Quantum Control of States of Light (2)

Optimization of information extraction from optical measurements

C. Fabre
Laboratoire Kastler Brossel
Université Pierre et Marie Curie-Paris6, ENS

Two levels in field quantization:

- Define a set of modes

Optimization of the mode : coherent control

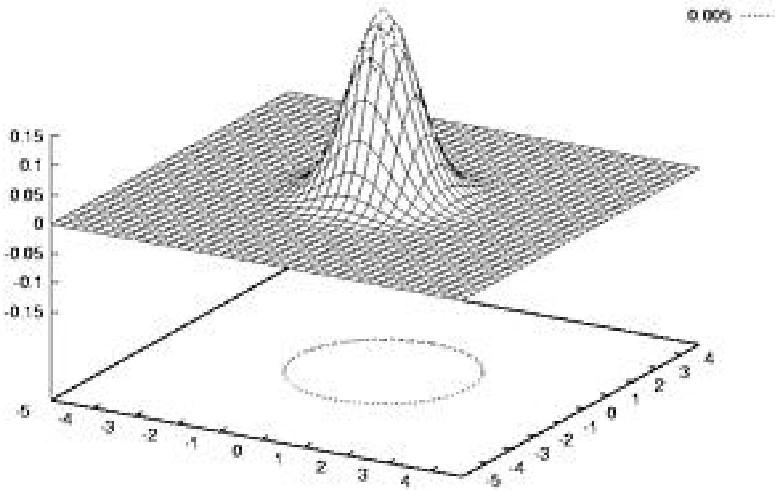
- Define a quantum state in each mode

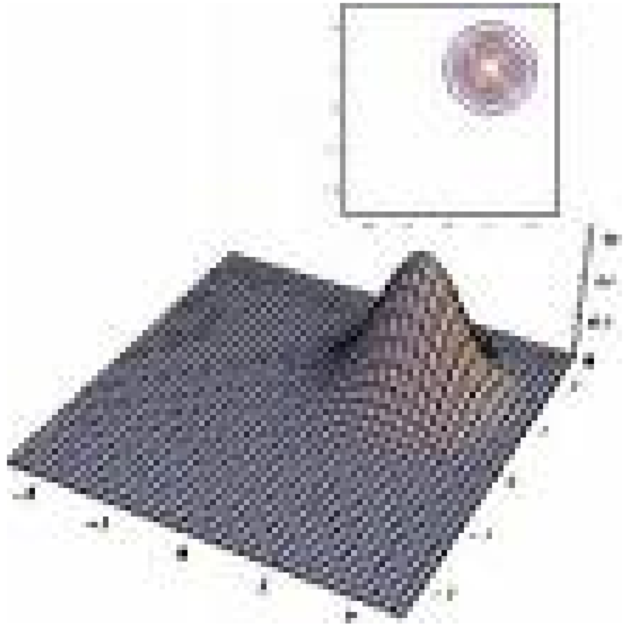
Optimization of the quantum state :
« pure quantum » control

Will not change the mean value of the field,
therefore not the mean atomic populations

Will change the variance and the correlations

Optimization of measurements and extraction of information



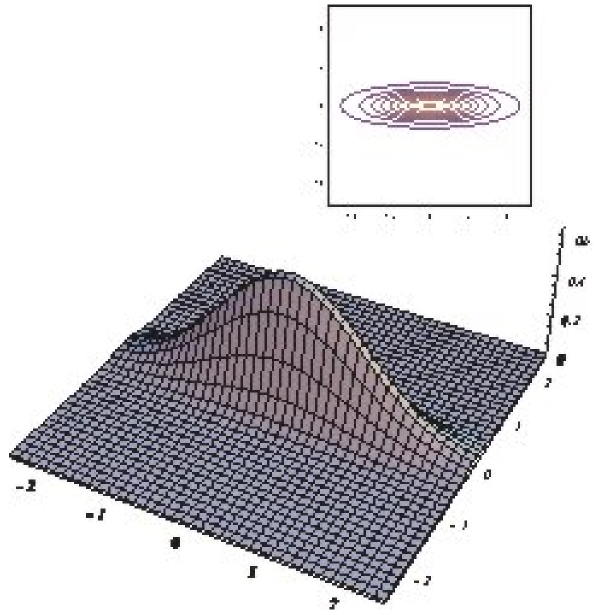


Vacuum

Coherent state
produced by a good laser

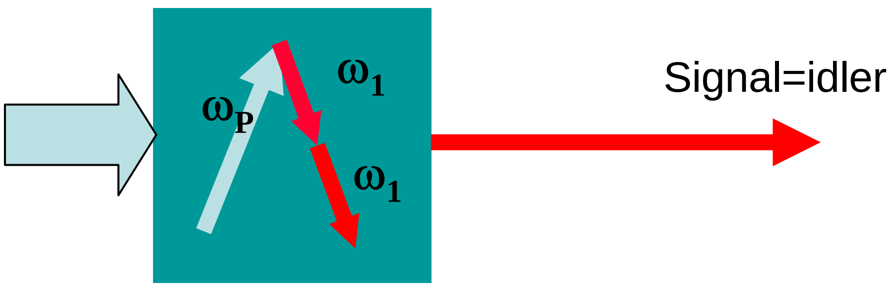
$$\Delta N = \sqrt{N}$$

Intensity noise : shot noise
Or « standard quantum limit »



Vacuum squeezed state:
less noise than in vacuum for one quadrature

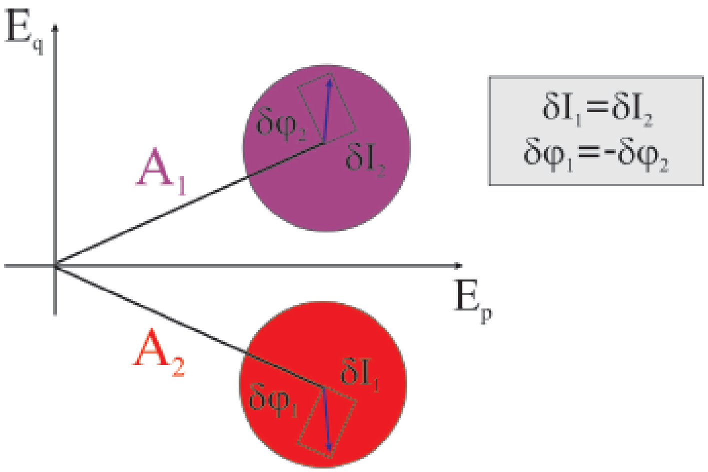
Produced by degenerate parametric down-conversion



Two-mode entangled twin-photon state

$$\left|\Psi\right\rangle = \sum_n c_n \left|\omega_1 : n, \quad \omega_2 : n\right\rangle$$

Produced by degenerate parametric down-conversion



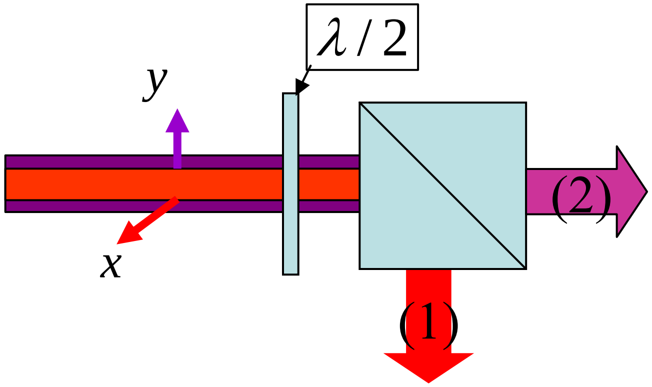
(with intense pump or resonant cavity)

Strong correlations on one quadrature component

Strong anti-correlations on the other quadrature

Einstein-Podolsky-Rosen entangled state

Example of change of mode basis
Polarization modes



in the basis of polarization modes x and y:

A tensor product of two vacuum squeezed states

in the basis of 45°, -45° polarization modes

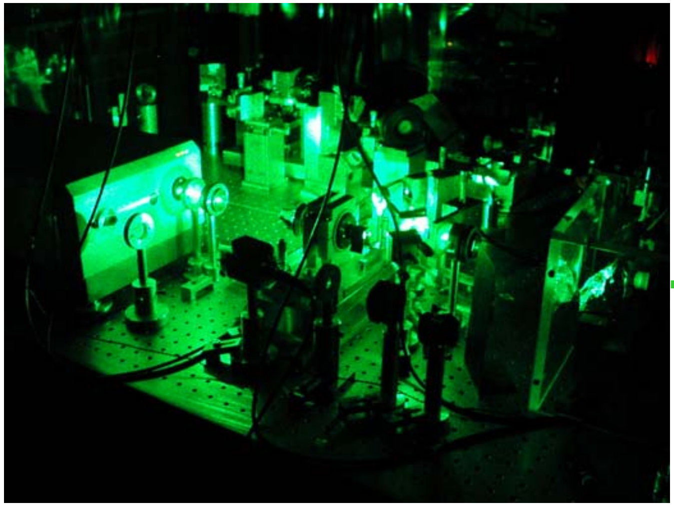
An EPR entangled state

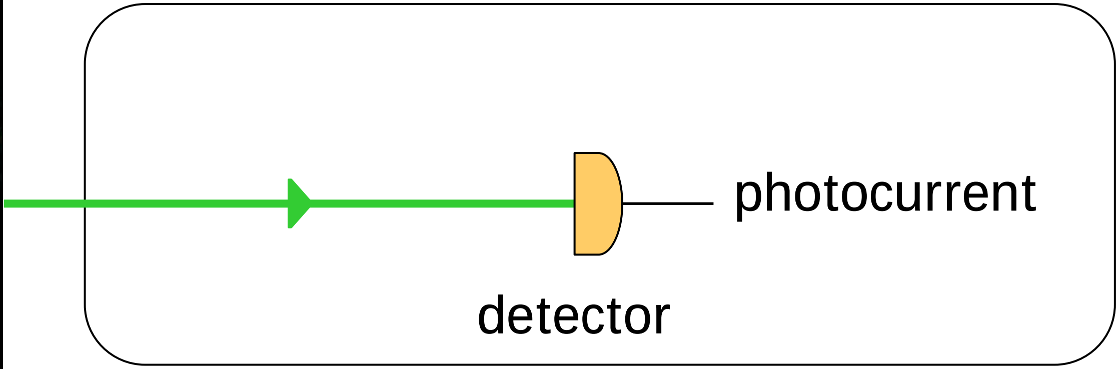
OPTIMIZATION

OF PARAMETER AND INFORMATION EXTRACTION

IN OPTICAL MEASUREMENTS

Light is one of the best carriers of information





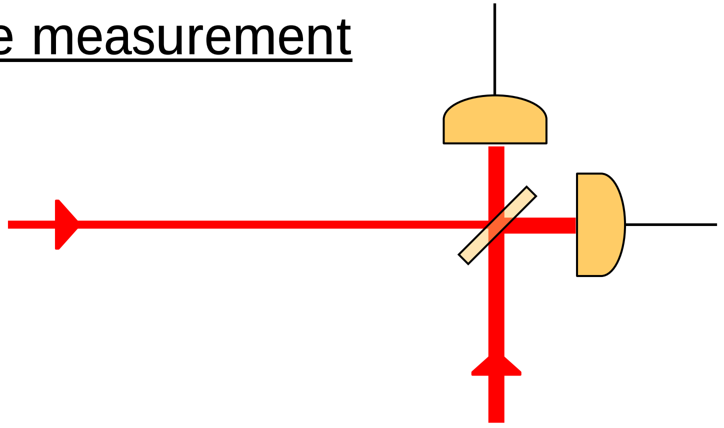
- Unavoidable existence of **« quantum noise »**

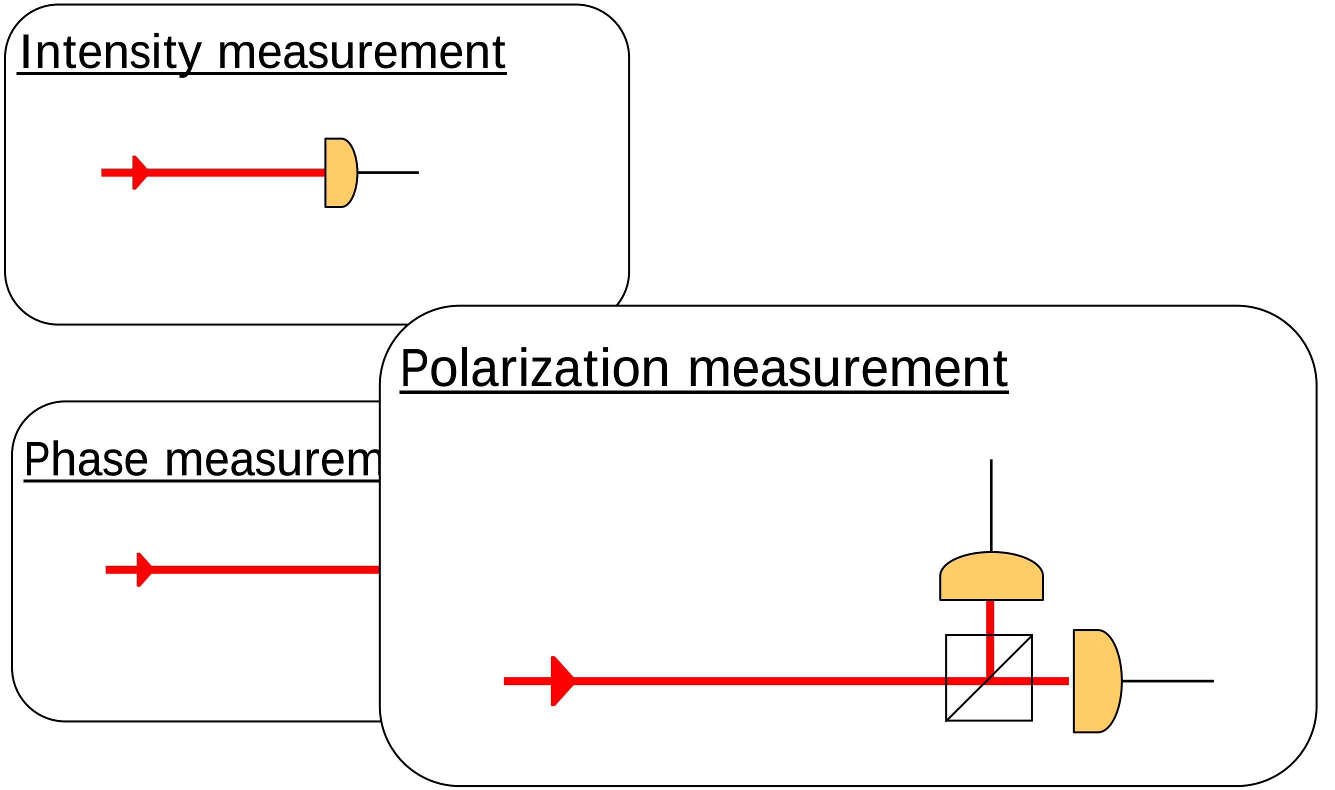
- Possibility of **quantum correlations** between measurements

