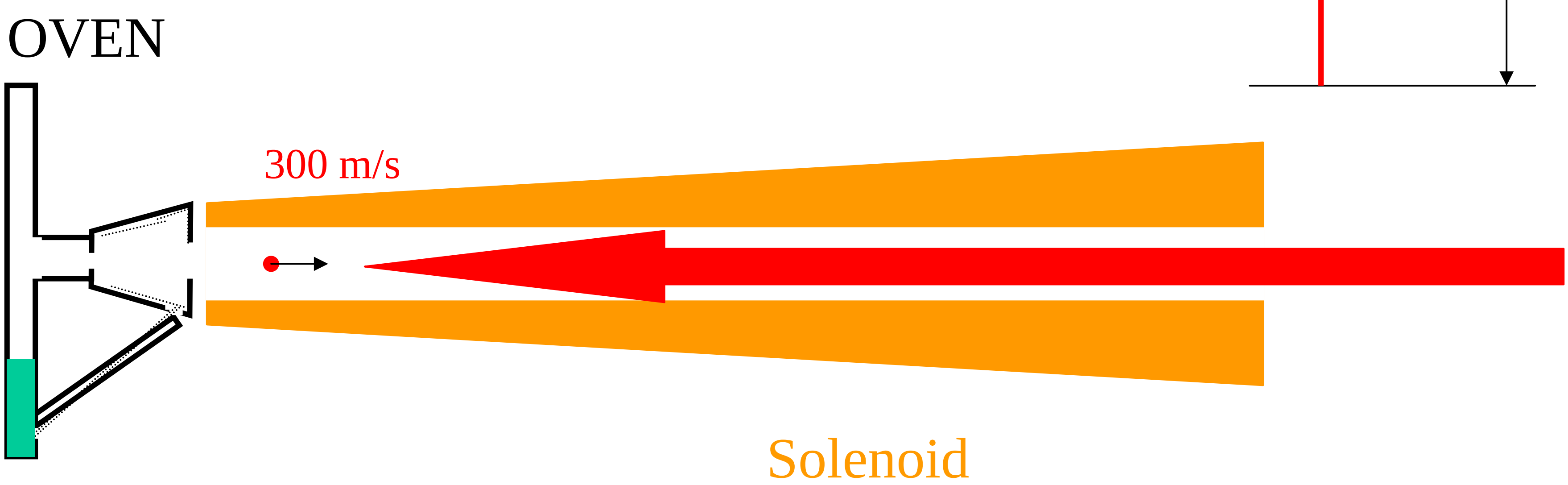
Force: $F_{\mathrm{RP}} = \hbar k \, \gamma_{\mathrm{fluo}}$ maximal value: $F_{\mathrm{RP,max}} = \hbar k \frac{\Gamma}{2}$

For sodium atoms, $a_{\mathrm{max.}}$ is 100 000 times larger than gravity

➡ Atoms moving at 100 m/s can be stopped over 1cm!

Slowing down an atomic beam



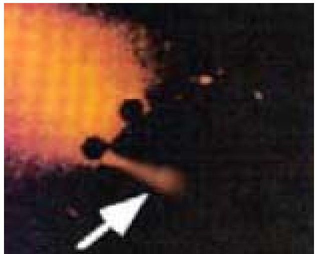


The atom is progressively out of resonance because of Doppler effect

300 m/s = 50000 x 6 mm/s
The atoms are stopped
After a one meter interaction
With the laser

$\omega_L + kv$ $\quad -\mu B(z)$

Zeeman slower



A stopped sodium beam

W. Phillips, 1985

The dipole force at low intensity

The expression for the dipole potential can be simplified when $s \ll 1$:

$$U_{\text{dip}}(\boldsymbol{R}) = \frac{\hbar\delta}{2}\ \ln(1 + s(\boldsymbol{R})) \simeq \frac{\hbar\delta}{2} s(\boldsymbol{R}) \qquad s = \frac{\Omega_1^2/2}{\delta^2 + \Gamma^2/4}$$

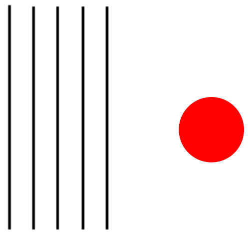
Consider in addition a detuning δ much larger than the natural linewidth Γ

$$s(\boldsymbol{R}) \simeq \frac{\Omega_1^2(\boldsymbol{R})}{2\delta^2} \quad \Longrightarrow \quad U_{\text{dip}}(\boldsymbol{R}) \simeq \frac{\hbar\Omega_1^2(\boldsymbol{R})}{4\delta}$$

- Depending on the sign of δ, the potential can be attractive or repulsive
- The scattering rate of photons γ_{fluo} varies as Ω_1^2/δ^2

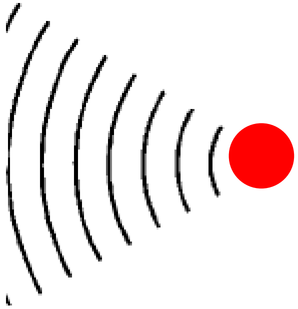
By going to large intensities and large detunings, one can keep U_{dip} constant and decrease γ_{fluo} to an arbitrary small value: **conservative force**

Physical interpretation #1 of the dipole force



Plane wave, no dipole force

absorption - stimulated emission occurs
but with no consequence on the atom motion



$$\vec{\nabla} I \neq \vec{0}$$

Dipole force is exerted on the atom

Many wave vectors are involved

Physical interpretation #1 of the dipole force

If the laser wave is a superposition of several plane waves

$$\vec{k}_i,\ \omega_i = \omega_L \qquad (i = 1, 2, 3, \ldots)$$

the atom can absorb one photon in the wave *i* and emit, in a stimulated way, one photon in another wave $i \neq j$

No energy is absorbed from the laser wave in such a cycle since

$$\hbar\omega_i = \hbar\omega_j = \hbar\omega_L$$

But since $\hbar\vec{k}_i \neq \hbar\vec{k}_j$, the atomic momentum changes by an amount

$$\hbar\left(\vec{k}_i - \vec{k}_j\right)$$

The reactive force is thus a force due to a « redistribution » of photons between the various plane waves forming the laser wave. This is why it is called also « redistribution force »

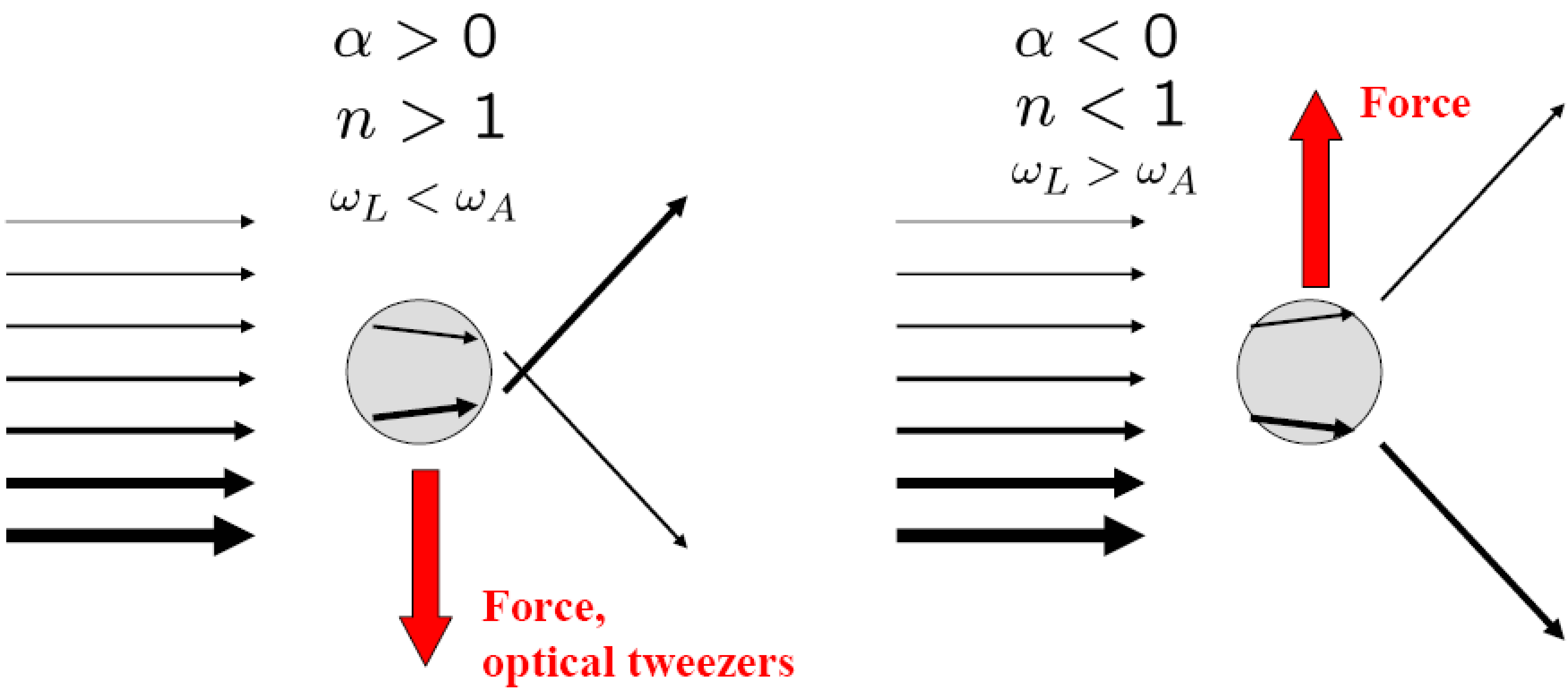
The sense of the redistribution $(i \to j \text{ or } j \to i)$ depends of the relative phase between the 2 waves i and j at the atom position.