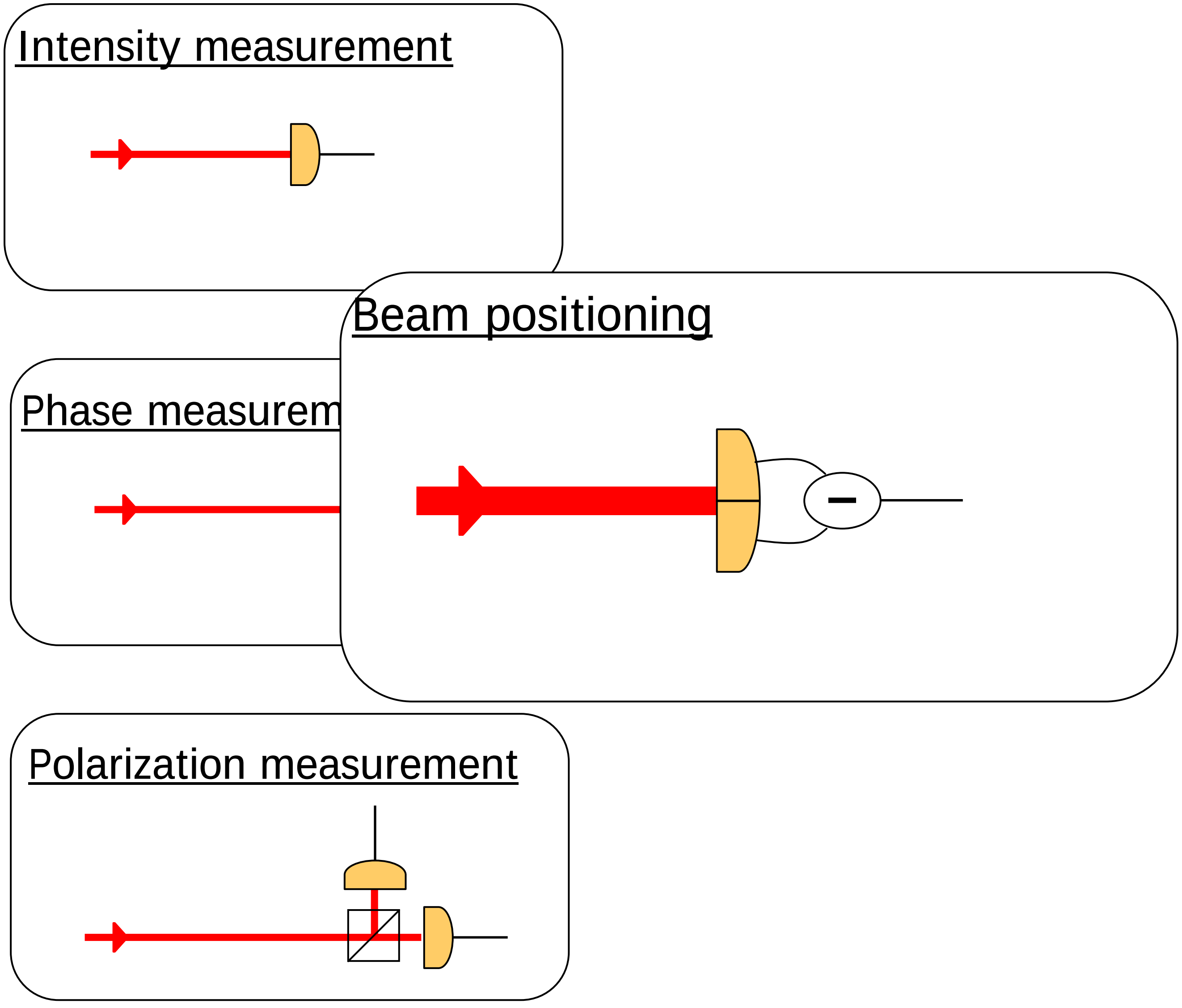
Intense beam regime : 1mW=10^{16} photons/second

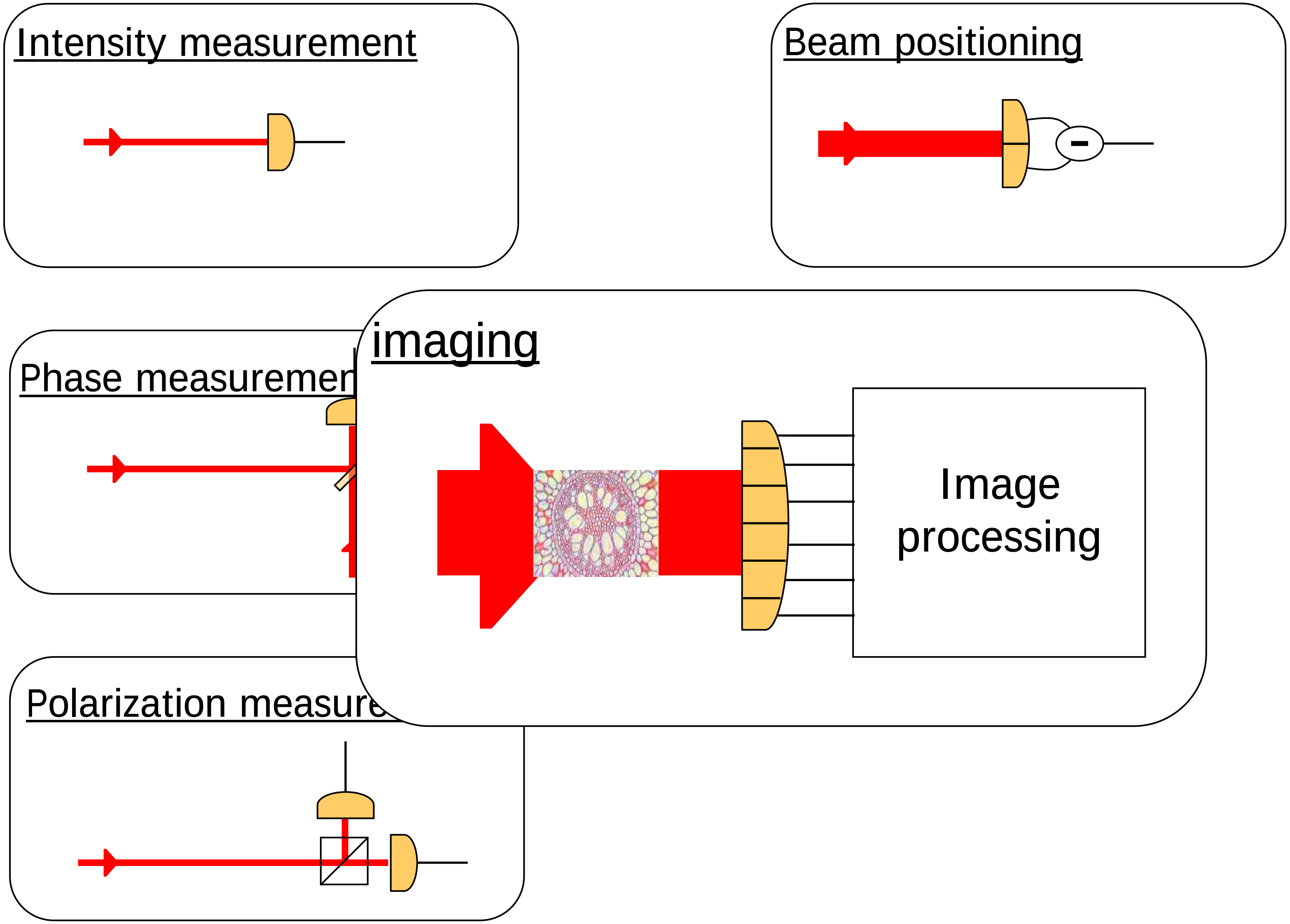
Intensity measurement

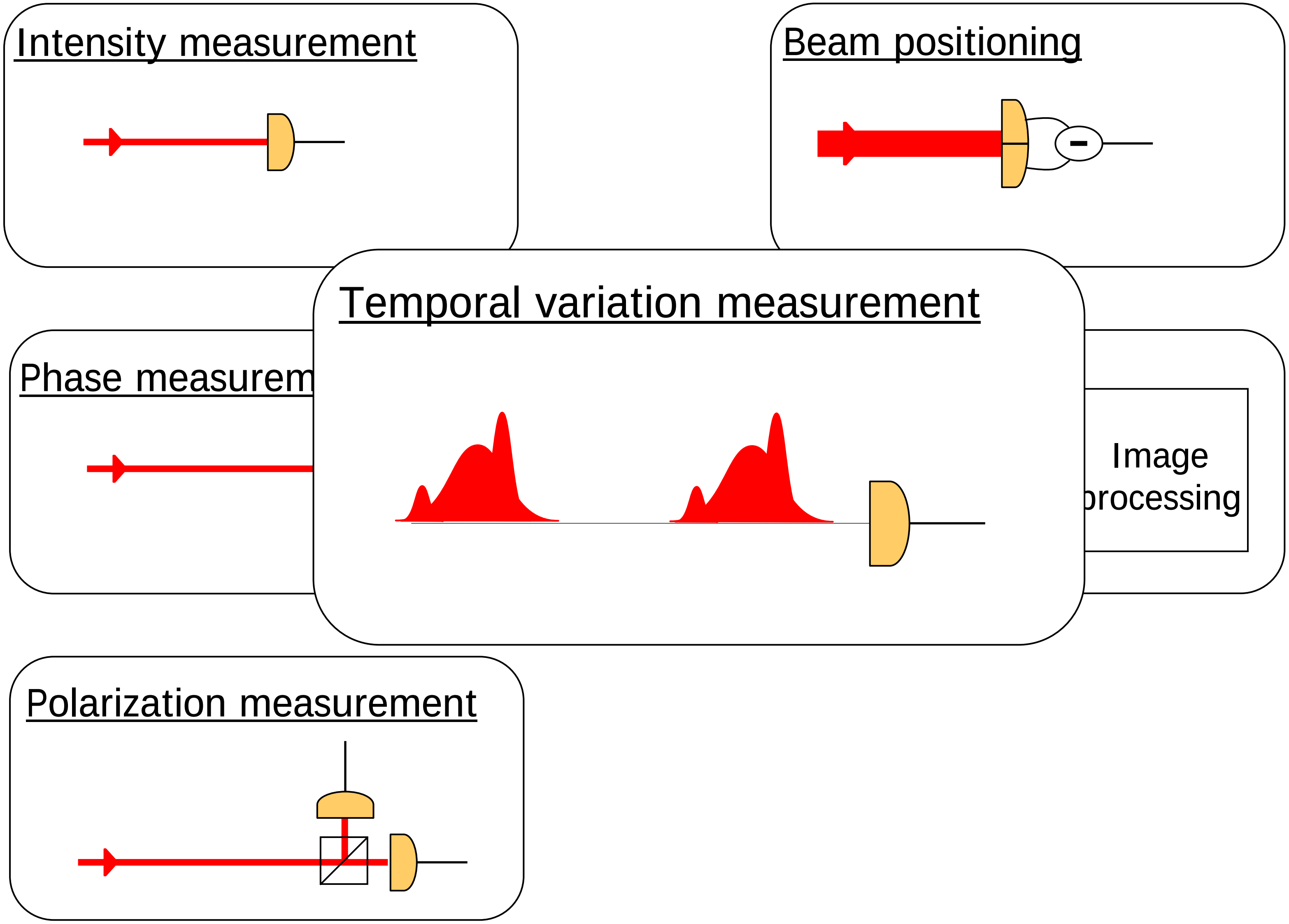
Phase measurement



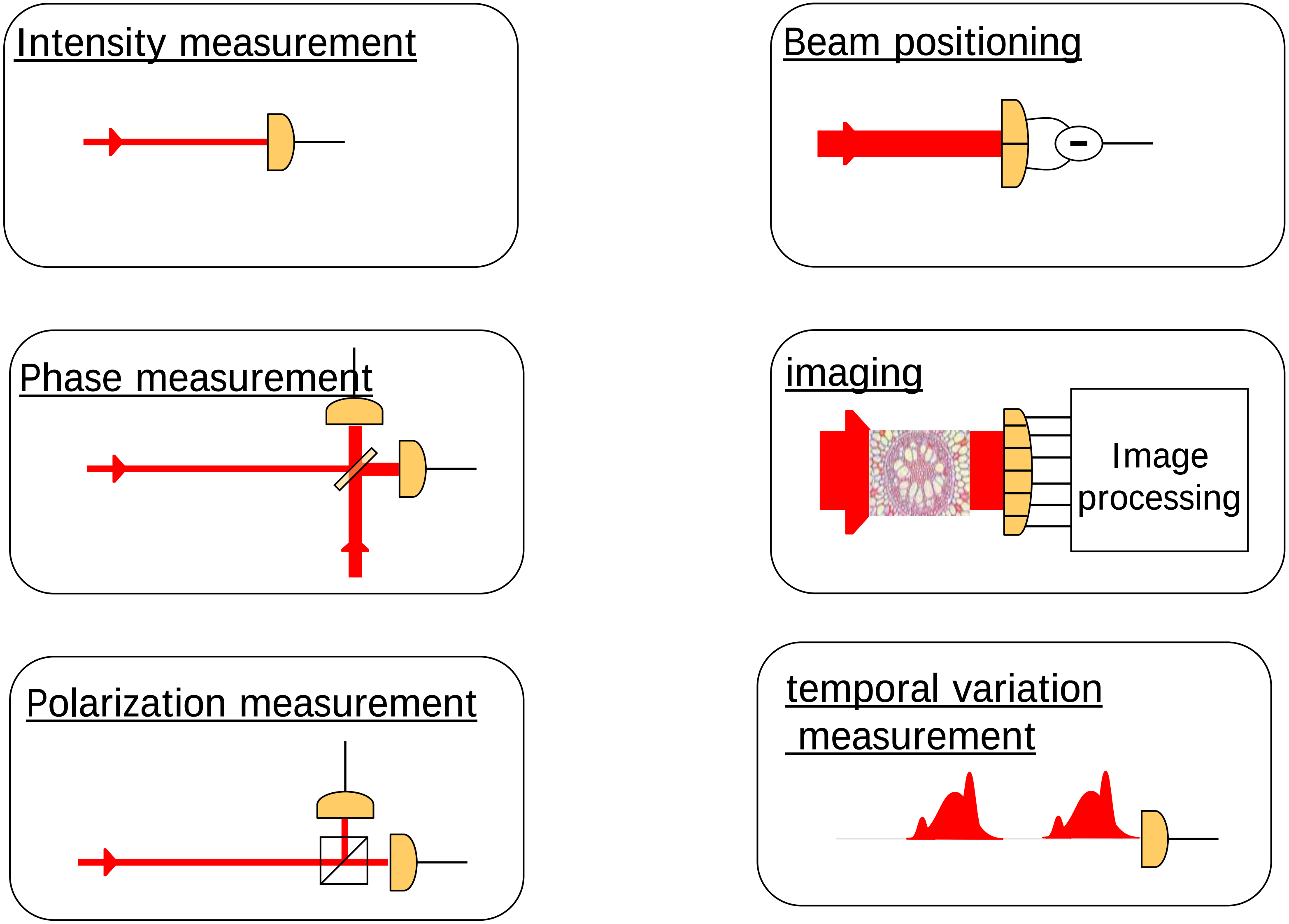

Intensity measurement
Phase measurem
Polarization measurement

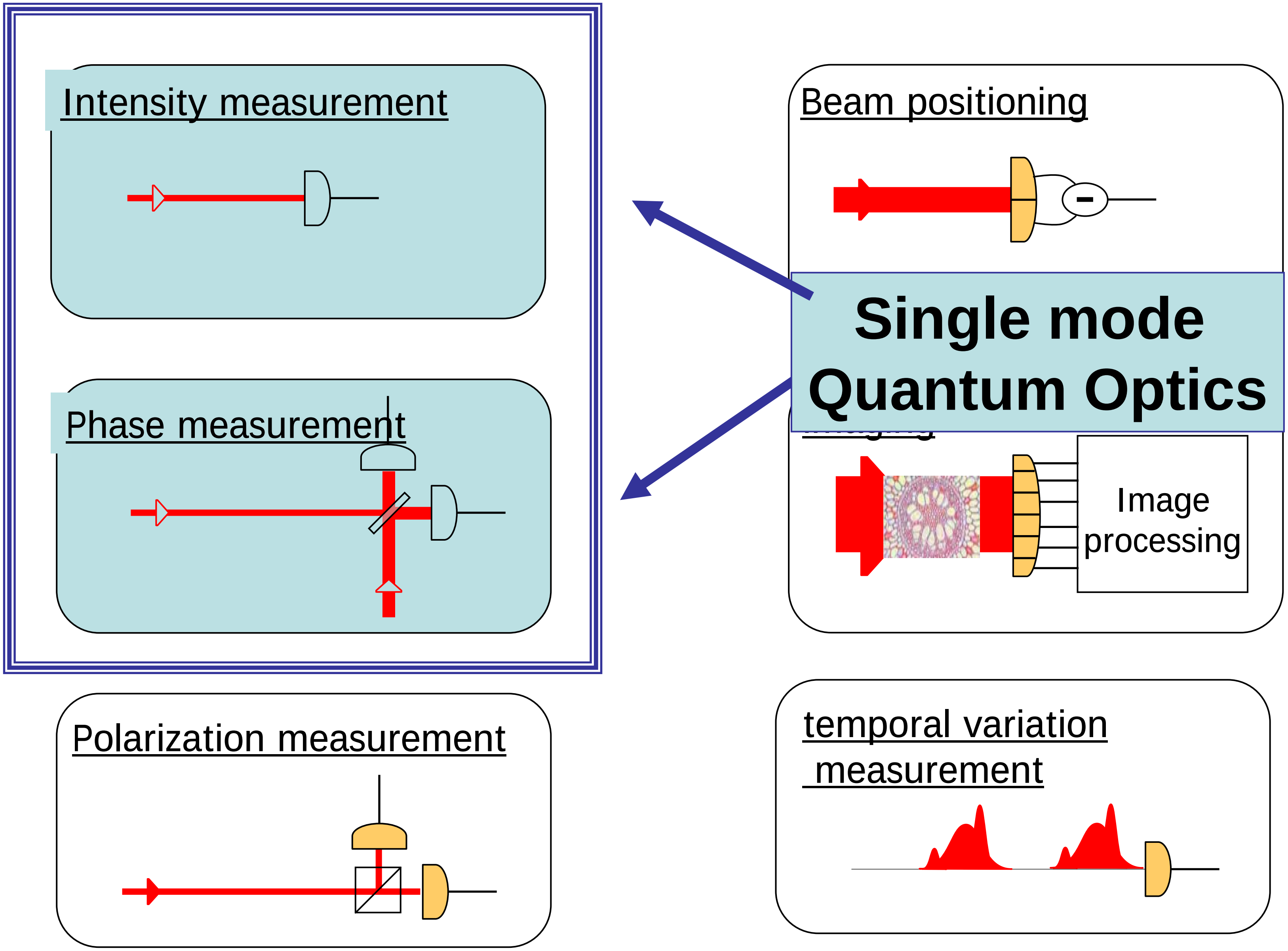

Intensity measurement
Phase measurem
Beam positioning
Polarization measurement

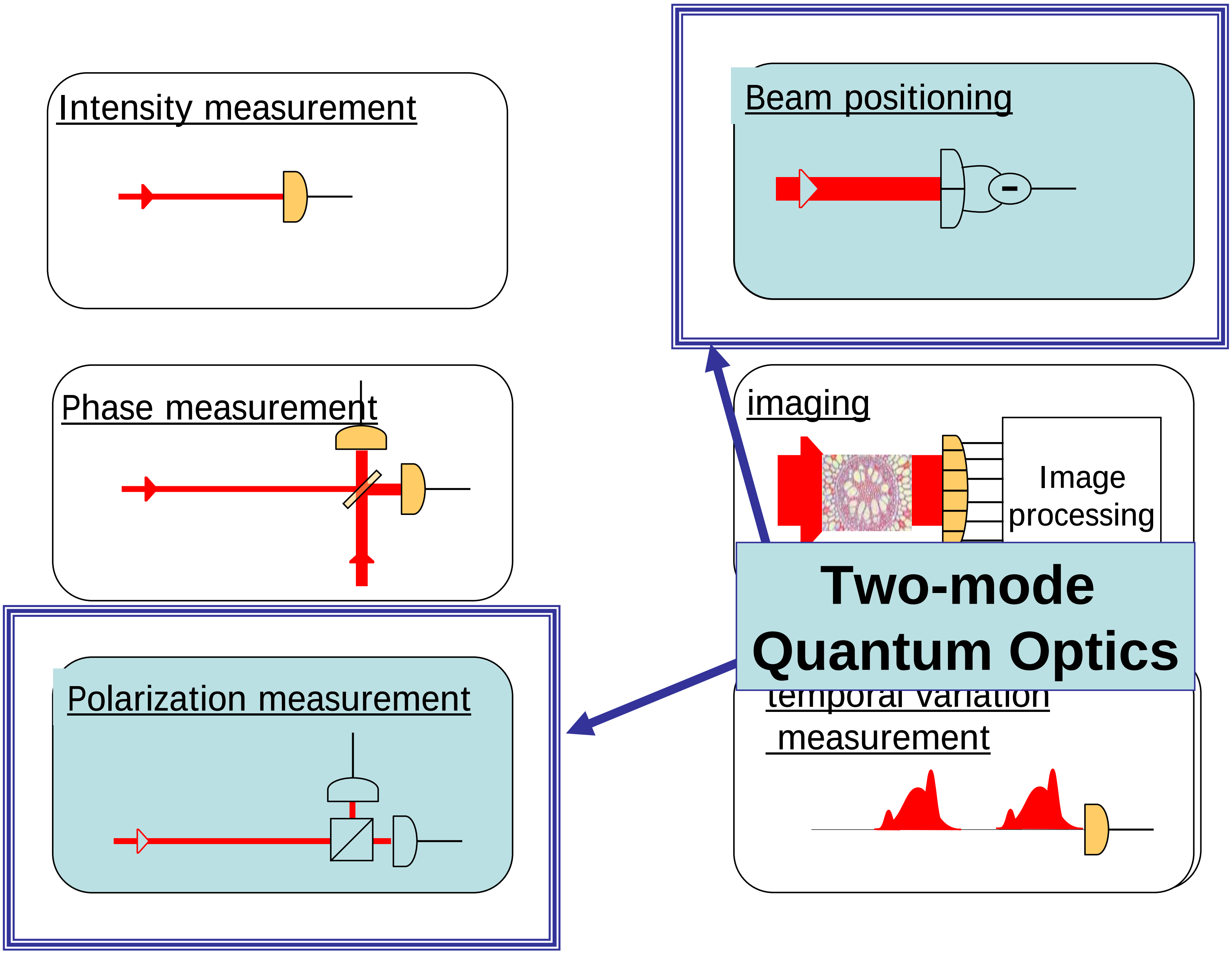

Intensity measurement
Beam positioning
Phase measuremen
imaging
Image processing
Polarization measure



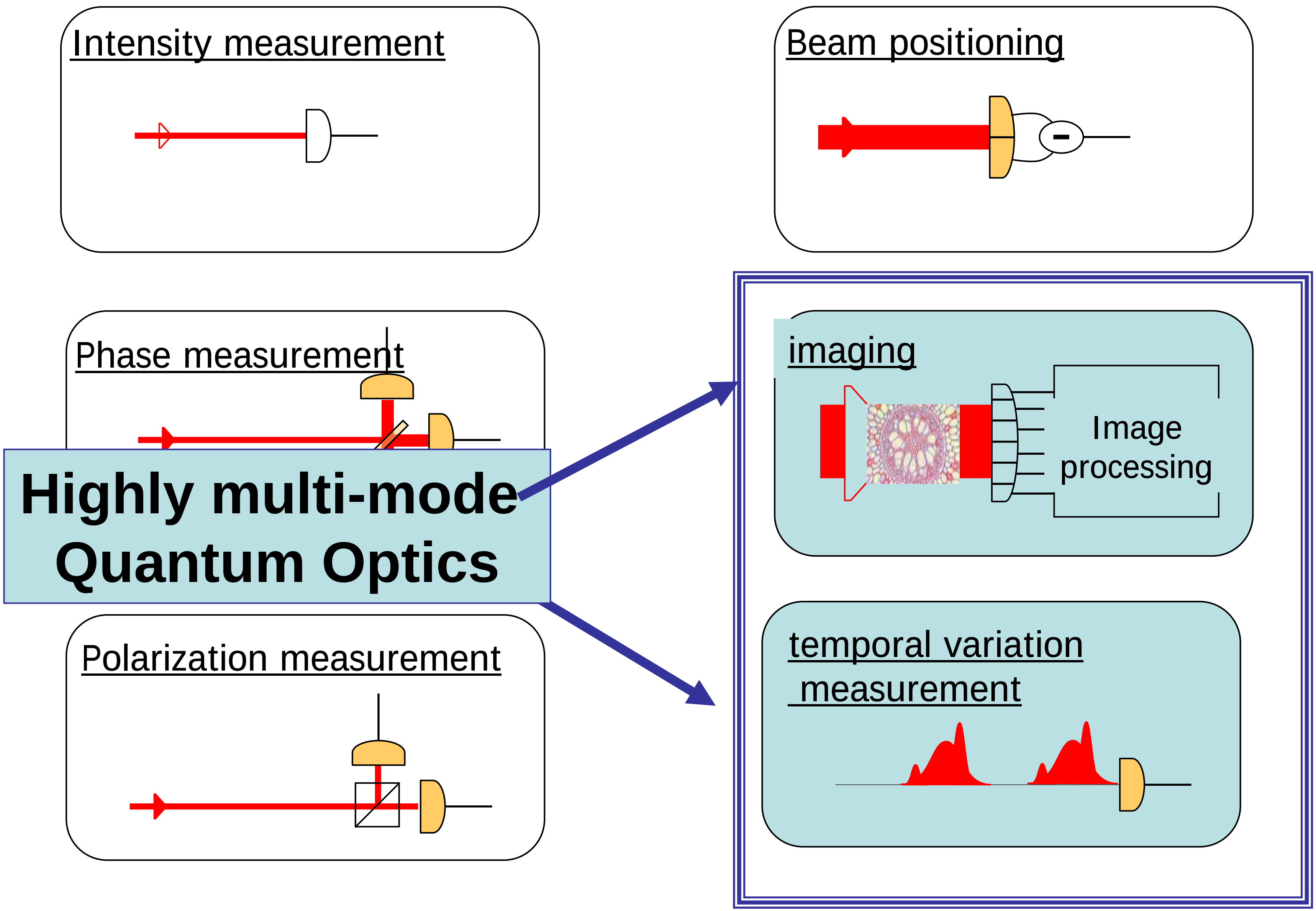
Intensity measurement
Beam positioning
Temporal variation measurement
Phase measurem
Image
processing
Polarization measurement


Intensity measurement
Beam positioning
Phase measurement
imaging
Image processing
Polarization measurement
temporal variation measurement


Intensity measurement
Phase measurement
Polarization measurement
Beam positioning
Single mode
Quantum Optics
Imaging
Image
processing
temporal variation
measurement



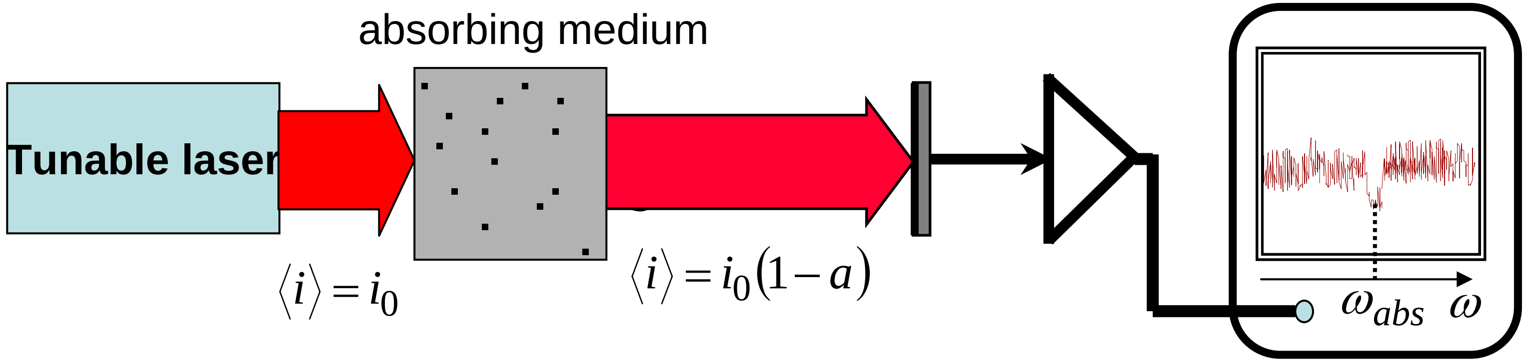
Intensity measurement
Beam positioning
Phase measurement
imaging
Image processing
Two-mode Quantum Optics
Polarization measurement
temporal variation measurement



Intensity measurement
Beam positioning
Phase measurement
Highly multi-mode Quantum Optics
imaging
Image processing
Polarization measurement
temporal variation measurement

INTENSITY MEASUREMENTS

Weak absorption measurement



Limit due to the intensity noise of the beam,
i.e. shot noise for a coherent beam

Shot noise limit of absorption measurement :

$$a_{SQL} = \sqrt{\frac{1}{\langle N \rangle}} = \sqrt{\frac{1}{\langle \Phi \rangle T}}$$

Absorption measurement using sub-Poisonian source

F. Marin, A. Bramati, V. Jost, E. Giacobino, Optics Communications ***140****, 146 (1997)*

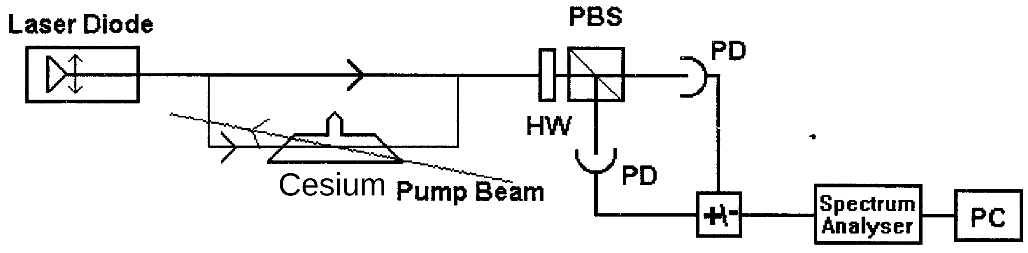
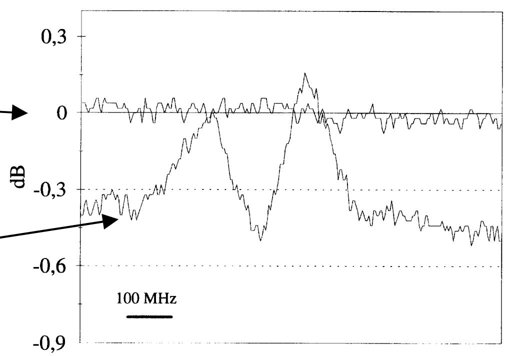


Fig. 3. Experimental apparatus for FM spectroscopy. PBS – polarizing beam splitter; HW – half-wave plate; PD – photodiode.

Saturated absorption experiment

shot noise level

modulated absorption signal



RBW : 1MHz, VBW 3 kHz

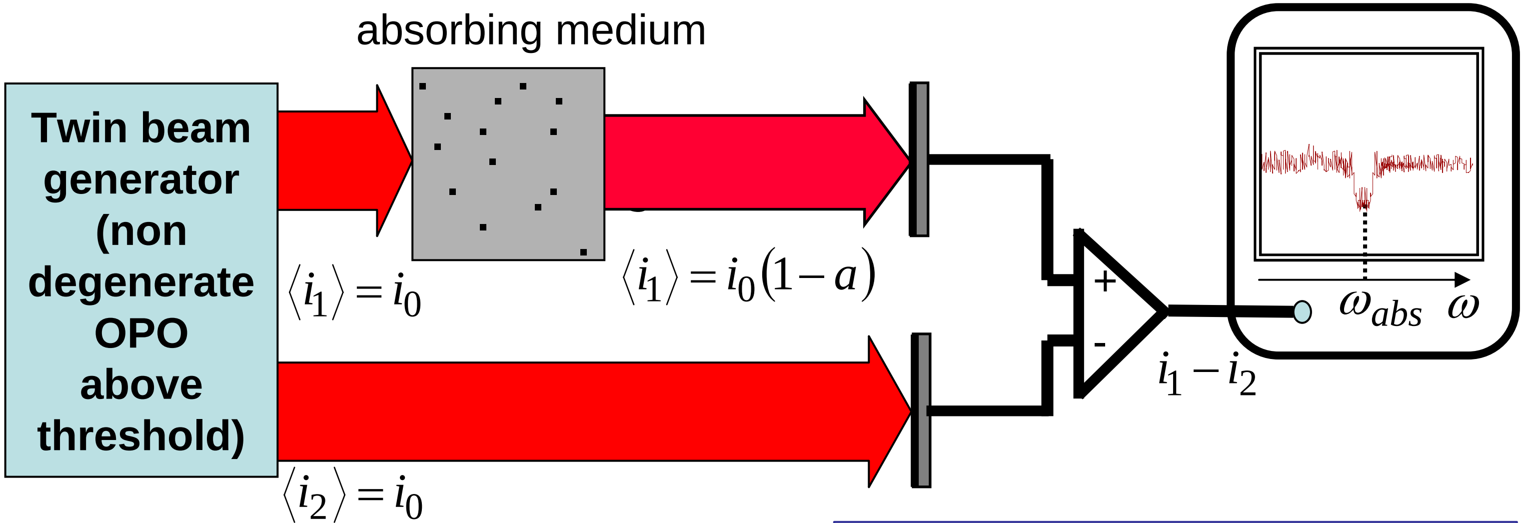
minimum detected absorption:

10^{-8} for 10 Hz bandwidth

0.8 db improvement (20%)

Absorption measurement using twin beams

*P. Souto-Ribeiro, C. Schwob, A. Maître, C. Fabre Opt. Letters **22**, 1893 (1997)*



Improvement
of signal to noise ratio :
1.9 dB (35%)
minimum detectable absorption
5.10^{-8} for 3Hz bandwidth