Redistribution is a coherent process

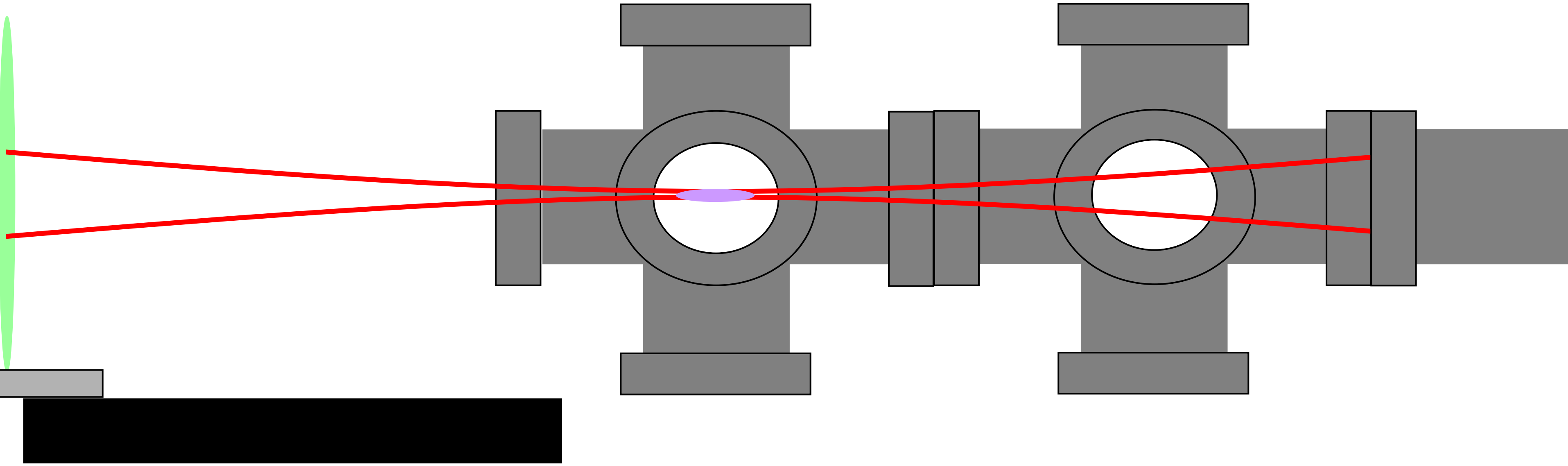
Physical interpretation #2 of the dipole force

Induced dipole $\vec{D} = \alpha(\omega)\ \vec{E}$

➡ Interaction energy: $V(\vec{r}) = -\frac{1}{2}\alpha(\omega)\ E^2(\vec{r})$

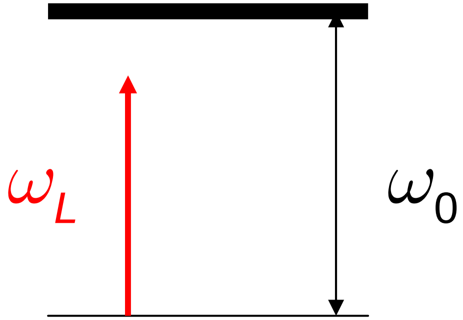


Transport of a packet of cold atoms



Yb-fiber laser: 100 W

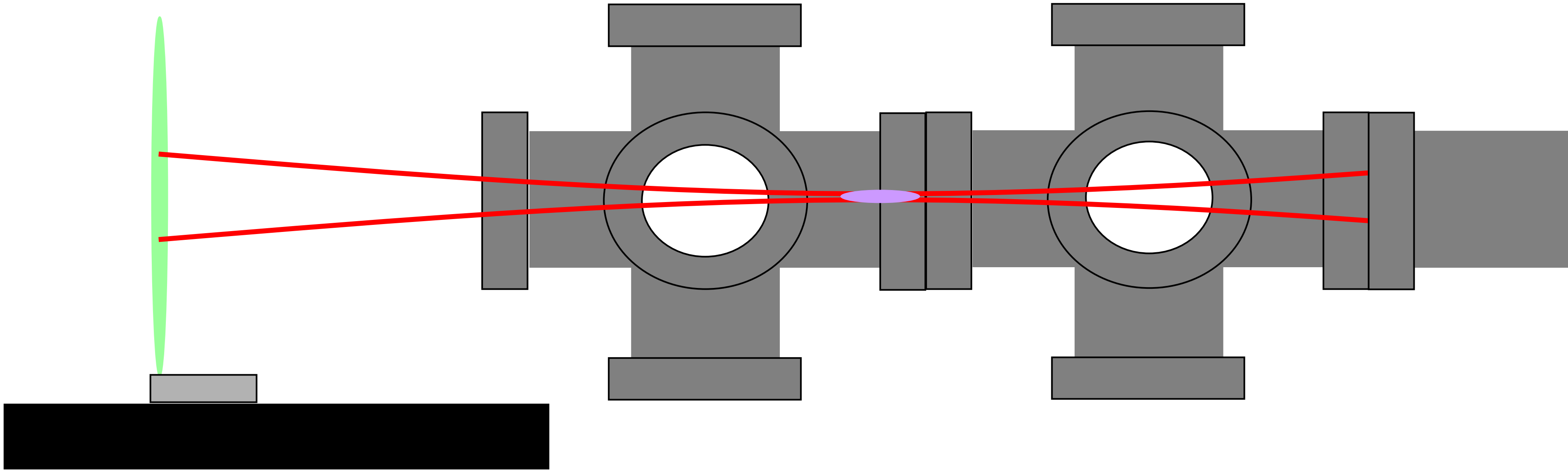
$\lambda_L = 1070$ nm



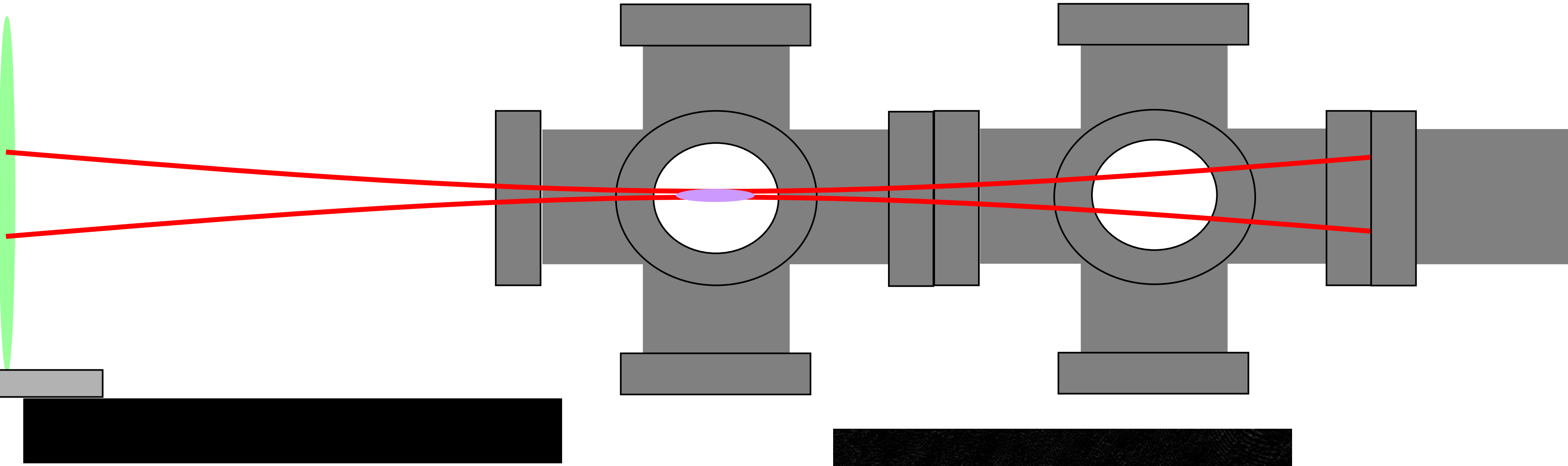
Rubidium atoms

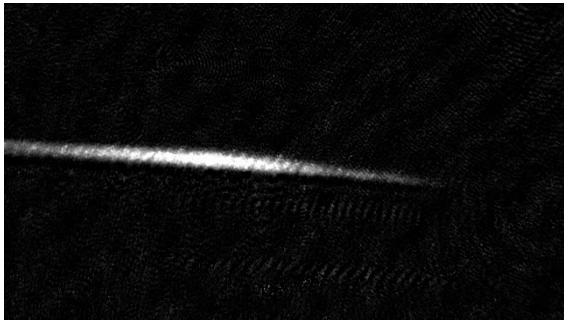
$\lambda_0 = 780$ nm

Transport of a packet of cold atoms



Transport of a packet of cold atoms





Non adiabatic transport:

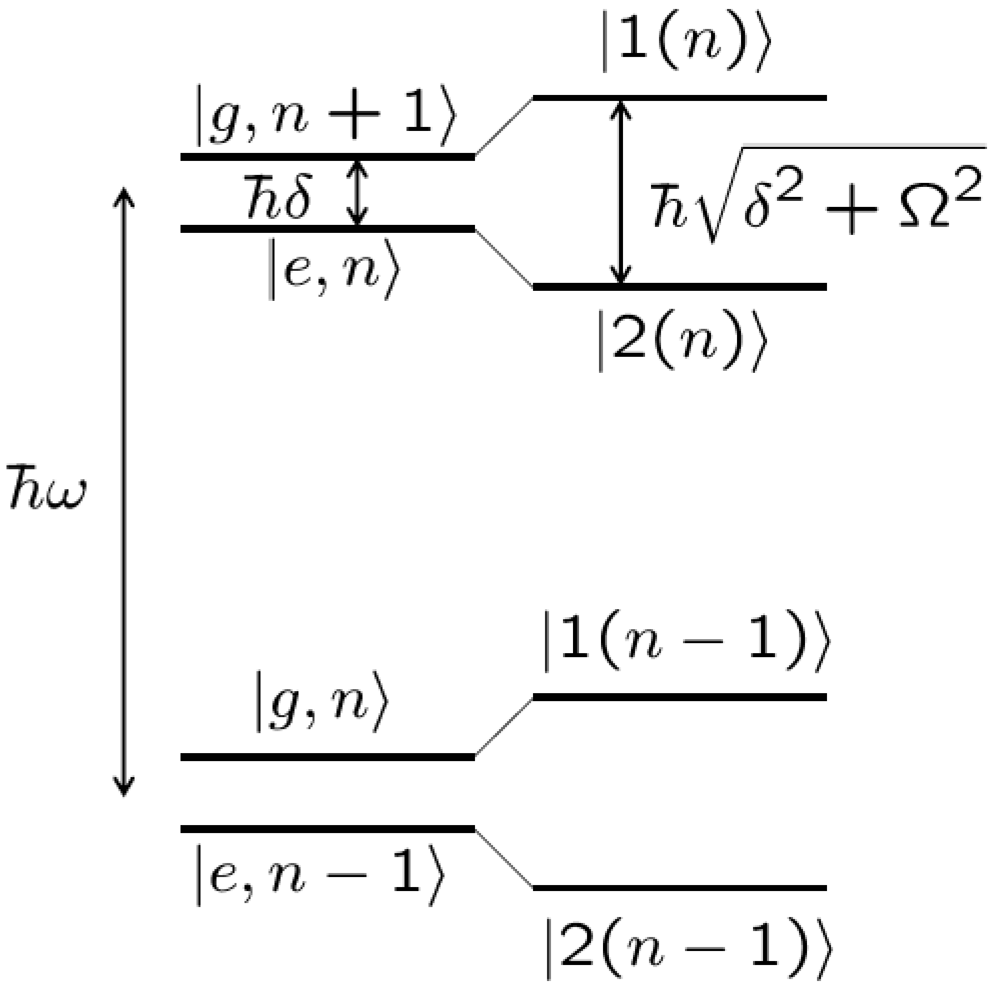
A. Couvert, T. Kawalec, G. Reinaudi and D. Guéry-Odelin, Europhys. Lett. **83**, 13001 (2008).