Gao JR, Cui F, Xue C, Xie C, Peng K
*Opt. Letters **23**, 870 (1998)*

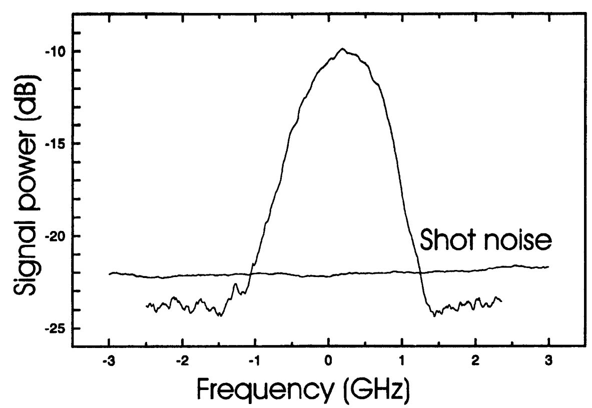


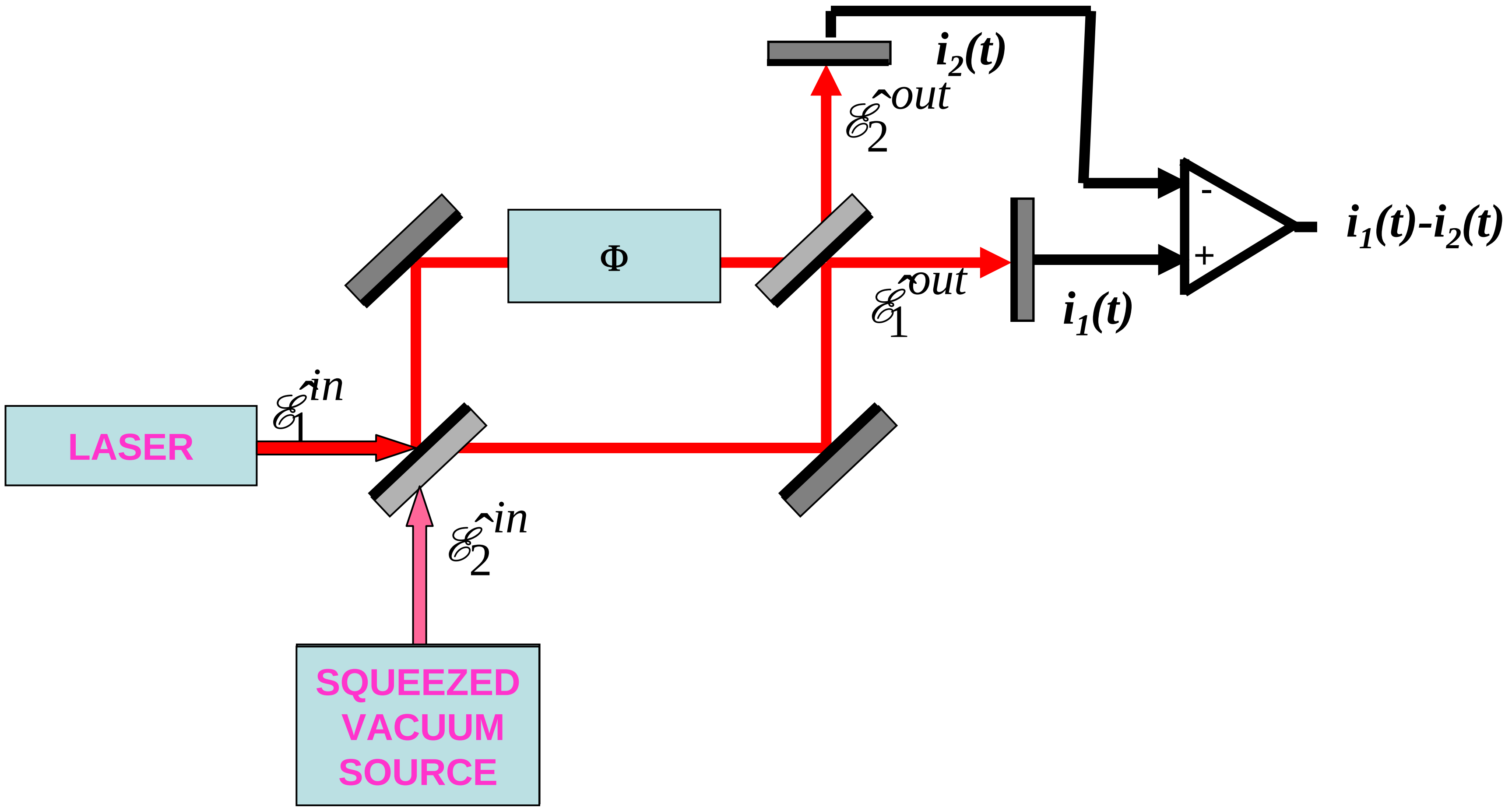
Fig. 3. Two-photon absorption signal as a function of the diode laser frequency, with the OPO signal beam in logarithmic power units. The flat trace shows the shot-noise level recorded with a YAG laser of identical power.

7dB improvement using dilute dye molecules

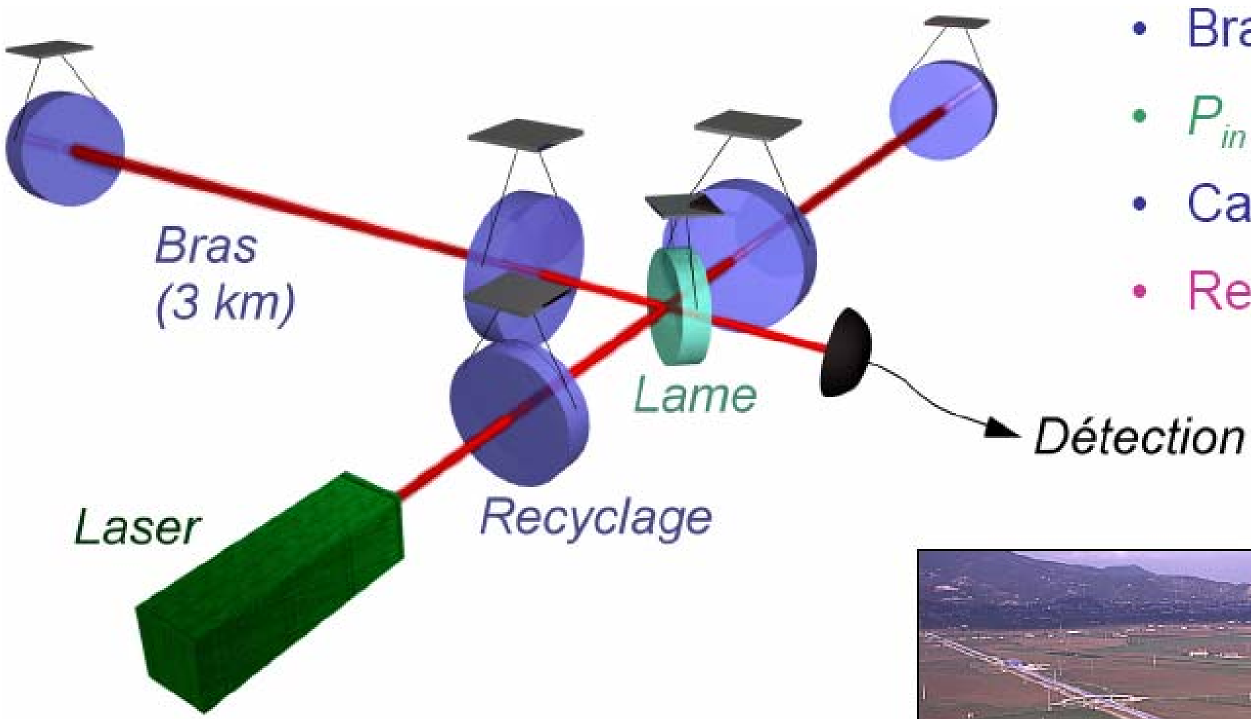
INTERFEROMETRIC

MEASUREMENTS

phase measurement using squeezed vacuum



Interferometric detectors of gravitational waves LIGO VIRGO



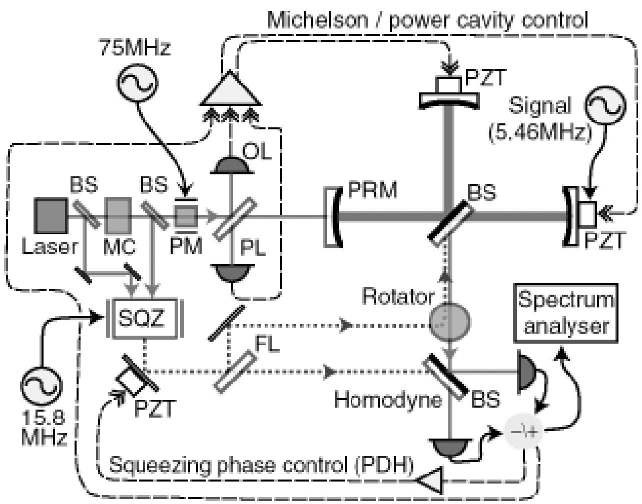
- Bras 3 km
- P_{in} = 10 W
- Cavités Fabry-Perot
- Recyclage (P = 15 kW)

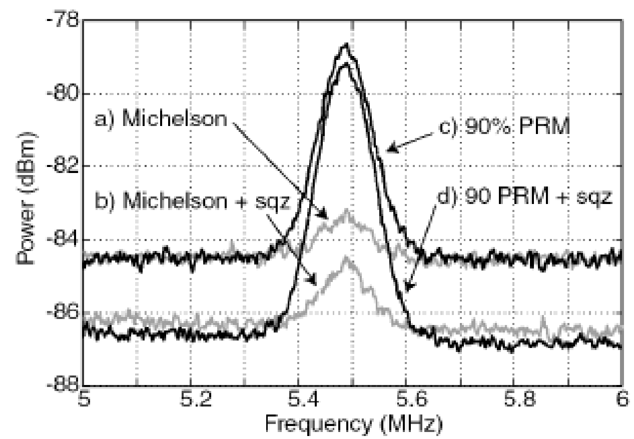
Virgo, Ligo, Geo600, TAMA, …

Sensibilité meilleure que 10^{-19} m

Démonstrations expérimentales

Injection d'un état comprimé dans un interféromètre
avec miroir de recyclage de la pompe :





K. McKenzie … D.E. McClelland, *Phys. Rev. Lett.* ***88***, *231102* (2002)

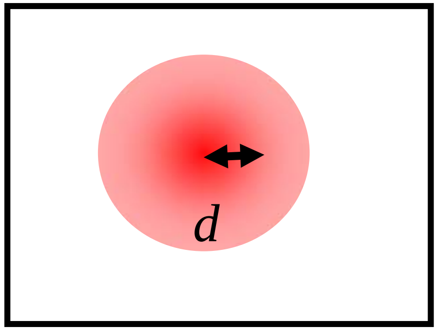
Egalement démontré avec double recyclage (pompe + signal) :
R. Schnabel *et al.* (2005)

MEASUREMENT

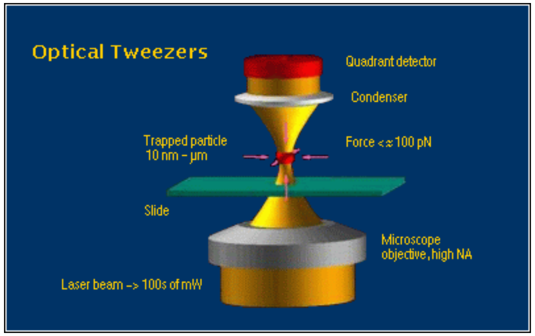
OF THE TRANSVERSE DISPLACEMENT

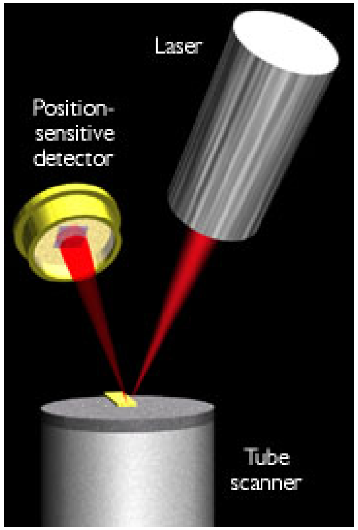
OF A LIGHT BEAM

(NANOPOSITIONING)



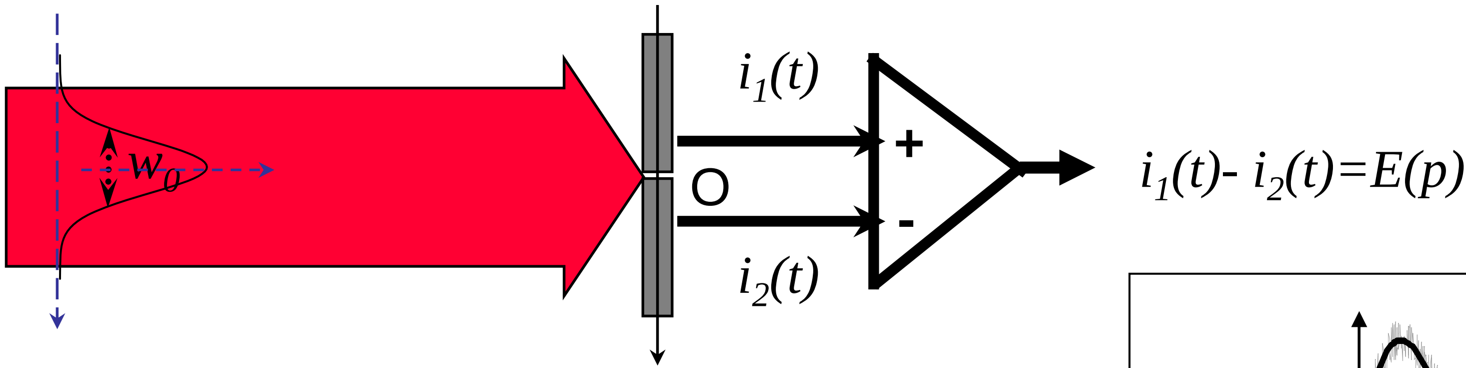
Optical tweezers

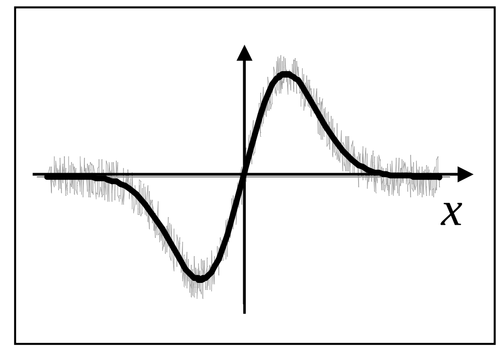




Atomic Force Microscope

Nano-positioning using a coherent TEM_{00} beam





Uncorrelated shot noise
on the two detectors

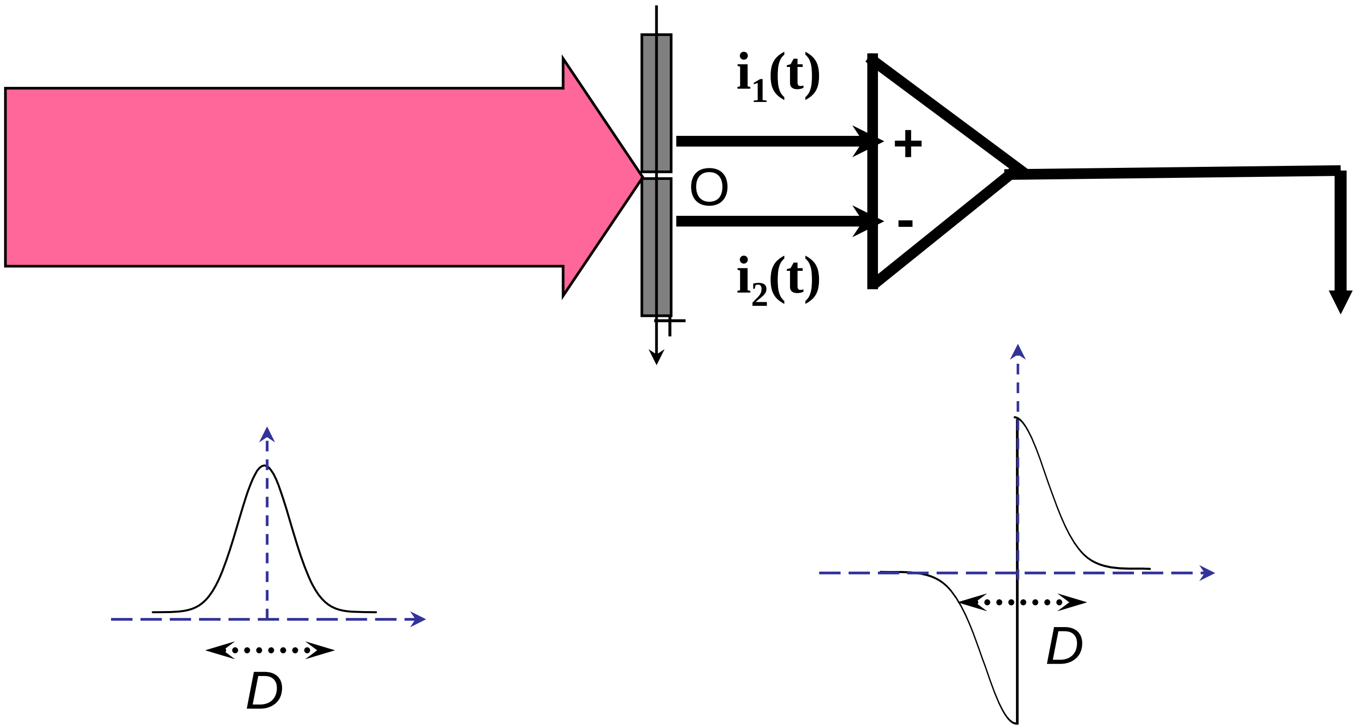
$$(\Delta x)_{Split-Standard} = \frac{\sqrt{8}}{\pi} \frac{w_0}{\sqrt{N}}$$

Can be very small (sub-nanometer range)

Is it possible to go beyond such a limit ?

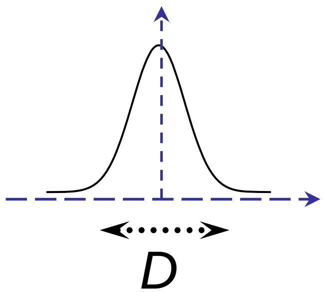
Use of non-classical state in transverse nano-positioning

The noise comes from a single « noise mode »: the « flipped mode »

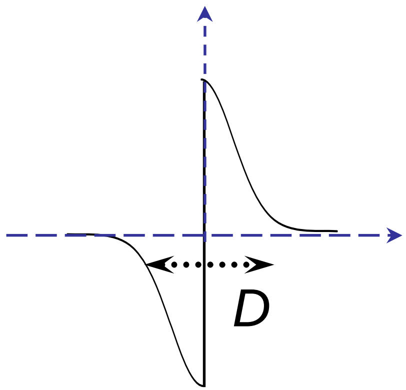


C. Fabre, J.-B. Fouet, A. Maître, Optics Letters 25, 76 (2000) to go beyond the standard quantum limit

one must use a two-mode state :



illumination mode
Gaussian mode TEM_{00}

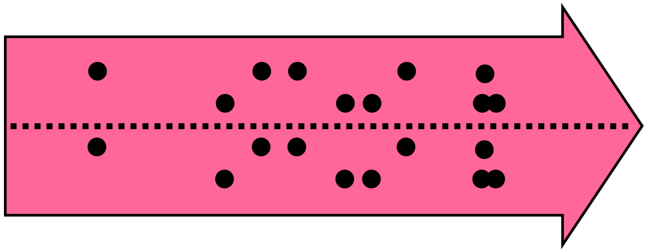


noise mode

intense coherent state In illumination mode	$\otimes$	squeezed vacuum in noise mode

The superposition of the two modes,
one coherent, the other squeezed,
gives rise to quantum correlations
when one uses another mode basis

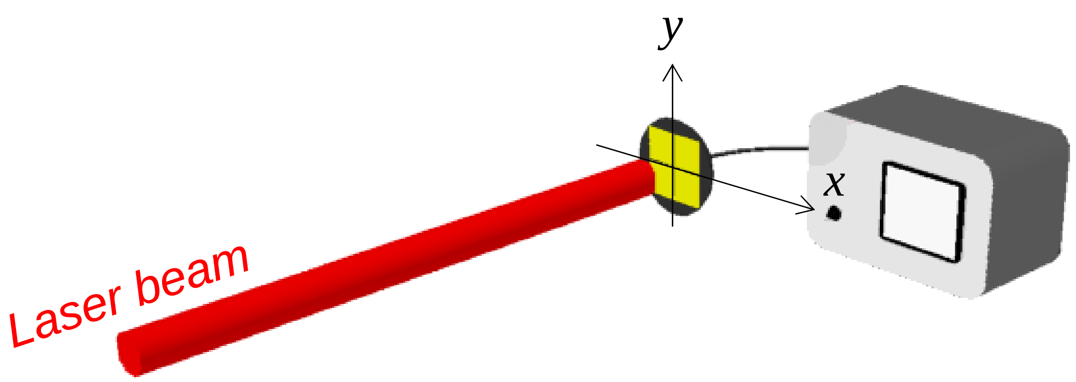
Photons detected in the two halves of the split detector are **correlated**

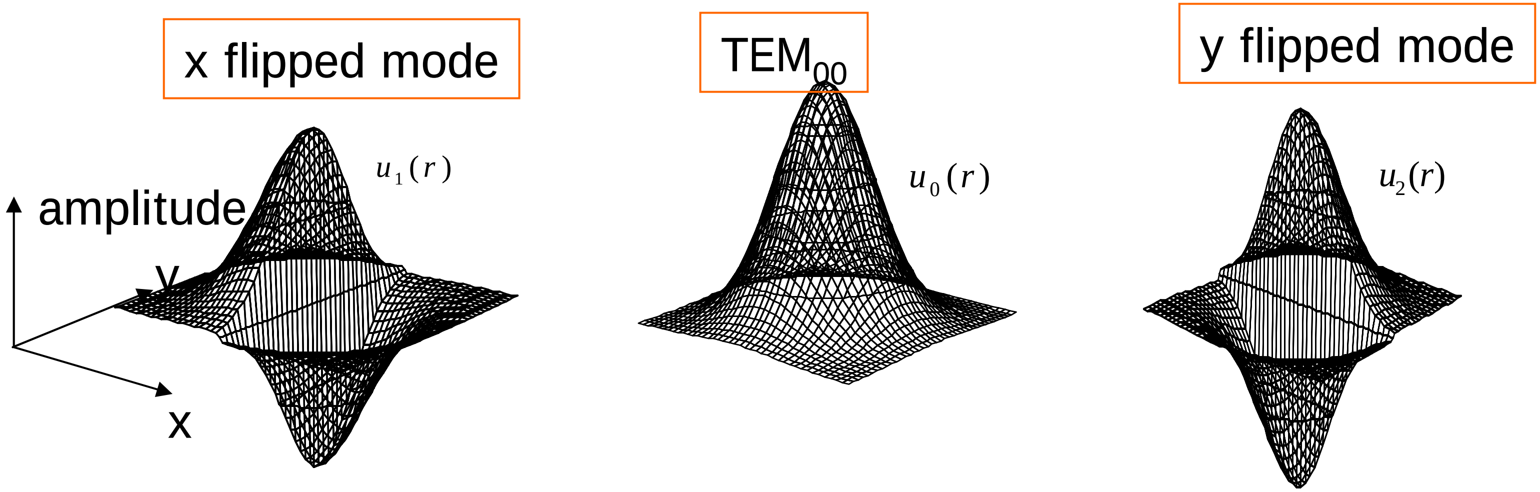


spatial ordering of photons

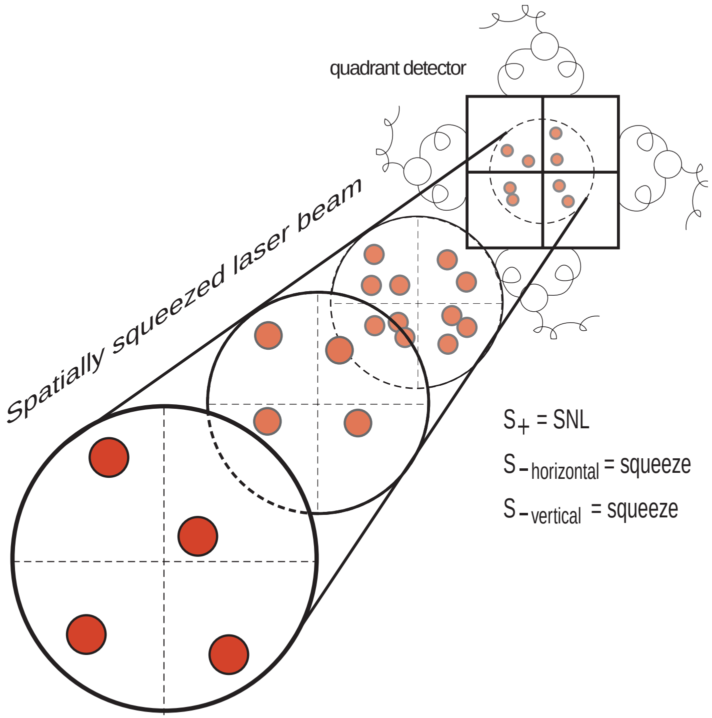
N. Treps, U. Andersen, B. Buchler, P.K. Lam, A. Maître, H. Bachor, C. Fabre, Phys. Rev. Letters **88** 203601 (2002)

2 D nano-positioning





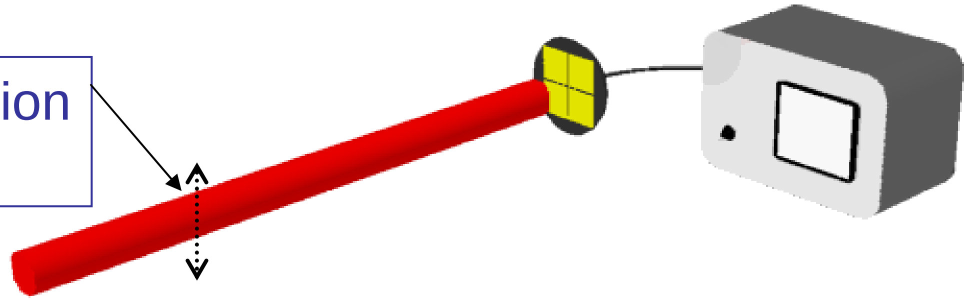
squeezed vacuum $\otimes$ *coherent state* $\otimes$ *squeezed vacuum*



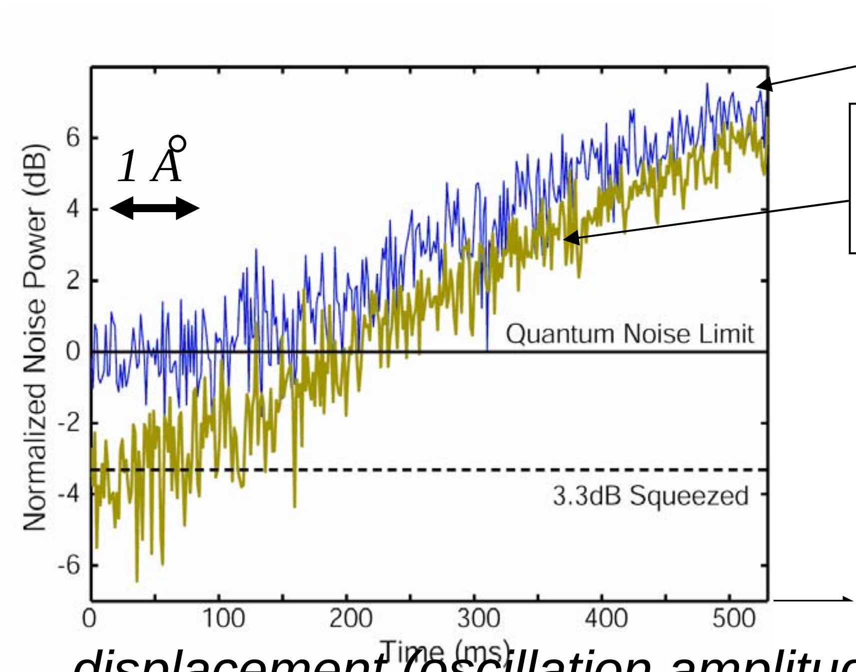
non-classical state in which photons are "ordered 4 by 4"

Experimental implementation
(collaboration with H. Bachor, Australia)

very small oscillation at 5 MHz



Intensity difference



displacement (oscillation amplitude)

Coherent laser beam

Three-mode beam “quantum laser pointer”

improvement to with respect to Standard Quantum Limit :

70% horizontal,
60% vertical

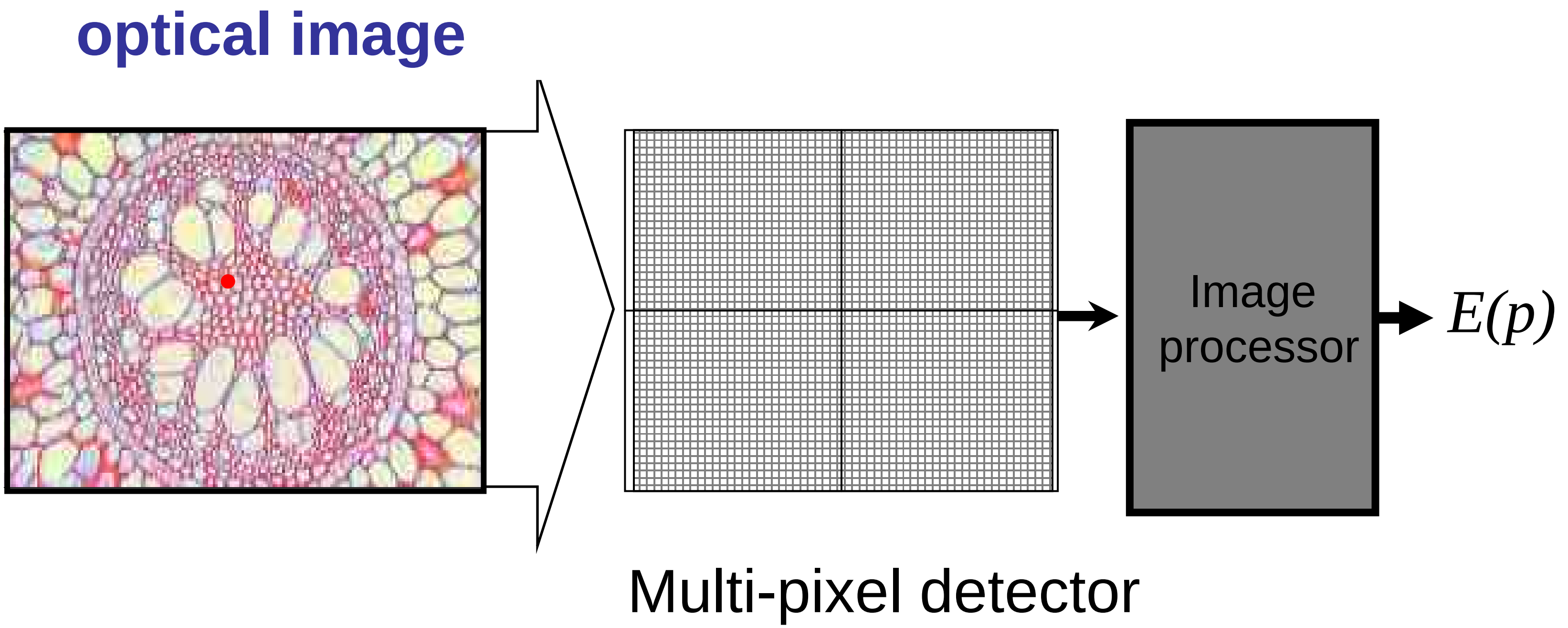
N. Treps, N. Grosse, W. Bowen, C. Fabre, H. Bachor, P.K. Lam, Science, **301**, 940 (2003)