Physical interpretation #3 of the dipole force: dressed atom approach

Consider the combined system formed by the atom+the laser mode

$0 < \delta = \omega - \omega_A \ll \omega$

$\Omega = d\mathcal{E}/\hbar$: Rabi frequency



For $\Omega \ll \delta$

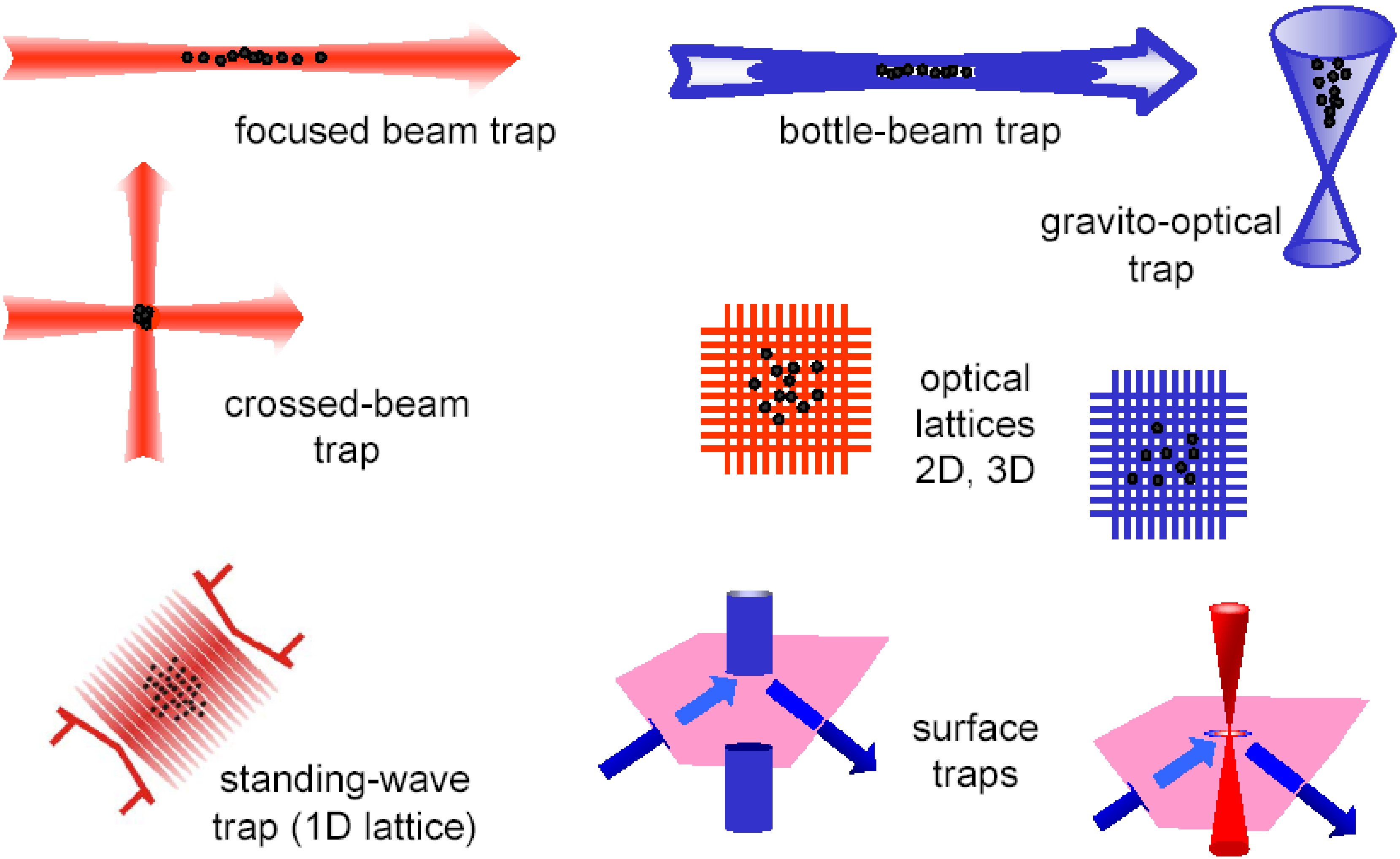
$$|1(n)\rangle \simeq |g, n+1\rangle$$

and the shift is:

$$\Delta E_g = \frac{\hbar}{2}\left(\sqrt{\delta^2+\Omega^2} - \delta\right)$$

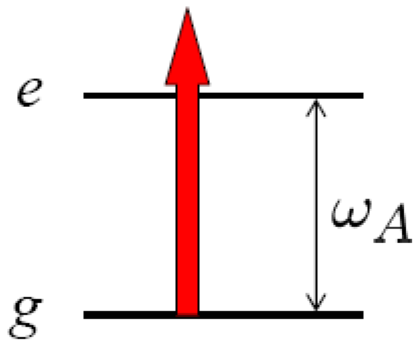
$$\simeq \frac{\hbar\Omega^2}{4\delta} \propto \frac{\text{intensity}}{\text{detuning}}$$

Dipole trap gallery



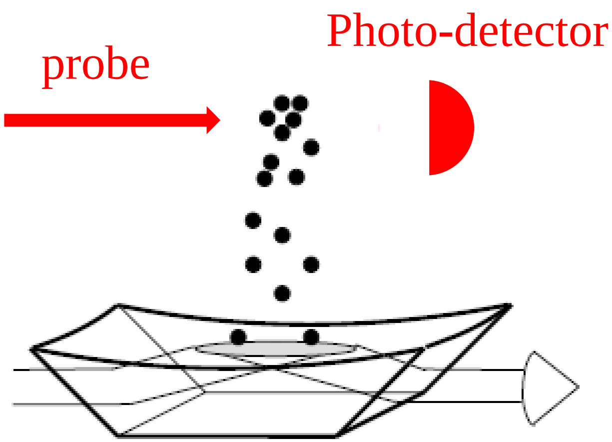
extremely versatile tools → many interesting applications !!!

Atomic mirror with blue detuned evanescent light

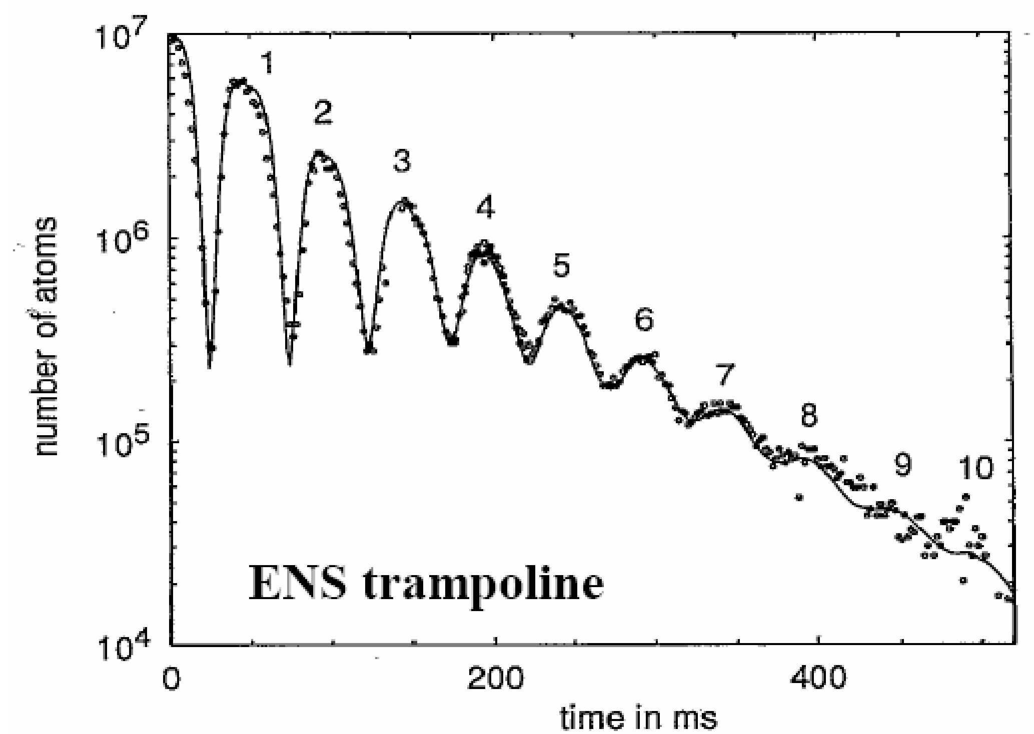


$$\omega > \omega_A$$

$$U_{\text{dip}}(\boldsymbol{R}) \simeq \frac{\hbar\Omega_1^2(\boldsymbol{R})}{4\delta}$$



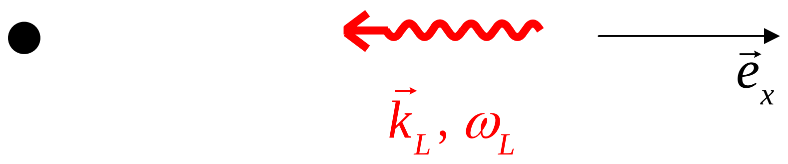
Blue evanescent wave
at the surface of the prism



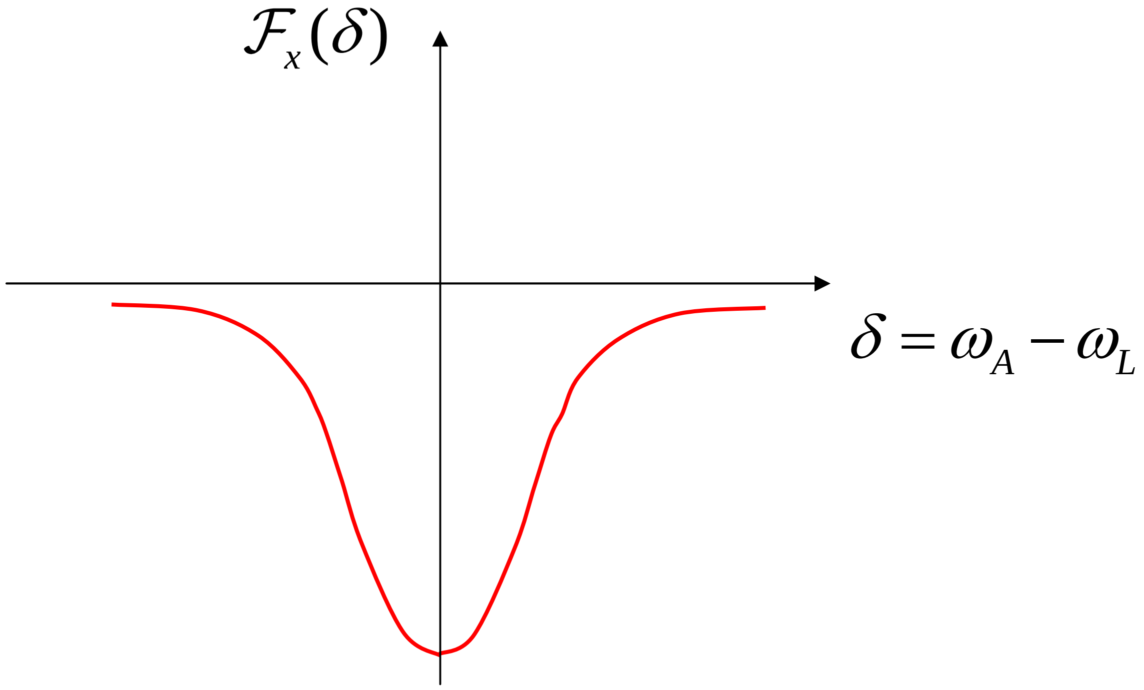
C. G. Aminoff, A. M. Steane, P. Bouyer, P. Desbiolles, J. Dalibard, and C. Cohen-Tannoudji, Phys. Rev. Lett. **71**, 3083 (1993)

Radiation pressure force

Plane wave



$$\vec{\mathcal{F}} = \langle \vec{F} \rangle = \frac{\hbar \vec{k}_L \Gamma}{2} s(\delta)$$



$$s(\delta) = \frac{\Omega_1^2 / 2}{\delta^2 + \Gamma^2 / 4}$$

Atom moving in a plane wave. Doppler effect

The atomic center of mass moves in a plane wave.