EXTRACTION OF INFORMATION

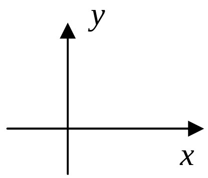
THROUGH

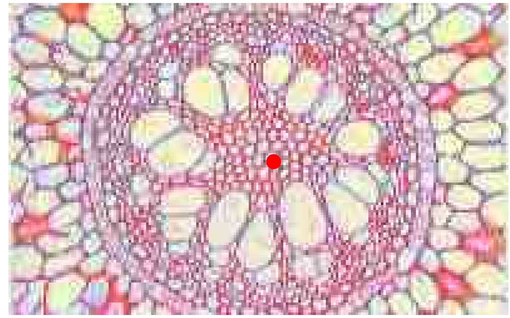
IMAGE PROCESSING

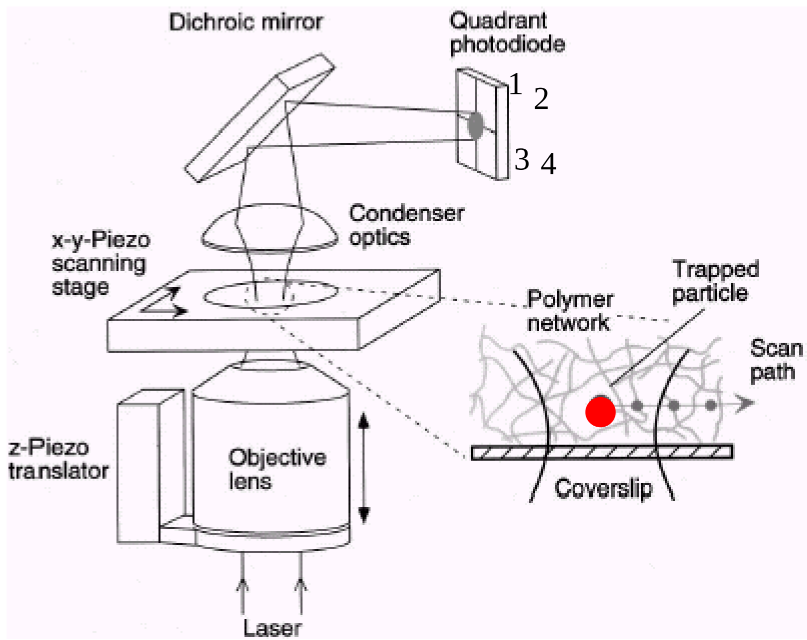
Extraction of a single parameter *p* by image processing



localisation of a very small scattering object







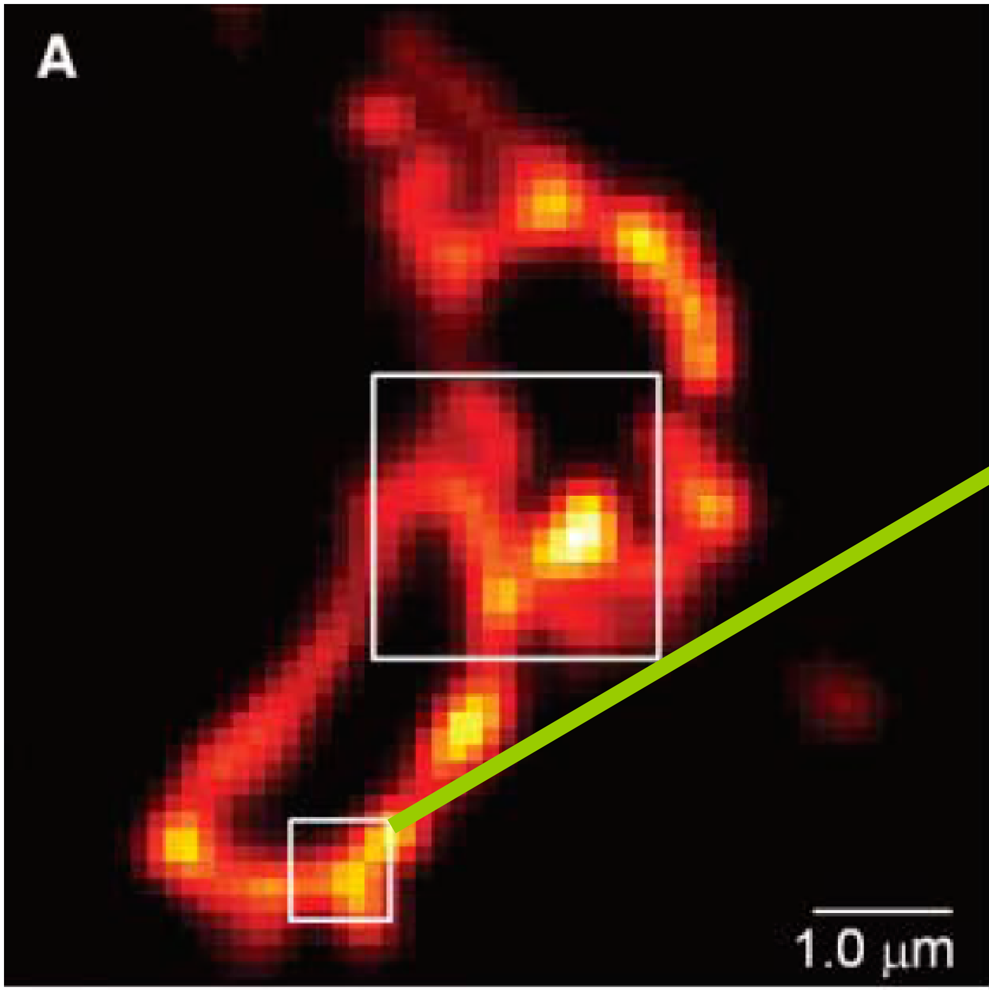
C. Tischer et al, Appl. Physics Letters, **79**, 3878 (2001)

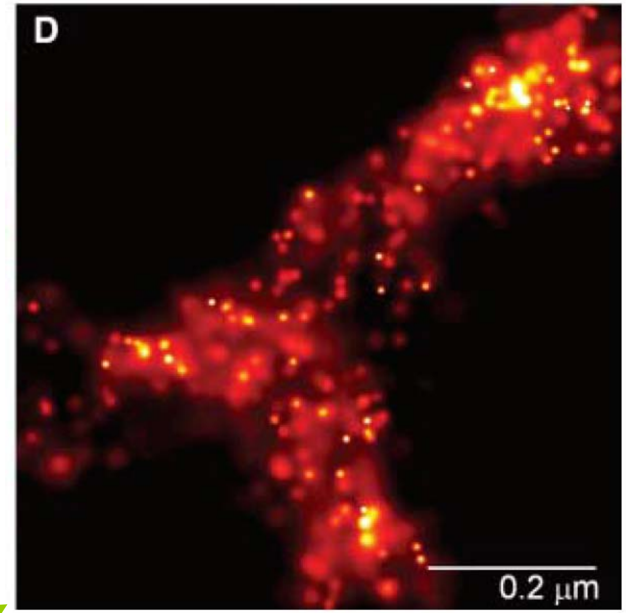
$$S(x) = i_1 + i_3 - i_2 - i_4$$

$$S'(y) = i_1 + i_2 - i_3 - i_4$$

Super-resolution techniques based on image processing

Image of fluorescent proteins by conventional high resolution microscopy

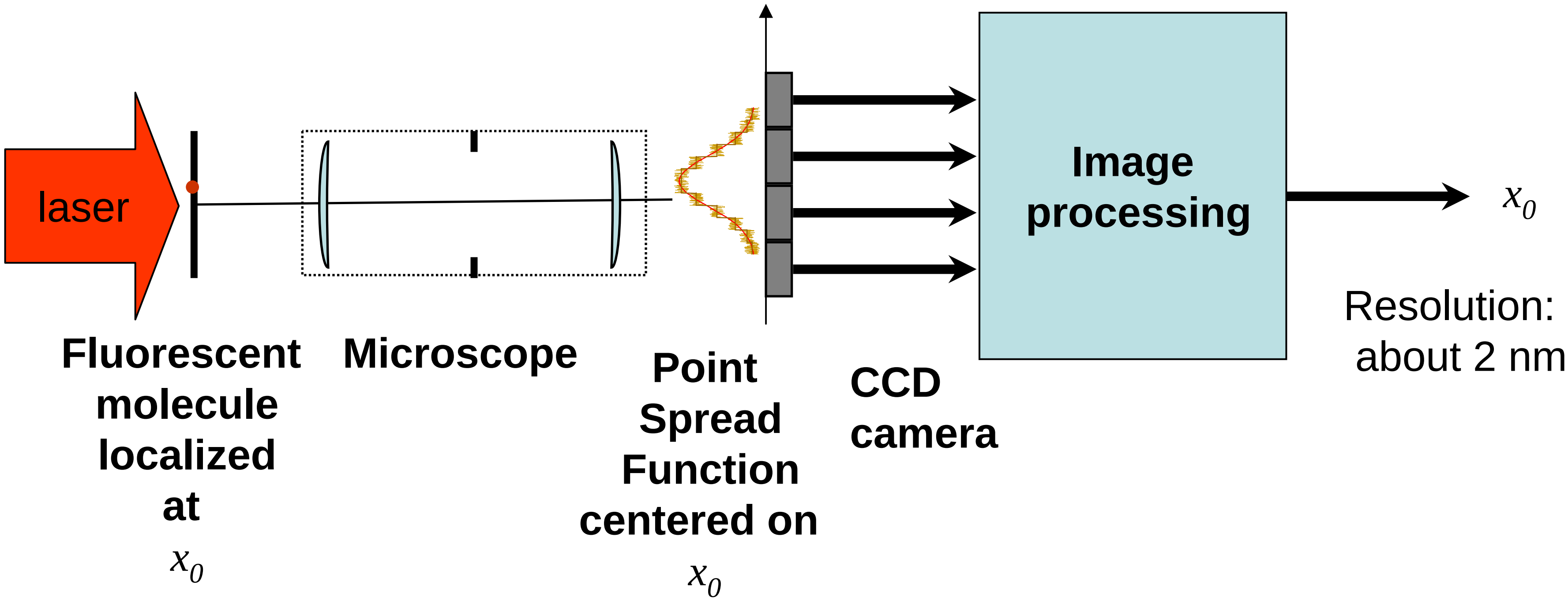




Super-resolution image

Imaging Intracellular Fluorescent Proteins at Nanometer Resolution
E. Betzig et al Science **313** 1642 (2006)

How is superresolution obtained ?



What is the ultimate limit on the accuracy of determination of x_0 ?

Related to the **quantum fluctuations** of the signals recorded by the pixels of the CCD camera

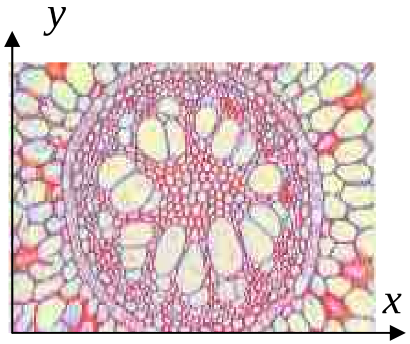
General case

p determined from the value of an **estimator** $E(p)$

obtained by **image processing**

Information on p is extracted from the local field amplitude

normalized field amplitude distribution in the image plane $u_0(x, y, p)$



$$E(p) = \iint dxdyF(x, y, u_0(x, y, p))$$

Often : $$E(p) = \iint dxdyg(x, y)\left|u_0(x, y, p)\right|^2$$

The quantum noise on the local field is the ultimate limit for the accuracy of the determination of p

What is the **higher limit**

imposed by quantum noise

to the **Signal to Noise ratio S/N**

on the determination

of parameter p ?

-**Maximum signal S** :
optimize the image processing protocol

-**Minimum noise N** :
reduce quantum fluctuations
on the estimator $E(p)$

Answer given by information theory

(collaboration with P. Réfrégier, Marseille)

The minimum variance of any (unbiased) estimator of p

is given by the **Cramer-Rao bound** $(\Delta p)_{CRb}$

- It is valid independently of the precise strategy used in the extraction

- It depends only on the statistical distribution of the noise affecting the measured quantities

1) Intensity measurement:

V. Delaubert, N. Treps, C. Fabre, H. Bachor, P. Réfrégier, Europhys. Letters **81** 44001 (2008)

Poisson noise on each pixel

$$(\Delta p)_{S-CRb} = \frac{p_0}{2\sqrt{N}}$$

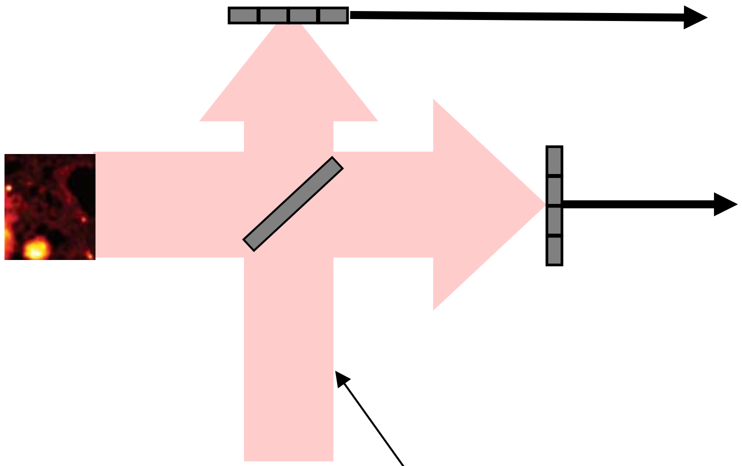
N : total number of photons measured

p_0 : characteristic parameter value

$$\frac{1}{p_0^2} = \iint dxdy \left(\frac{\partial u_0}{\partial p} \right)^2$$

$u_0(x, y, p)$ normalized field amplitude distribution in the image plane

2) Amplitude measurement:



local oscillator of arbitrary shape

homogeneous Gaussian noise on each pixel

$$(\Delta p)_{S-CRb} = \frac{p_0}{2\sqrt{N}}$$

Same expression as for intensity measurement

$$\left(\Delta p\right)_{S-CRb} = \frac{p_0}{2\sqrt{N}}$$

is an « absolute limit »:

No measurement can do better

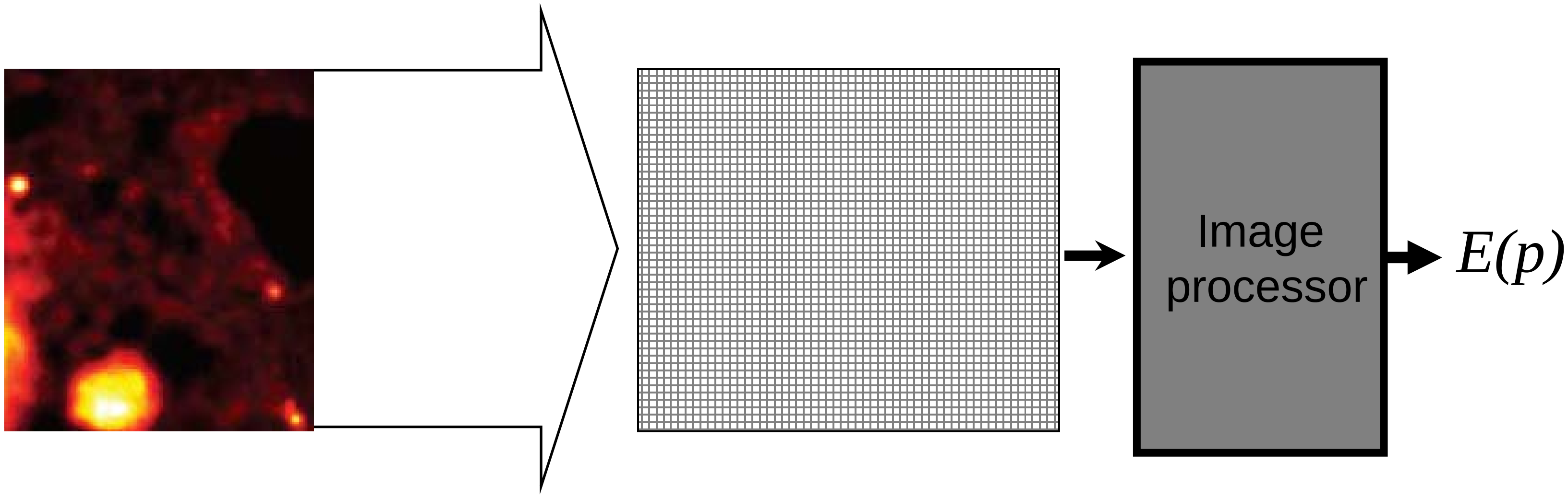
on a shot noise limited image

is there a way to reach

the Standard Cramer Rao bound ?