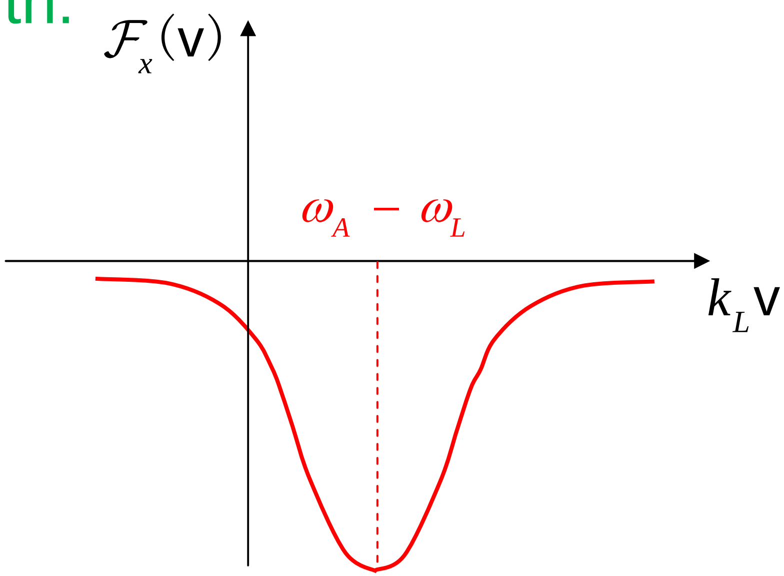
$$\vec{R} = \vec{r}_0 + \vec{v}\, t = \vec{v}\, t \qquad (\text{we take } \vec{r}_0 = \vec{0})$$

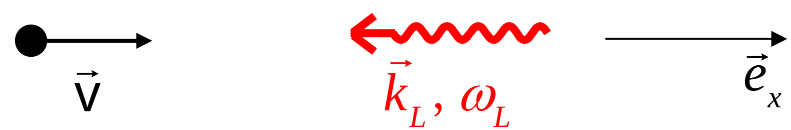
The velocity v is considered as fixed.

<u>Laser wave</u> $\omega_L \rightarrow \omega_L - \vec{k}_L.\vec{v}$ Doppler effect

$$\vec{E}_L\left(\vec{R},t\right) = \vec{e}_L\ \mathcal{E}_L \cos\left[\omega_L t - \vec{k}_L.\vec{R}\right] = \vec{e}_L\ \mathcal{E}_L \cos\left(\omega_L - \vec{k}_L.\vec{v}\right)t$$

Optical Bloch equations keep the same form as for an atom at rest with:





Couterpropagating atom and laser

$$\vec{k}_L = -k_L\ \vec{e}_x$$
$$\vec{k}_L.\vec{v} = -k_L\ \mathrm{v}$$

Resonant excitation when:

$$\omega_L - \vec{k}_L.\vec{v} = \omega_L + k_L \mathrm{v} = \omega_A$$

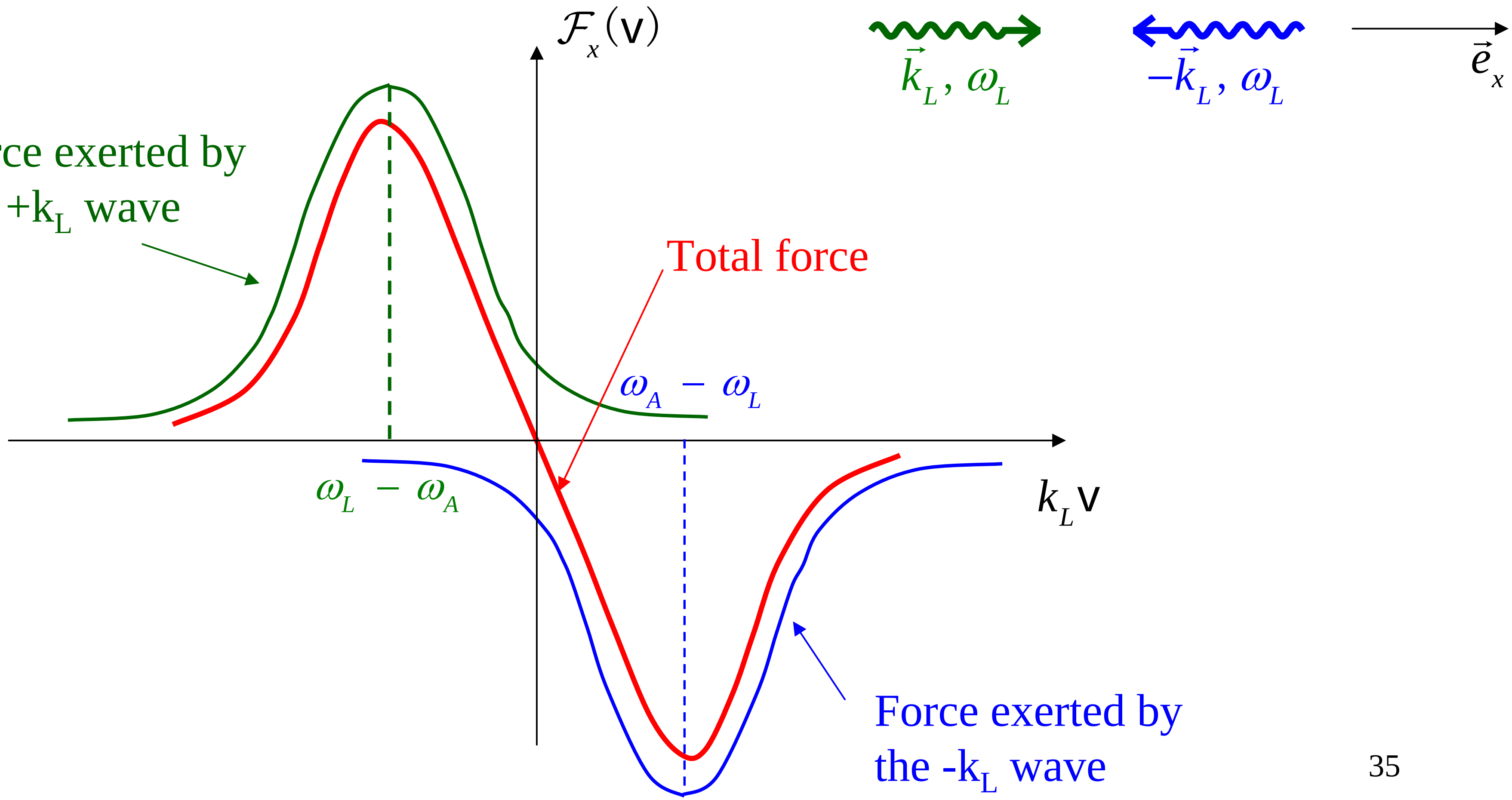
We suppose here : $\omega_L < \omega_A$
(red detuning)

Hänsch-Schawlow
Wineland-Dehmelt
1975

Principle of Doppler cooling

(for a red detuning : $\delta = \omega_L - \omega_A < 0$)

One supposes that one can add independently the radiation pressure forces of the 2 waves.



Principle of Doppler cooling

$$\vec{\mathcal{F}} = \langle \vec{F} \rangle = \frac{\hbar \vec{k}_L \Gamma}{2}\left(s_+(\vec{v}) - s_-(\vec{v})\right) \simeq -\alpha \vec{v} \quad \text{with} \quad s_\pm(\vec{v}) = \frac{\Omega_1^2/2}{(\delta \mp k_L.v)^2 + \Gamma^2/4}$$

Friction coefficient

$$s \ll 1 \quad \rightarrow \quad \alpha = 2\hbar k_L^2 s \frac{-\delta\,\Gamma}{\delta^2 + \left(\Gamma^2/4\right)}$$

$$\alpha = 2\hbar k_L^2 s \quad \text{when } \delta = -\Gamma/2$$

Order of magnitude of the velocity damping time

$$\tau_{\text{Damp}} = M/\alpha = M/2\hbar k_L^2 s \qquad \tau_{\text{Damp}} = 10M/\hbar k_L^2 \ \text{if } s = 0.1$$

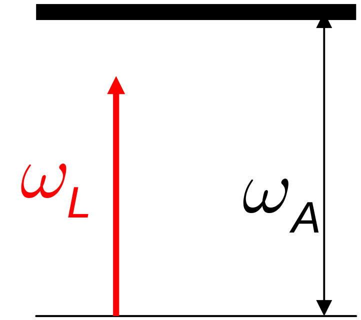
For Rb atoms and s=0.1, one finds $\tau_{\text{Damp}} \sim 100\ \mu\text{s}$

Very short damping time

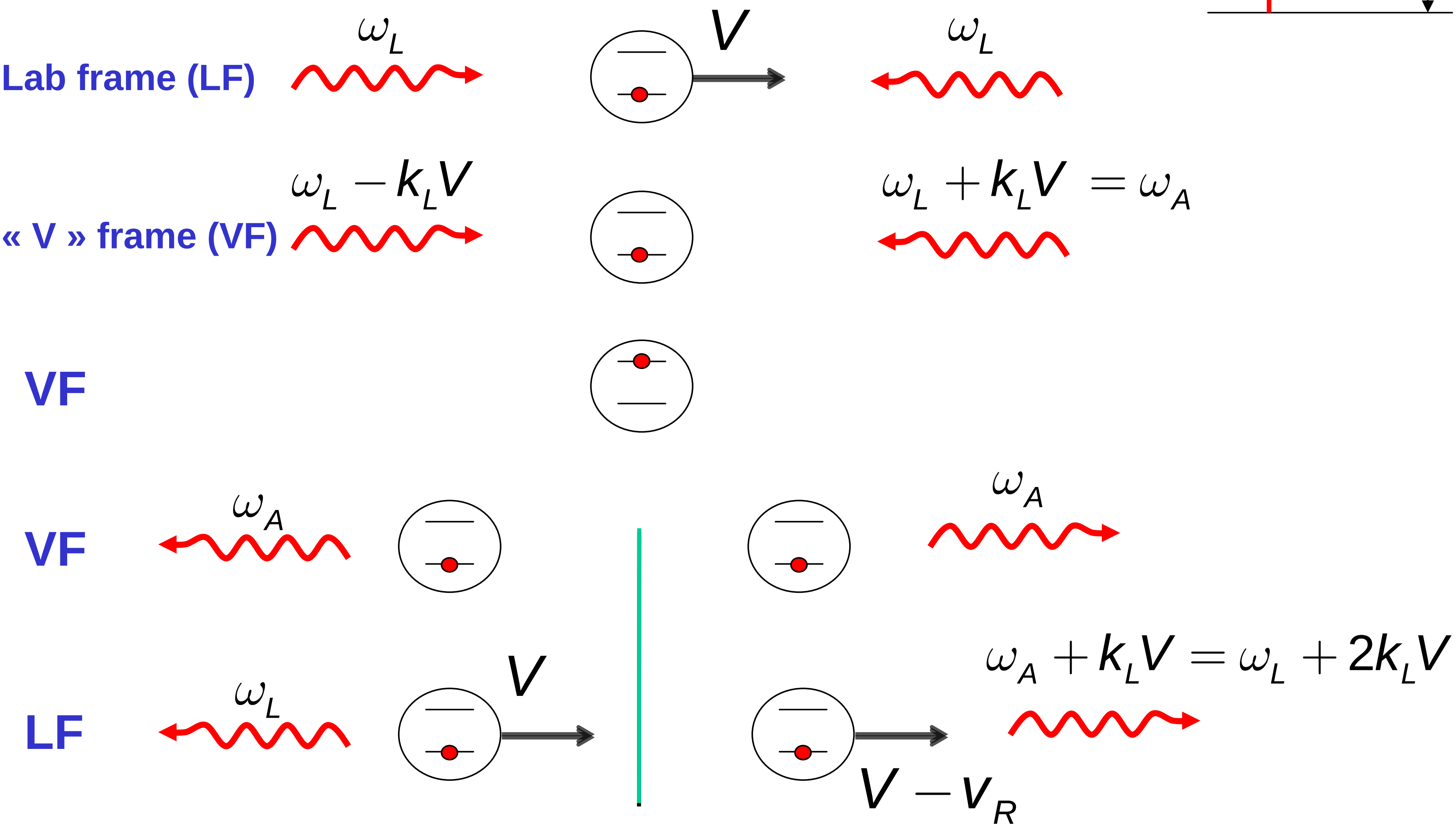
Velocity capture range

For $\delta=-\Gamma/2$, the variation with $k_L v$ of the total force shows that the velocity interval v_{capt} over which the force is appreciable is given by:

Physical interpretation



Quasi-resonant, red-detuned counter propagating beams



Energy balance and entropy considerations

Energy balance

The energy of the reemitted photon is on average larger than the energy of the absorbed one.

The energy of the radiation field increases while the energy of the atoms decreases.

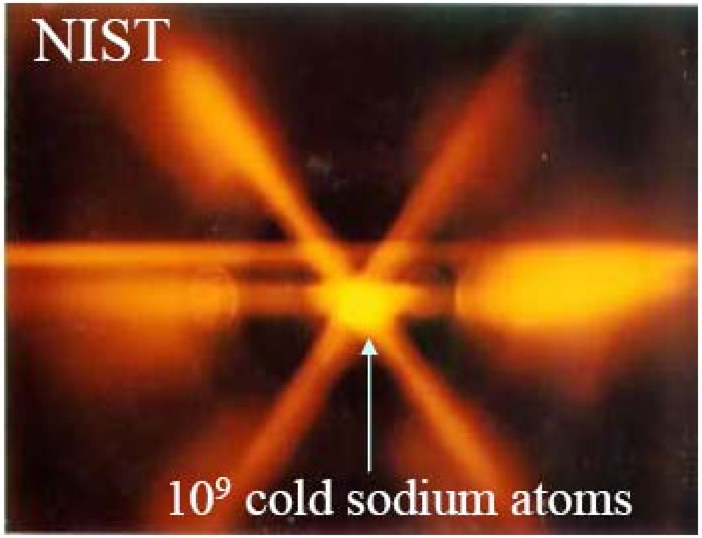
Qualitative considerations about the entropy

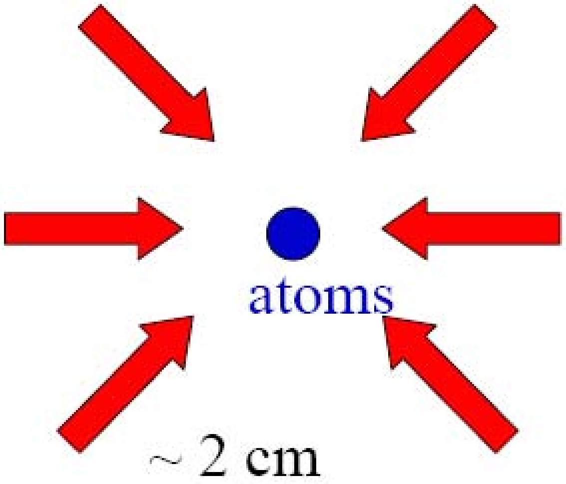
Cooling of atoms results in a decrease of the entropy of the atoms.

But photons are absorbed from the laser beam, which is a low entropy system and transformed into fluorescence photons emitted in all possible directions. The fluorescence field is a disordered system with a high entropy.

The entropy of the radiation field thus increases while the entropy of the atoms decreases.

3D version





1st optical molasses: Bell Labs (S. Chu et al), 1985

S. Chu *et al*. PRL **55**, 48 (1985).

Does an optical molasses deserve its name?

Consider an atom embedded in a 3D optical molasses. How this point distribution expand?

According to the equation of motion, the atom "loose" the memory of its initial velocity after the damping time $\tau = M/\alpha$

The spatial diffusion can be described by a random walk with a step size $\ell \sim v_{rms}\tau$ given by the product of the rms velocity by the damping time

The randow walk model gives $\langle r^2 \rangle = 2\ell^2 \mathcal{N}$ where $\mathcal{N}$ is the number of steps, after a time T, $\mathcal{N} = T/\tau$

$$\langle r^2 \rangle = 2D_x T \text{ with } D_x = v_{rms}^2 \tau = \frac{Mk_B T}{\tau} = \frac{D}{\alpha^2}$$

Numerical application: $\sqrt{\langle r^2 \rangle} \sim 0.5$ cm, requires, for Cs atoms, a time diffusion $T = 1$ second !

The Doppler temperature

$$\frac{d\langle \mathbf{P}^2 \rangle}{dt} = -\langle m\gamma \mathbf{V}.\mathbf{P} \rangle + 2R\hbar^2 k_L^2$$

radiation pressure cooling force

Each scattering event represent two random walk steps

$$m\langle V^2 \rangle = 3k_B T = \frac{2R}{m\gamma}\hbar^2 k_L^2 = \frac{\hbar\Gamma}{4}\left(\frac{2|\delta|}{\Gamma} + \frac{\Gamma}{2|\delta|}\right)$$

$k_B T_{\min} = \frac{\hbar\Gamma}{2}$ for $\delta = -\Gamma/2$ ex.: Rubidium T_{min} = 140 μK

The Doppler temperature

$$k_B T_{\min} = \hbar\Gamma / 2$$

The corresponding value of the velocity dispersion is given by:

$$M \left(\Delta \mathrm{v}\right)^2_{\min} \sim \hbar\Gamma \quad \rightarrow \quad \left(\Delta \mathrm{v}\right)_{\min} \sim \sqrt{\hbar\Gamma / M}$$

The first estimations of $T_{\min}$ were mixing up $\left(\Delta \mathrm{v}\right)_{\min}$ and $\left(\Delta \mathrm{v}\right)_{\mathrm{capt}} \sim \Gamma / k_L$

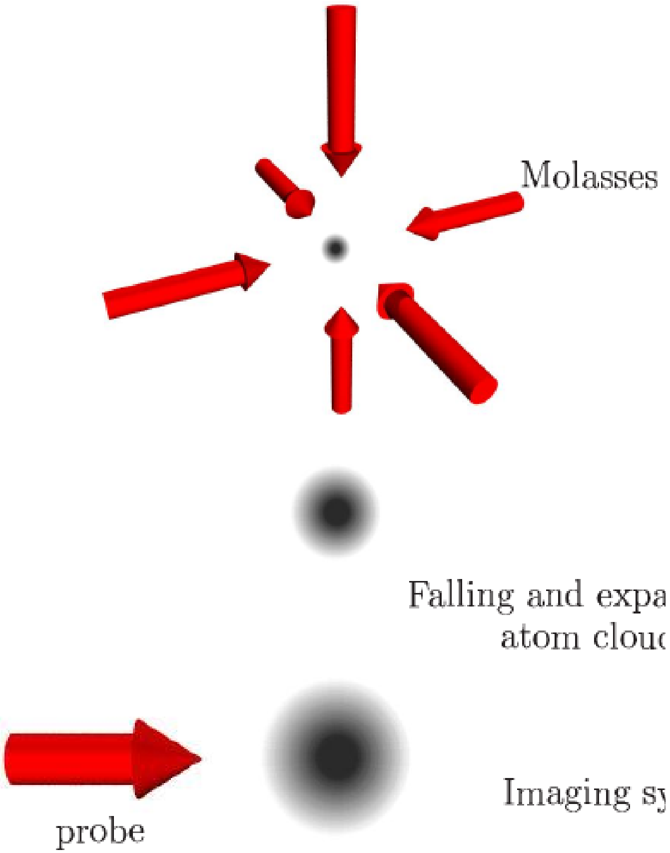
In fact, $\left(\Delta \mathrm{v}\right)_{\min} / \left(\Delta \mathrm{v}\right)_{\mathrm{capt}} \sim \left(\sqrt{\hbar\Gamma / M}\right) / \left(\Gamma / k_L\right) \sim \sqrt{\hbar\Gamma / E_{\mathrm{rec}}} \sim 1$