V. Delaubert, N. Treps, C. Fabre, H. Bachor, P. Réfrégier
Europhys. Letters **81** 44001 (2008)

1) Intensity measurement:



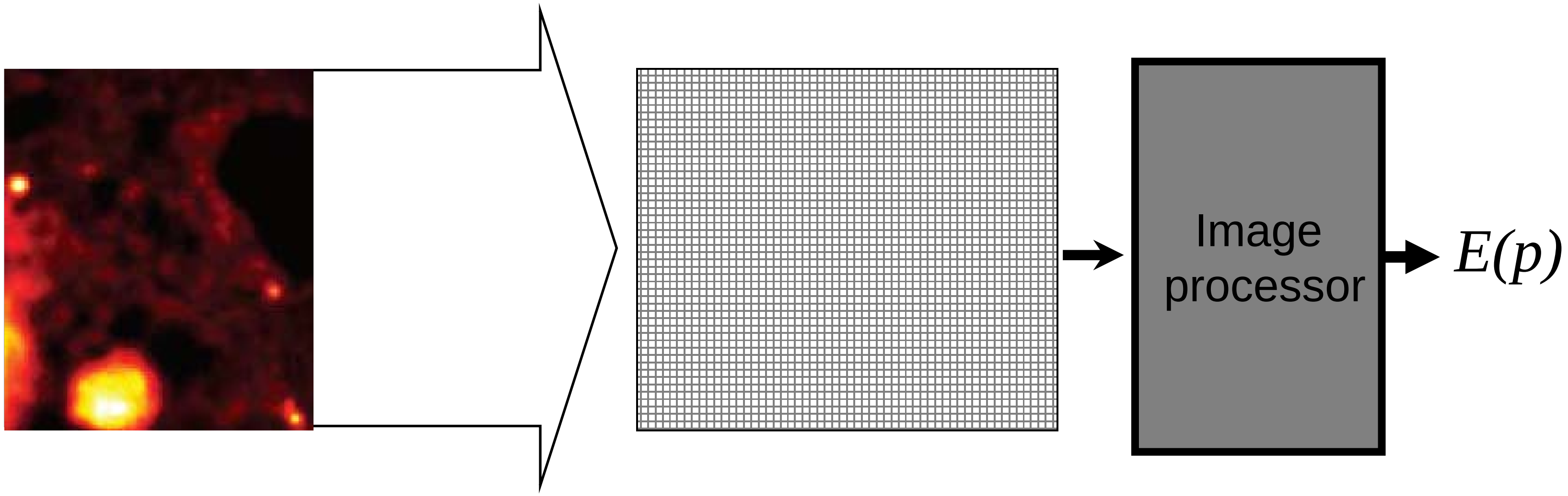
Linear processing of intensities:

$$E(p) = \sum_{pixel\ m} i_m(p)\, g_m = \iint dxdy\; i(x,y,p)\; g(x,y)$$

Optimum value of $g(x,y)$ for maximum signal to noise ratio:

$$g_{optimum} = \frac{1}{u_0(x,y,p)} \frac{\partial}{\partial p} u_0(x,y,p)$$

1) Intensity measurement:

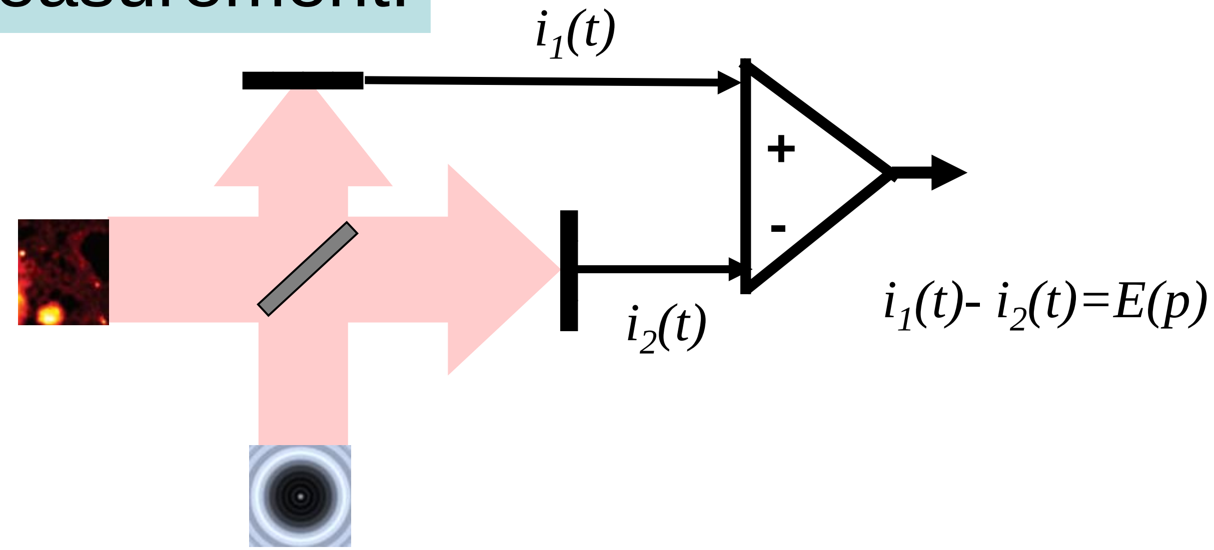


Minimum measurable value of p using linear processing (S=N) :

$$p_{\min} = \frac{p_0}{2\sqrt{N}}$$

Standard Cramer Rao bound reached !

1) Amplitude measurement:



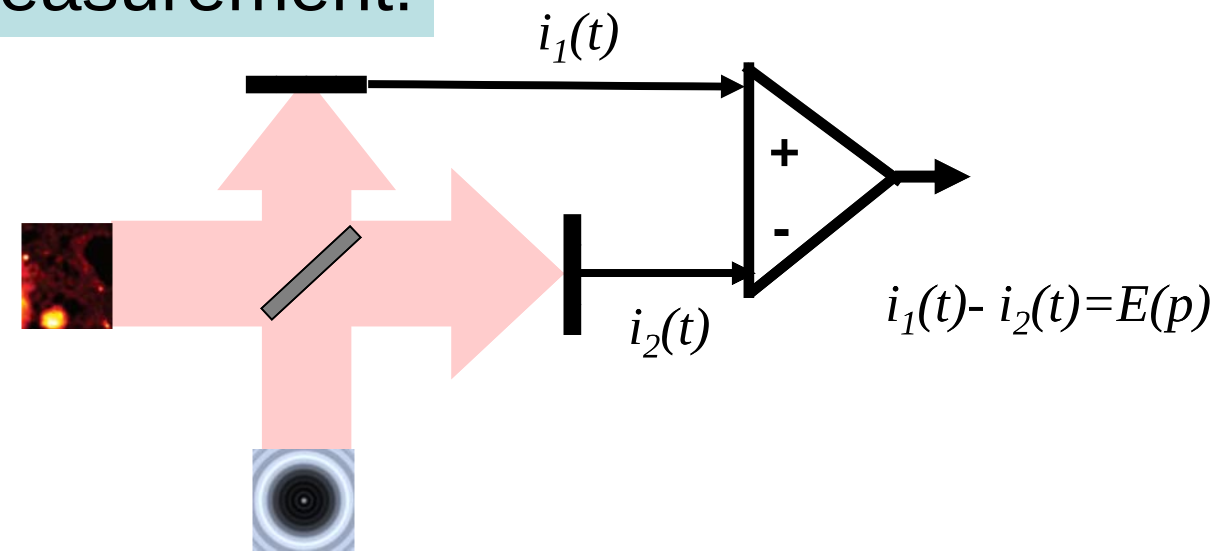
local oscillator $u_{LO}(x,y)$

detectors measure **total intensity**

$$E(p)=\iint dxdy\,u_0(x,y,p)\,u^*_{LO}(x,y)$$

Linear processing of amplitudes

1) Amplitude measurement:



local oscillator $u_{LO}(x,y)$

Optimum choice of the local oscillator amplitude:

$$u_{optimum}(x,y) = \left[\frac{\partial}{\partial p} u_0(x,y,p) \right]_{p=0}$$

Minimum measurable value of p (S=N) :

Standard Cramer Rao bound reached again !

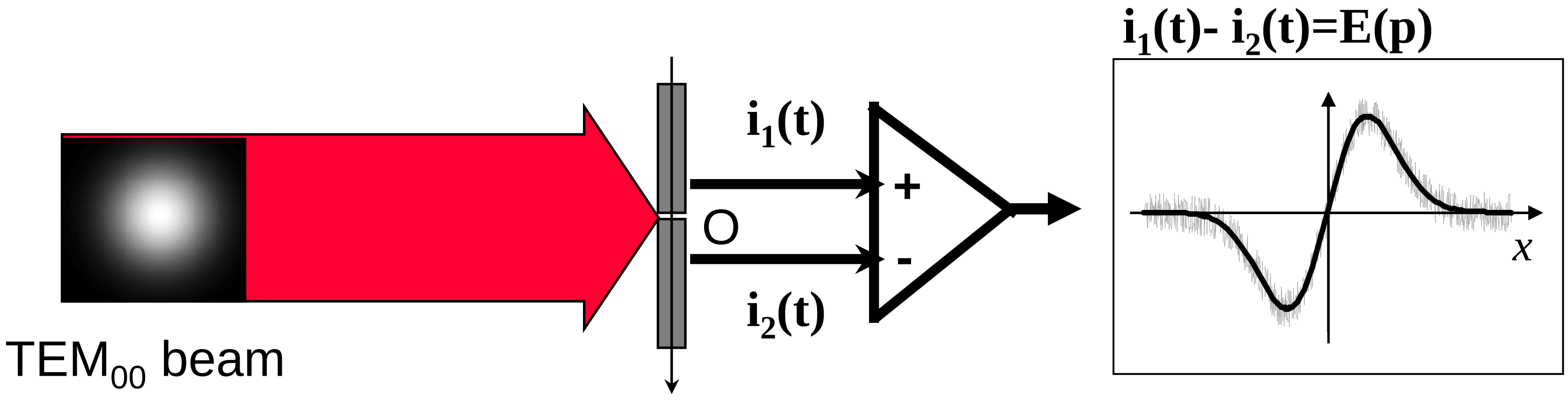
$p_{\min}$ $2\sqrt{N}$

Example

Laser beam nano-positioning

(two-pixel processing)

1) The standard technique : split detector

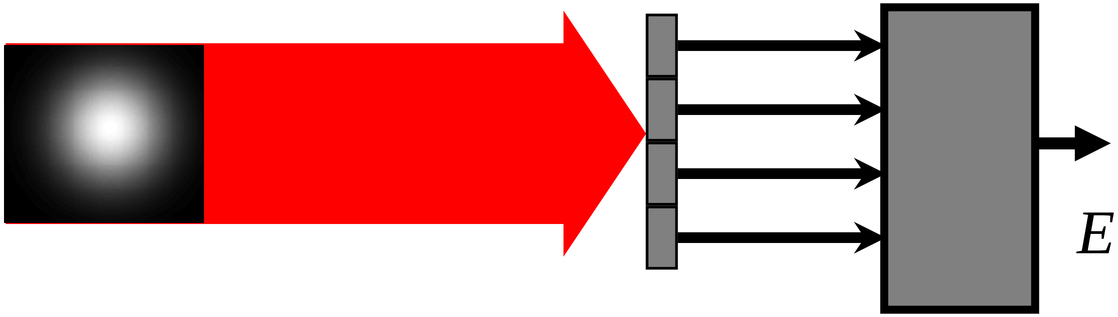


With coherent beam :

$$(\Delta p)_{Split-Standard} = \frac{\sqrt{8}}{\pi}\frac{w_0}{\sqrt{N}} \approx 1.2\ (\Delta p)_{S-CRb}$$

Split detector technique is not the best detection technique !

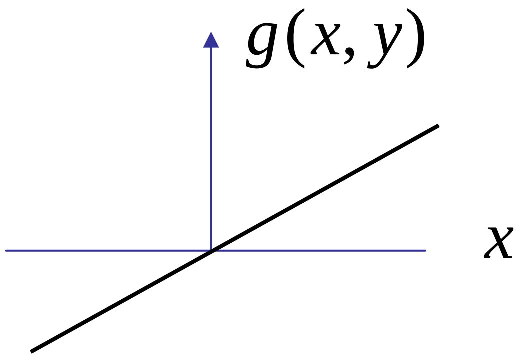
2) Optimized technique on intensity measurement:

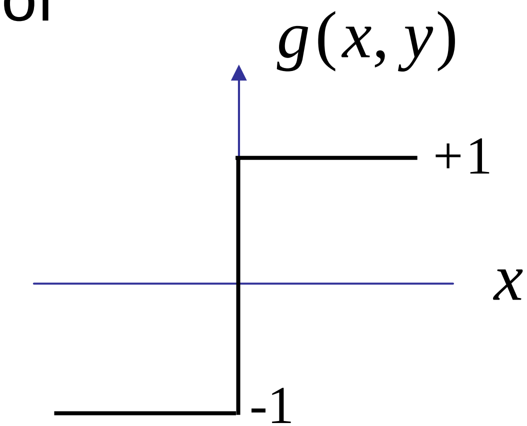


$$E(p) = \iint dx dy\, i(x, y, p) g(x, y)$$

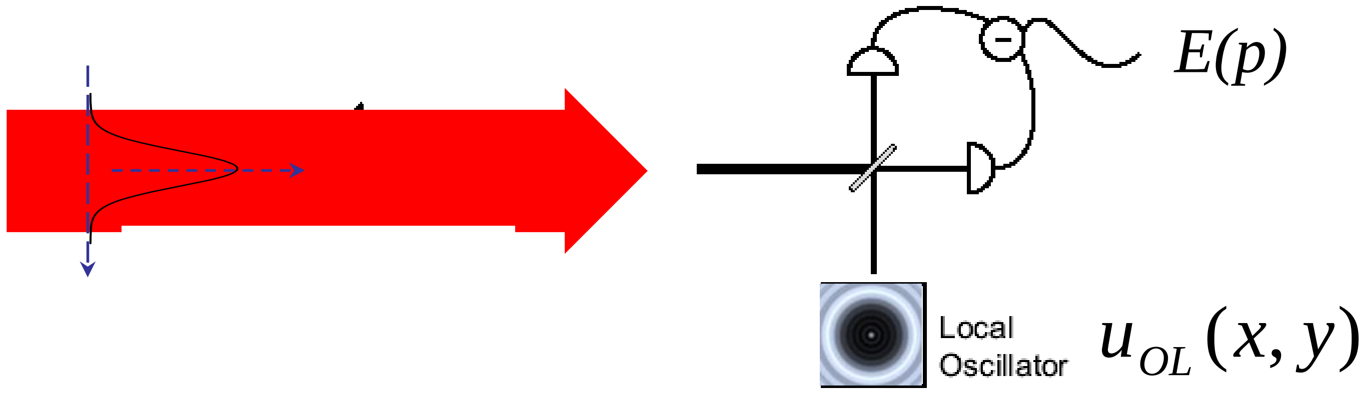
optimized choice of $g(x,y)$ for a TEM_{00} beam:

$g(x,y) = x$ instead of





3) Optimized technique on homodyne measurement:



(Detectors measure total intensity)

Local Oscillator shape $u_{OL}(x,y)$ to be found

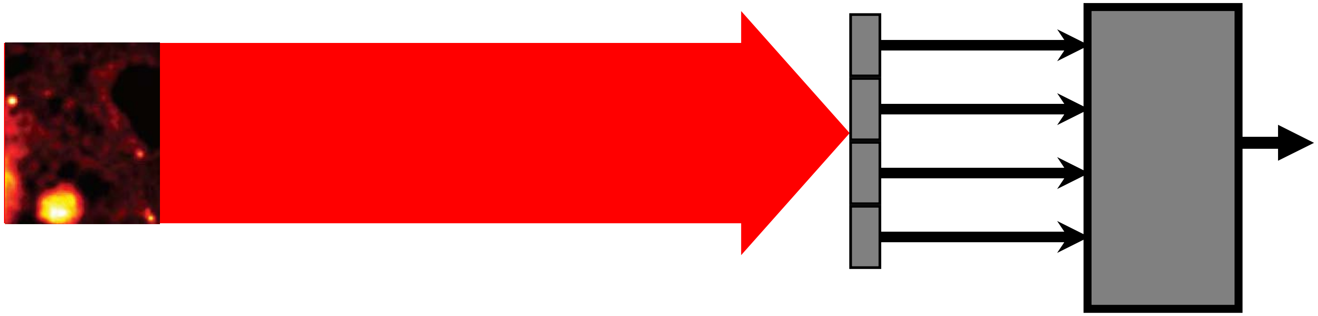
optimized choice of $u_{OL}(x,y)$ for a TEM_{00} beam:

V. Delaubert et al Phys. Rev A **74** 053823 (2006)

Standard Cramer Rao bound reached again !

How to go beyond ?