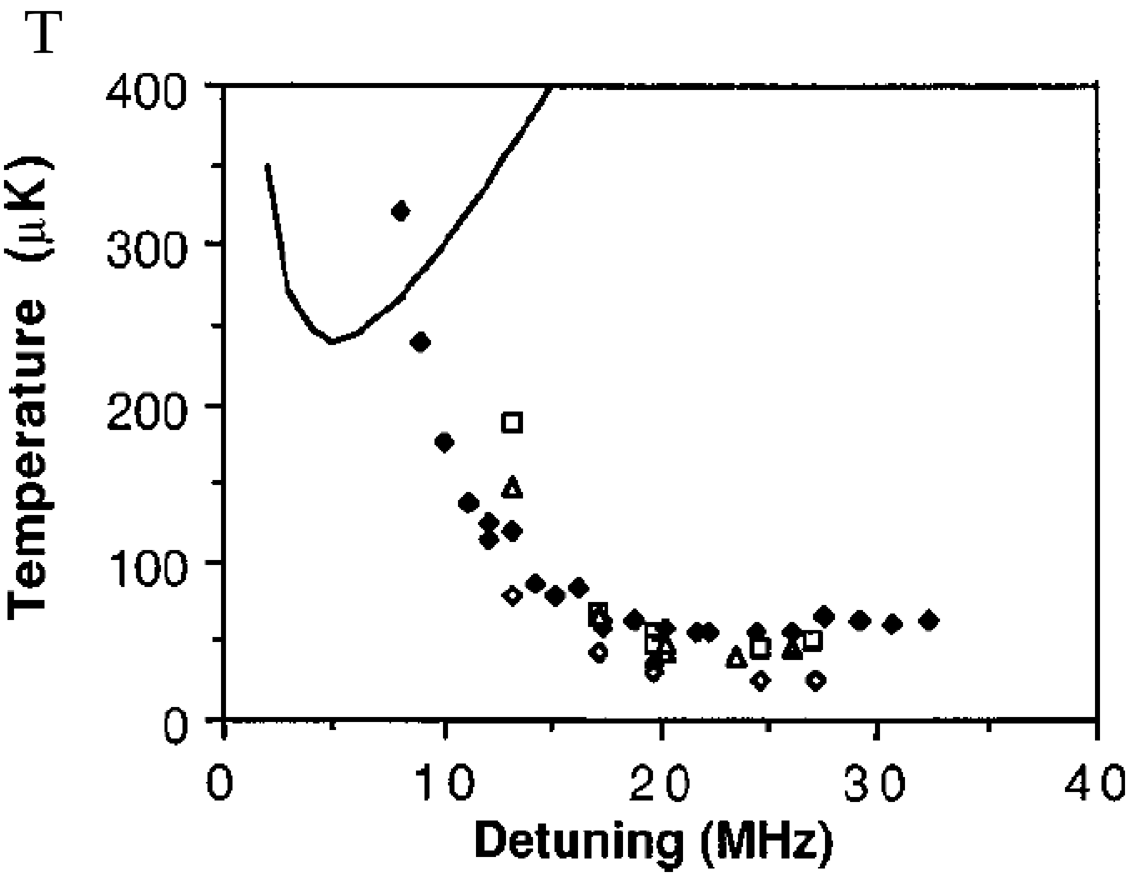
First correct calculation of the limits of Doppler cooling:

Moscow group :

V. S. Letokhov, V. G. Minogin and B. D. Pavlik, *Cooling and trapping of atoms and molecules by a resonant laser field,* Opt. Commun. **19**, 72 (1976).

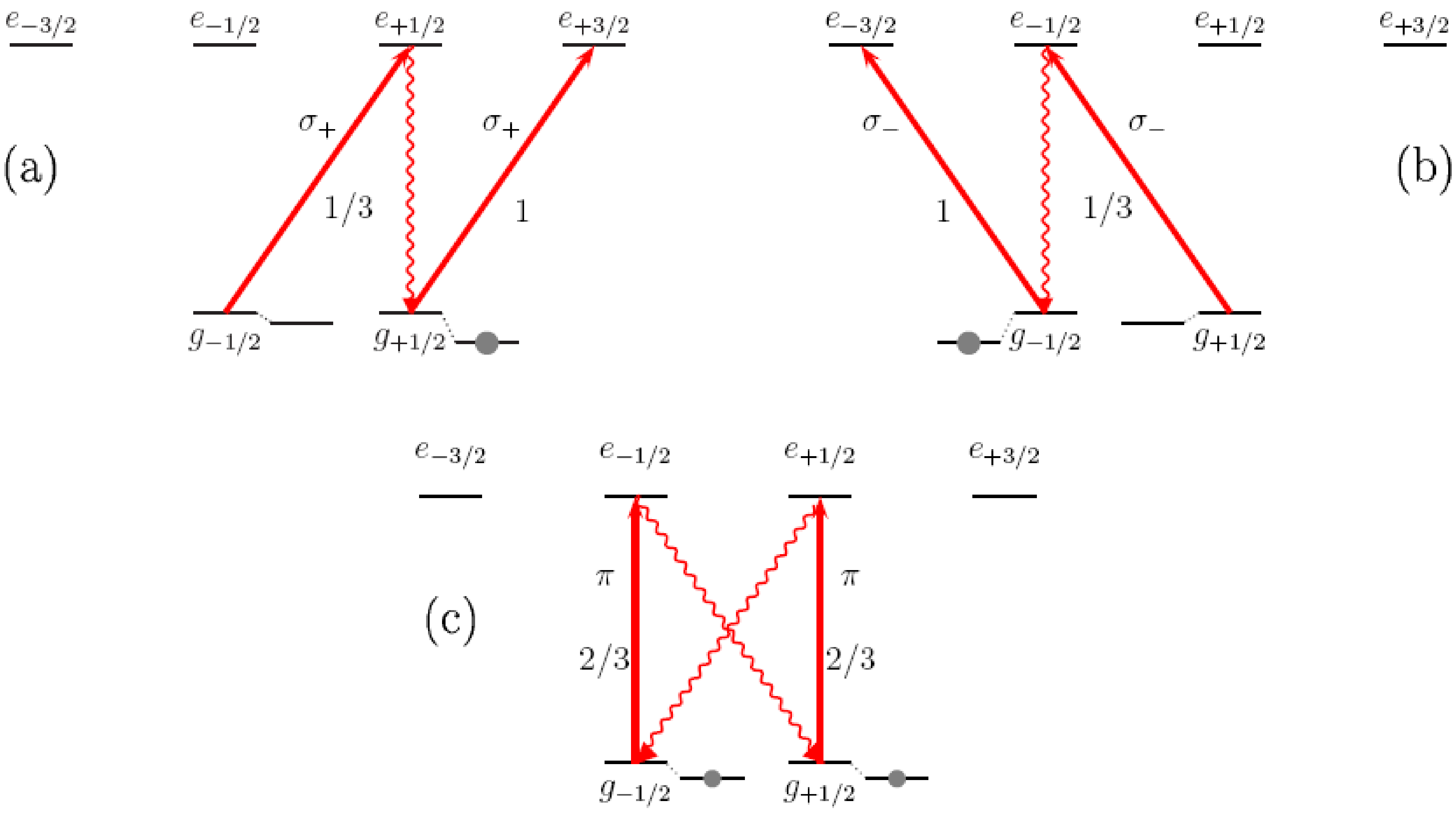
Doppler theory versus experiment …



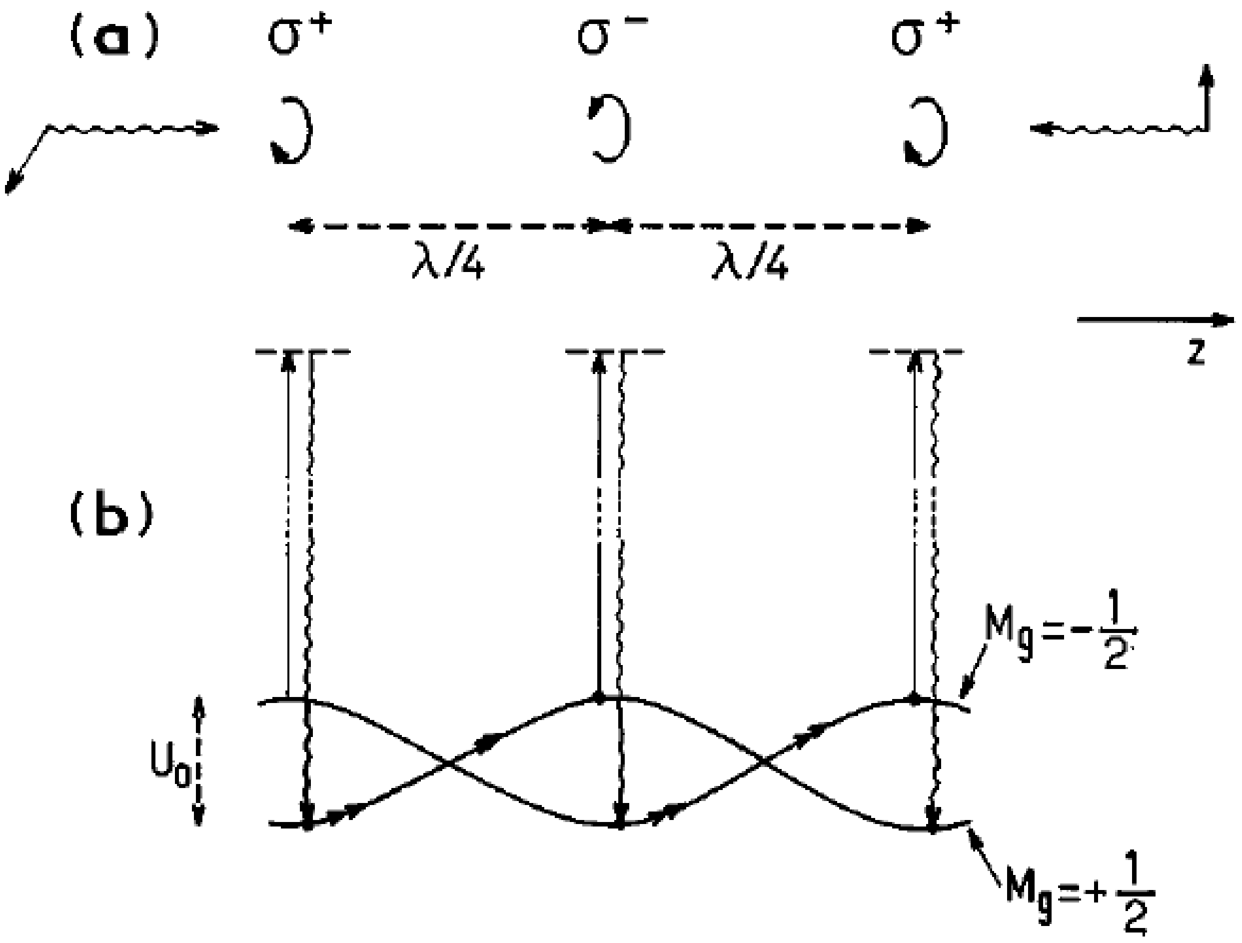


Temperatures much lower than $T_{doppler}$ have been reported, explanation relies on the sublevel structure of the ground state

Light shifts and polarization



Sisyphus cooling



Polarization Gradient

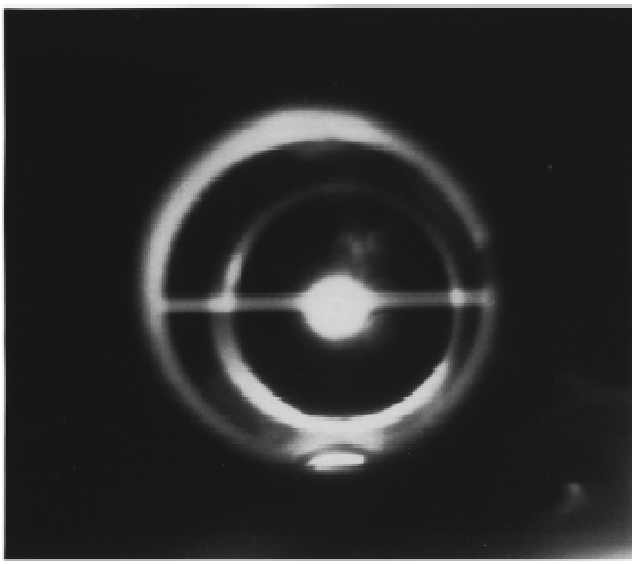
Degenerate Ground State

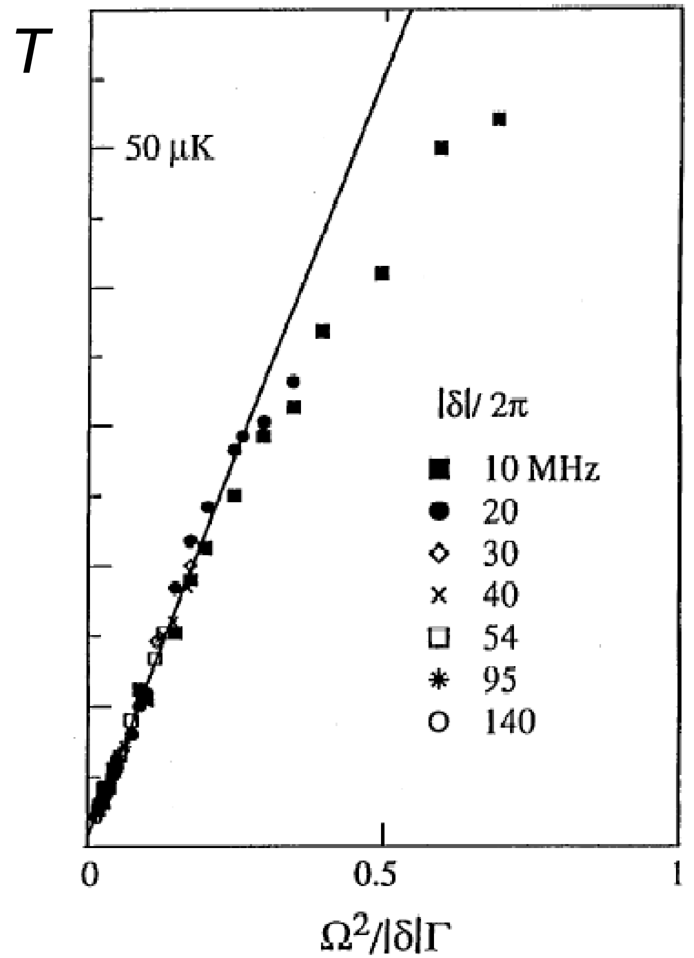
Interplay between dispersive and dissipative effect

Claude Cohen-Tannoudji, Review of Modern Physics **70**, 707 (1998)

Confrontation with experiment

$$T \propto U_0 = \text{Light Shift} \propto \frac{I}{\delta}$$

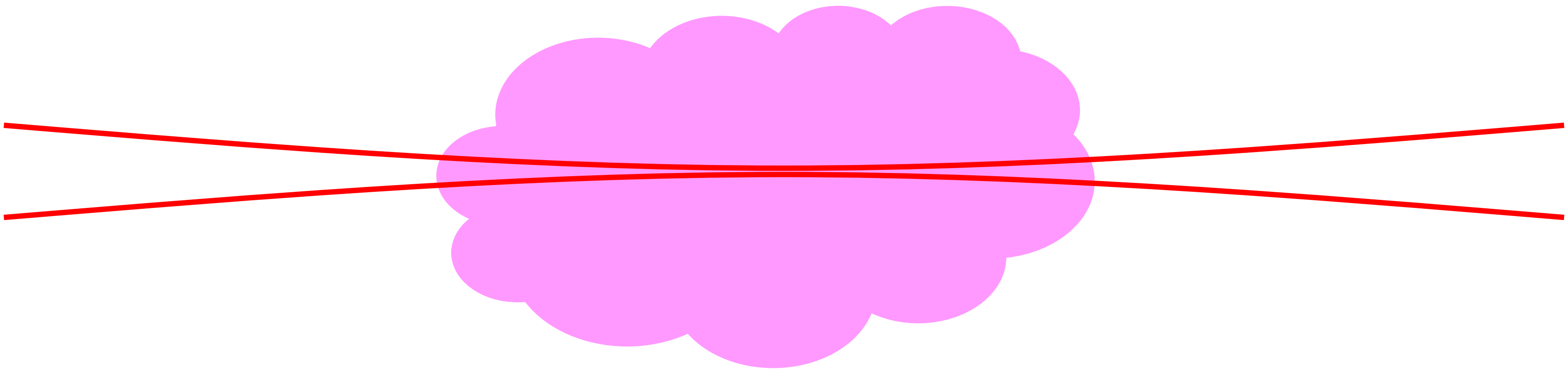




Europhys. Lett **12**, 683 (1990)

Use of optical molasses to load a dipole trap

Superimpose the dipole beam on the optical molasses



First experiment:

S. Chu, J. E. Bjorkholm, A. Ashkin, and A. Cable, *Experimental Observation of Optically Trapped Atoms*, Phys. Rev. Lett. **57**, 314 (1986).

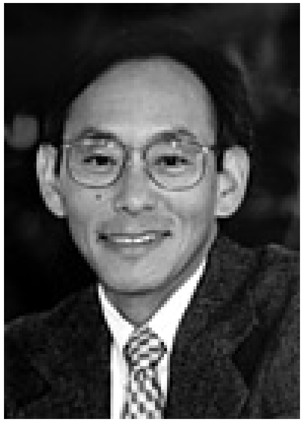
500 sodium atoms were trapped in the laser beam

Cooling and trapping of atoms

1997 Physics Nobel prize

W. Phillips, S. Chu and C. Cohen-Tannoudji

Zeeman Slower
Molasses



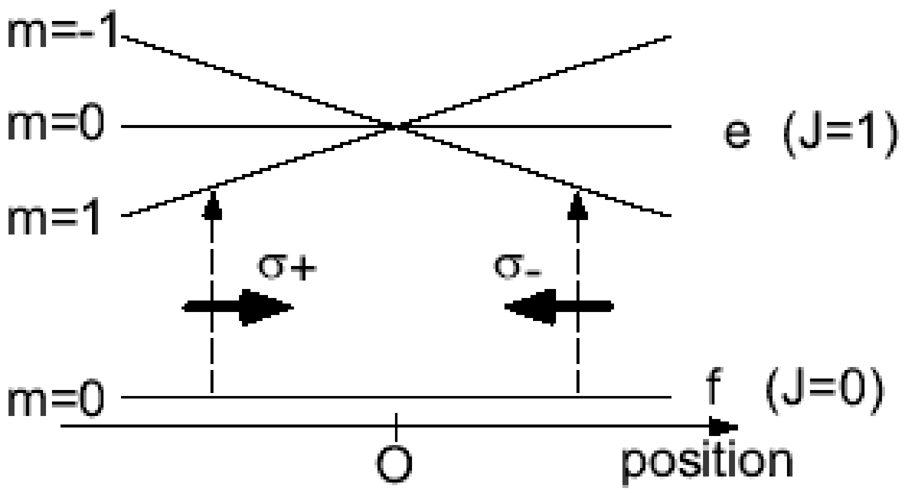
First realization of
the magneto-optical
trap

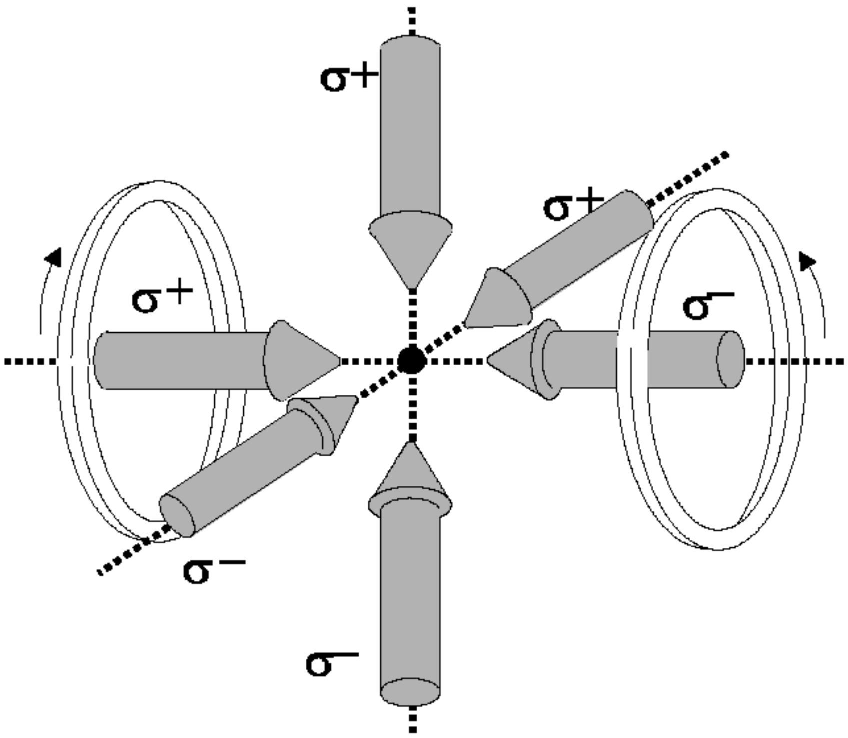
Sub-Doppler
cooling mechanism

"for development of methods to cool and trap atoms with laser light"

Radiation pressure trap: magneto-optical trap (MOT)

Idea proposed by Jean Dalibard 1987





b' = 10 Gauss / cm

I = a few mW per arm

Coils in AntiHelmholtz configuration

Force exerted on an atom in a MOT

$$F_{-z} = +\frac{\hbar k}{2}\Gamma \frac{\Omega^2/2}{\left(\Delta - kv_z - \frac{\mu_B}{\hbar}\frac{dB}{dz}z\right)^2 + (\Gamma/2)^2 + \Omega^2/2}$$

$$F_{\mathrm{MOT}} = F_{+z} + F_{-z} = -\alpha\dot{z} - Kz$$

Damping
Doppler effect

Trapping
Zeeman effect

The limit of small number of atoms

Spring constant of the trap?

➡ detuning including Doppler and Zeeman effects: $\delta \pm \mu b' x \pm kv$

$$\Rightarrow \quad \kappa = k\mu b' \, s_0 \frac{-2\Gamma\delta}{\delta^2 + \Gamma^2/4}$$

Expected size for the atomic cloud at equilibrium:

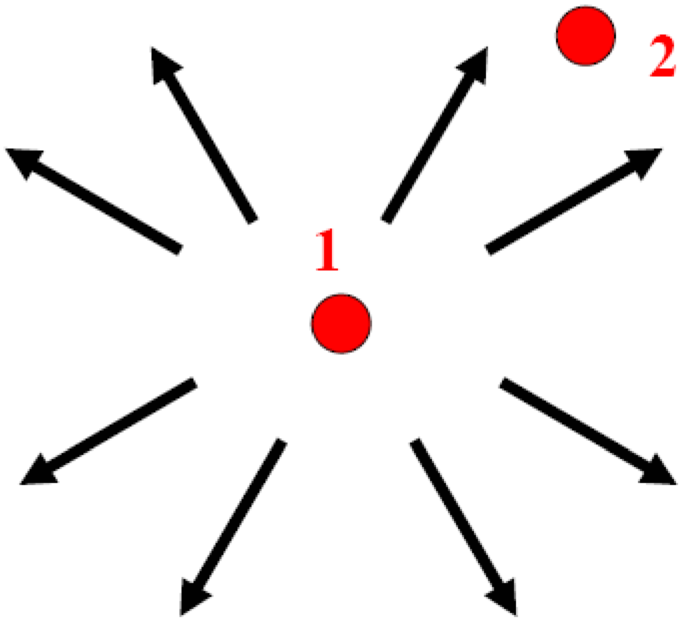
$$\langle z^2 \rangle = \frac{k_B T}{\kappa} \sim \frac{\hbar\Gamma}{k\mu b'}$$

For b'=10 G/cm, s_0=1/4, one get for rubidium atoms a few tens of micrometers.

Usual MOTs are much larger, why?

The MOT in the limit of large atom numbers (1)

Atom-atom repulsion due to photon scattering



Atom 1 scatters $6 \times (\Gamma s_0/2)$ photons/second

Atom 2 intercepts a fraction $\dfrac{\sigma}{4\pi r^2}$ of these photons

$$\sigma = \frac{3\lambda^2}{2\pi} \frac{\Gamma^2}{\Gamma^2 + 4\delta^2} \simeq \frac{3\lambda^2}{8\pi} \frac{\Gamma^2}{\delta^2}$$

Radiation pressure force formally identical to a Coulomb force with

$$\frac{q^2}{4\pi\epsilon_0 r^2} = 3\Gamma s_0 \hbar k \, \frac{\sigma}{4\pi r^2}$$

The MOT in the limit of large atom numbers (2)

In the limit of zero temperature, uniform density

$$n_0 = \frac{16\pi}{3}\frac{\mu b'}{\hbar\lambda^2}\frac{|\delta|}{\Gamma^2}$$

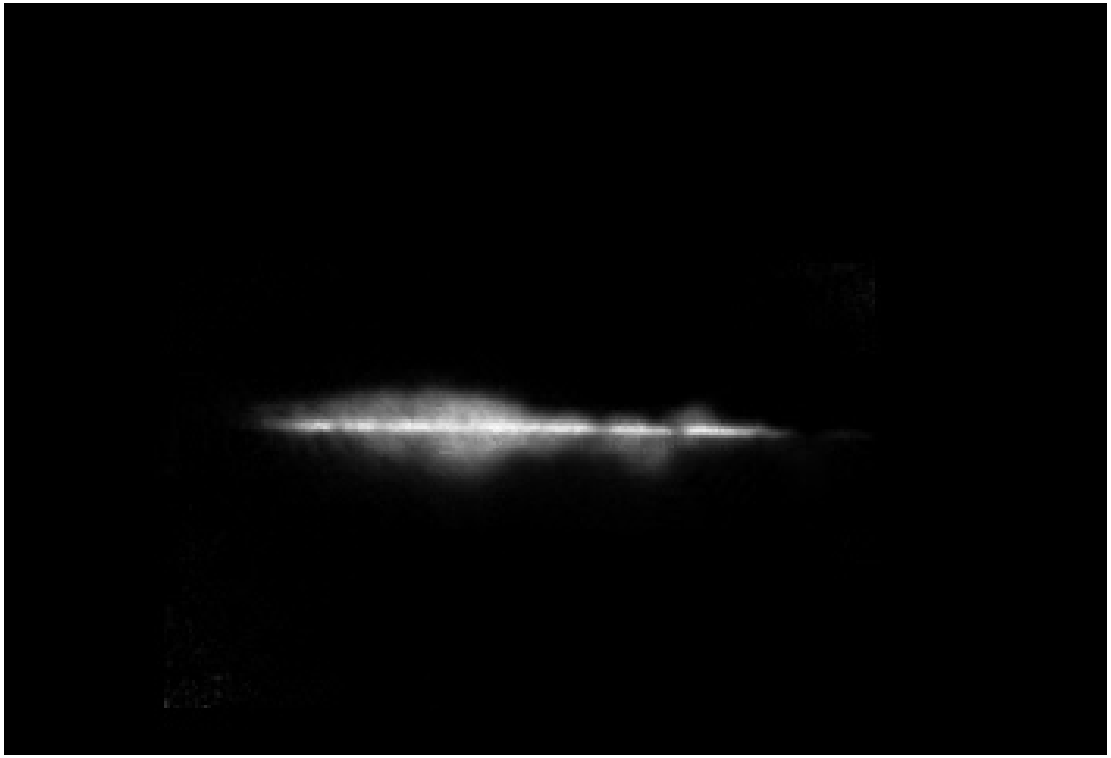
up to a radius such that: $N_{at} = \frac{4}{3}\pi R^3 n_0$

For typical parameters: n_0 = a few 10^{10} atoms/cm^3

radius of 2 mm for 10^9 atoms

very rich non linear dynamics…

Use of a MOT to load a dipole trap

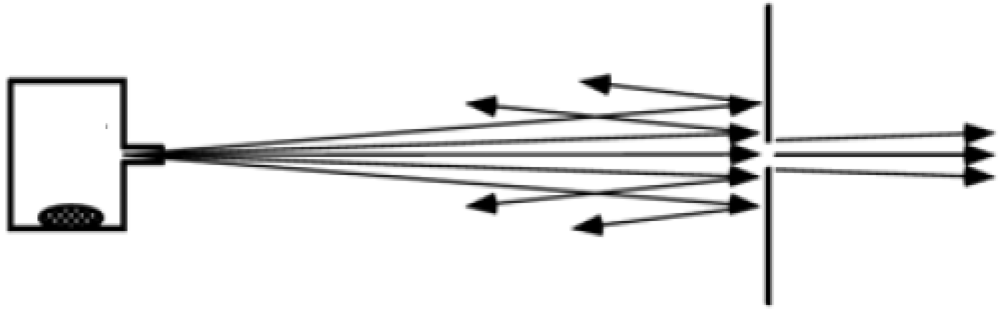


$5\ 10^7$ rubidium atoms are trapped in the laser beam

Can one cool without a dissipative force ? (1)

No, because of Liouville theorem, but…

A simple filtering:



Conversion of kinetic into potential energy:

$$\Delta v_f^2 = \Delta x_i^2 \, \omega'^2 = \Delta v_i^2 \, \frac{\omega'^2}{\omega^2} \ll \Delta v_i^2$$

Can one cool without a dissipative force ? (2)

If the N particles are independent, one cannot increase phase space density

- single particle density operator ρ
- this operator is hermitian and it can be diagonalized
- the largest eigenvalue λ_{max} gives the maximal phase space density $N\,\lambda_{max}$
- the eigenvalues of ρ are unchanged in during the time evolution, even if the hamiltonian depends explicitly on time

$$i\hbar\dot{\rho} = [H(t), \rho(t)]$$

unitary evolution

Ketterle-Pritchard

Not true for evaporative cooling, nor for stochastic cooling

Laser cooling: the proposals

[2.1] T. Hänsch and A. Schawlow, *Cooling of gases by laser radiation*, Opt. Commun. **13**, 68 (1975).
[2.2] D. Wineland and H. Dehmelt, *Proposed $10^{14}\Delta\nu<\nu$ laser fluorescence spectroscopy on TI^+ mono-ion oscillator III (side band cooling),''* Bull. Am. Phys. Soc. **20**, 637 (1975)**.**

Theory of Doppler cooling

[2.3] V. S. Letokhov, V. G. Minogin and B. D. Pavlik, *Cooling and trapping of atoms and molecules by a resonant laser field*, Opt. Commun. **19**, 72 (1976).
[2.4] D. Wineland and W. Itano, *Laser cooling of atoms*, Phys. Rev. A **20**, 1521 (1979).
[2.5] J. P. Gordon and A. Ashkin, *Motion of atoms in a radiation field*, Phys. Rev. A **21**, 1606 (1980).
[2.6] Y. Castin, H. Wallis and J. Dalibard, *Limit of Doppler cooling*, J. Opt. Soc. Am. B **6**, 2046 (1989).

Optical molasses

[2.7] S. Chu, L. Hollberg, J. Bjorkholm, A. Cable, and A. Ashkin, *Three-dimensional viscous confinement and cooling of atoms by resonance radiation pressure*, Phys. Rev. Lett. **55,** 48 (1985).
[2.8] V. S. Letokhov and V. G. Minogin, *Laser radiation pressure on free atoms*, Phys. Rep. **73**, 1 (1981).
[2.9.] T. W. Hodapp, C. Gerz, C. Furtlehner, C. I. Westbrook, W. D. Phillips and J. Dalibard, Three-dimensional spatial diffusion in optical molasses, Appl. Phys. B **60**, 135 (1995).

Nobel Lectures

[2.17] Steven Chu, Nobel Lecture: *The manipulation of neutral particles*, Rev. Mod. Phys. **70**, 685 (1998).

[2.18] Claude N. Cohen-Tannoudji, Nobel Lecture: *Manipulating atoms with photons* Rev. Mod. Phys. **70**, 707 (1998)

[2.19] William D. Phillips, Nobel Lecture: *Laser cooling and trapping of neutral atoms*, Rev. Mod. Phys. **70**, 721 (1998).