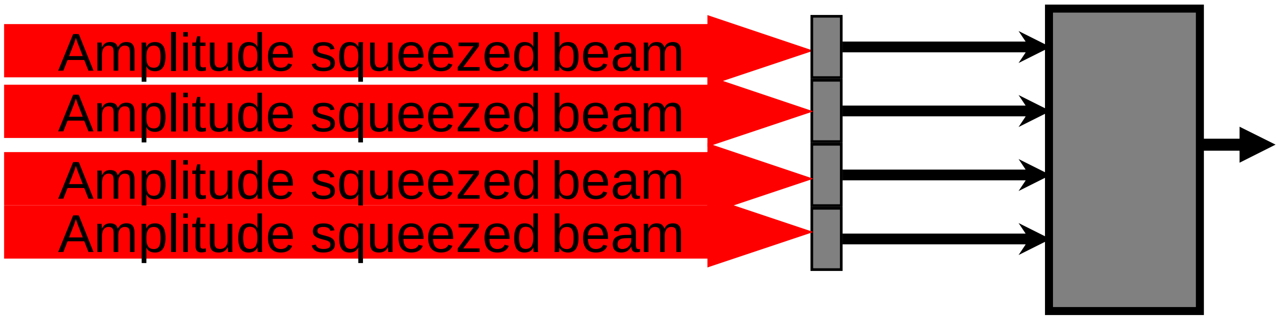
1) Squeeze the total image ?



Use amplitude squeezed state,
or even number state
in mode $u_0(x, y)$?

does not improve measurements of parameters
which do not change the total intensity

2) Squeeze each partial beam ?



New Cramer-Rao bound:

$$p_{\min} = S_q \frac{p_0}{2\sqrt{N}}$$

Common squeezing factor of all beams

Possible but difficult !

3) Squeeze the right mode

The quantum noise on the estimator $E(p)$ comes from a single « **noise mode** » $u_1(x,y)$

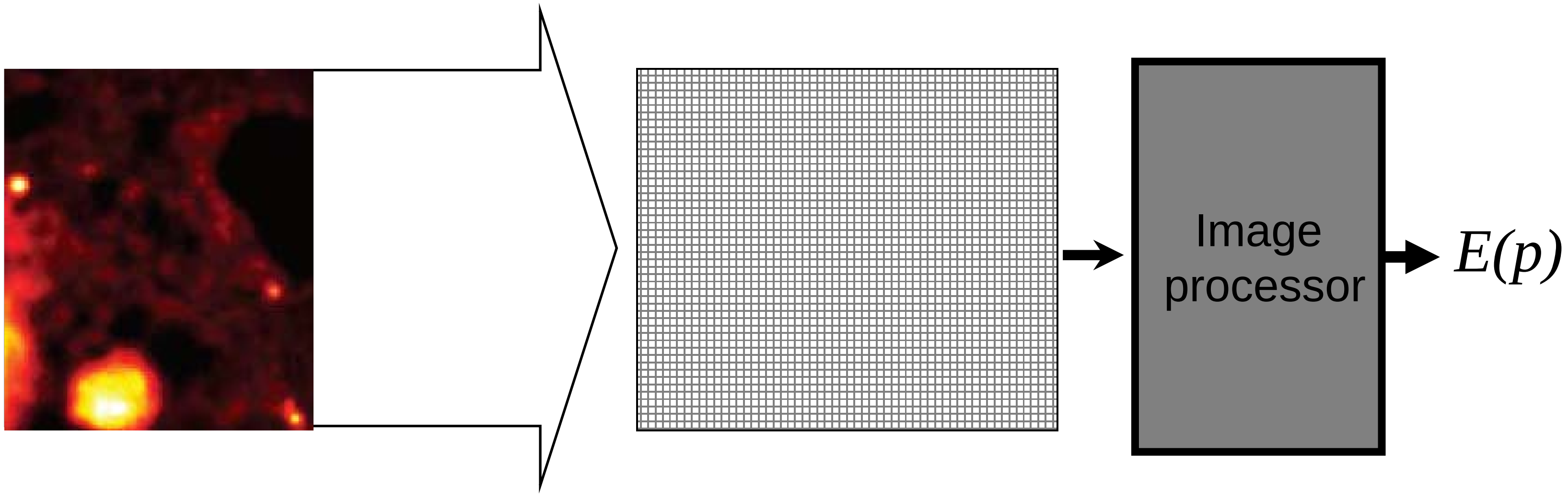
N. Treps et al Phys. Rev A**71** 013820 (2005)

One can build a basis of transverse functions starting with $u_0(x,y)$ $u_1(x,y)$

$$\{u_0(x,y), u_1(x,y), \ldots, u_n(x,y), \ldots\}$$

Quantum fluctuations on $E(p)$ come only from mode $u_1(x,y)$

1) Intensity measurement beyond the SCRB:

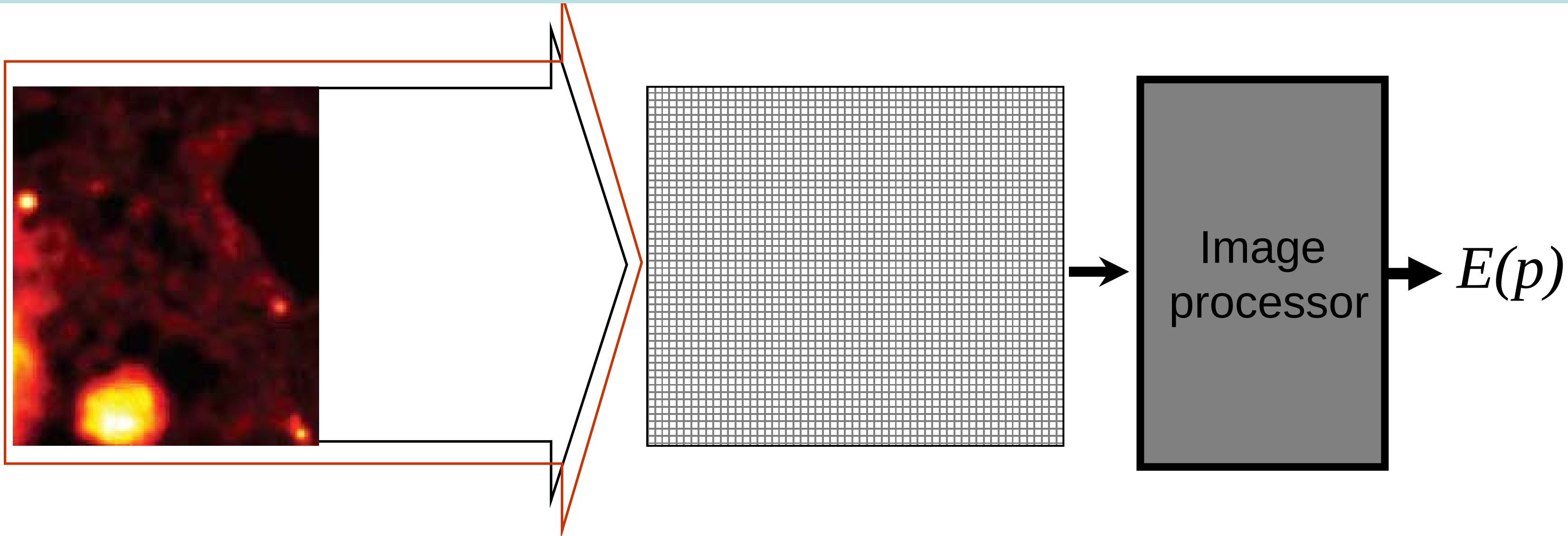


if $$E(p) = \iint dxdy\, i(x, y, p)\, g(x, y)$$

then : $$u_1(x, y) = u_0(x, y)\, g(x, y)$$

For optimum gain : $$u_1(x, y) = \frac{\partial}{\partial p} u_0(x, y, p)$$

non-classical beam for squeezed image processing:



Coherent state in mode $u_0(x,y)$

$\otimes$ Squeezed vacuum in mode $u_1(x,y)$

$$p_{\min} = S_q \frac{p_0}{2\sqrt{N}}$$

The superposition of these two modes creates anticorrelated quantum fluctuations on the different pixels,
which results in a noise cancellation in $E(p)$

Example

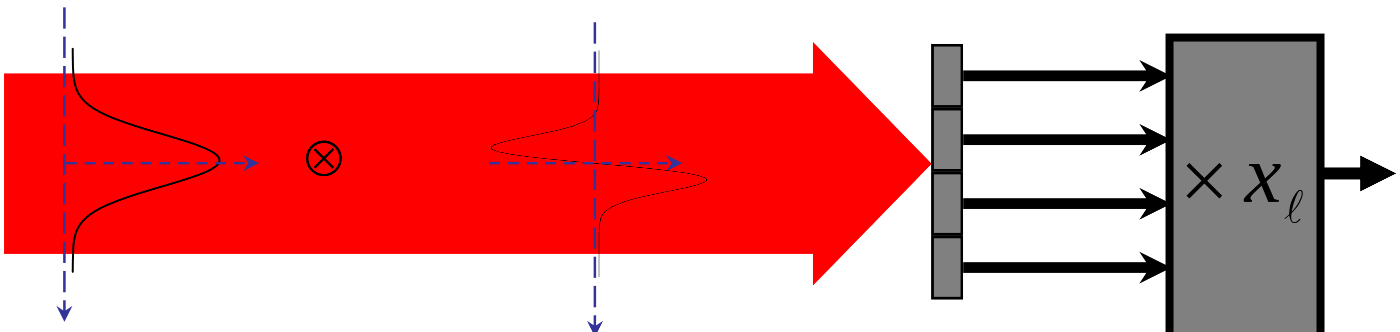
Beyond the SCRB in

laser beam nano-positioning

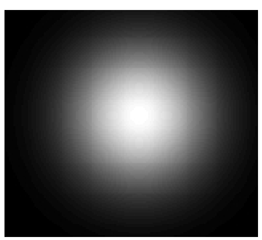
Intensity measurement

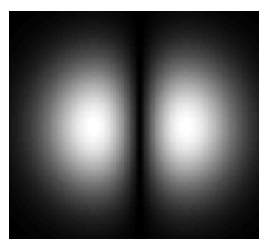
Noise mode for the optimized image processing:

$$u_1(x, y) = g(x, y)u_0(x, y) = xTEM_{00} = TEM_{10}$$

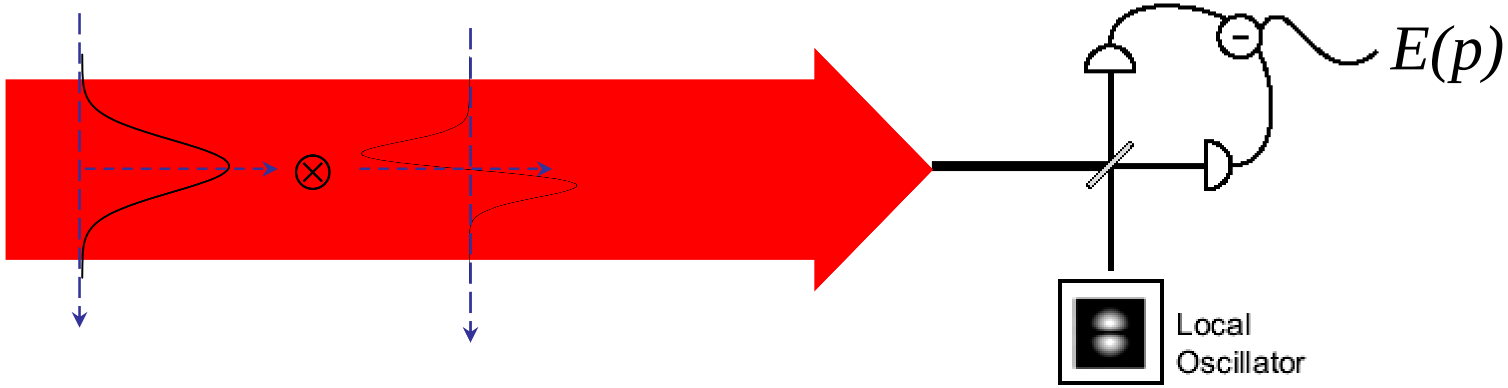


Coherent state ⊗ Squeezed vacuum

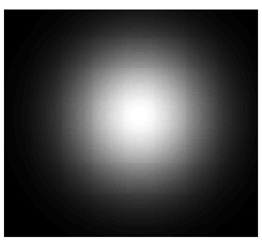


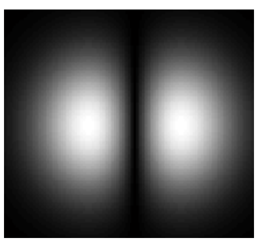


Homodyne measurement



Coherent state ⊗ Squeezed vacuum

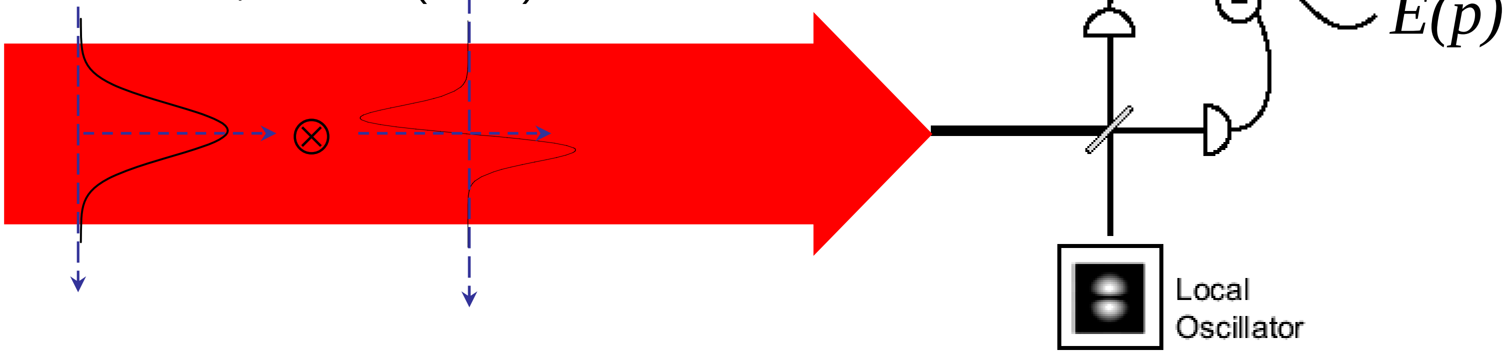




Same superposition of coherent state and squeezed state as previously

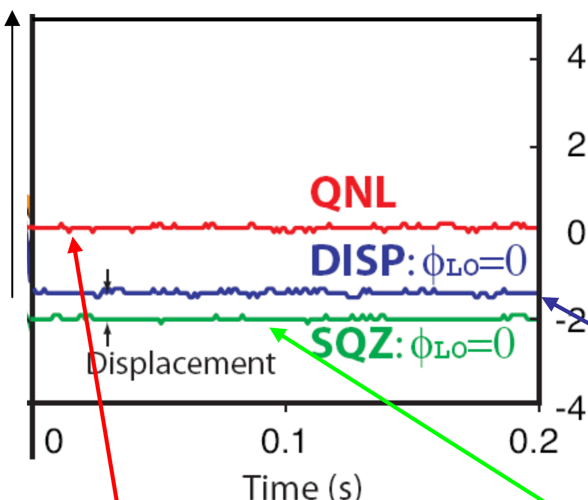
Experimental implementation

M. Lassen, V. Delaubert, J. Janousek, K. Wagner, H-A.Bachor, P.K.Lam, N.Treps, P.Buchhave, C.Fabre, C.C.Harb, Phys. Rev. Letters **98**, 083602 (2007)



Coherent state ⊗ Squeezed vacuum

Spectral power of E(p)



beam displacement smaller than Standard Cramer Rao bound

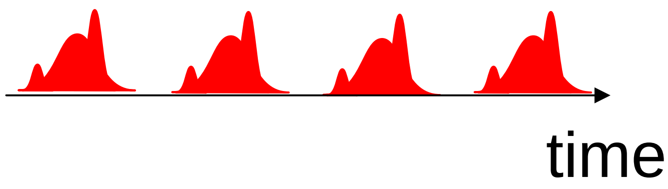
Standard Cramer Rao bound

Reduced noise with squeezed TEM_{10}

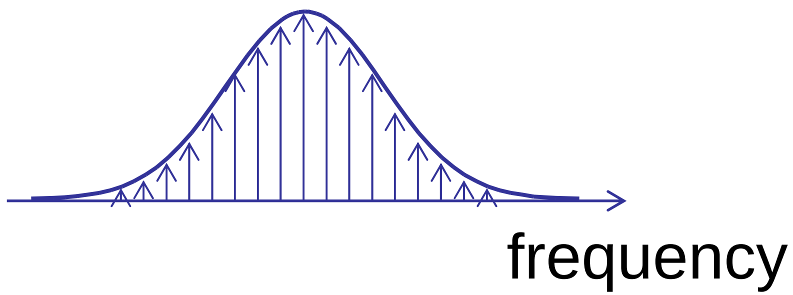
MEASUREMENTS IN THE TIME/FREQUENCY DOMAIN

From spatial quantum effects to temporal quantum effects

Trains of pulses of arbitrary shape



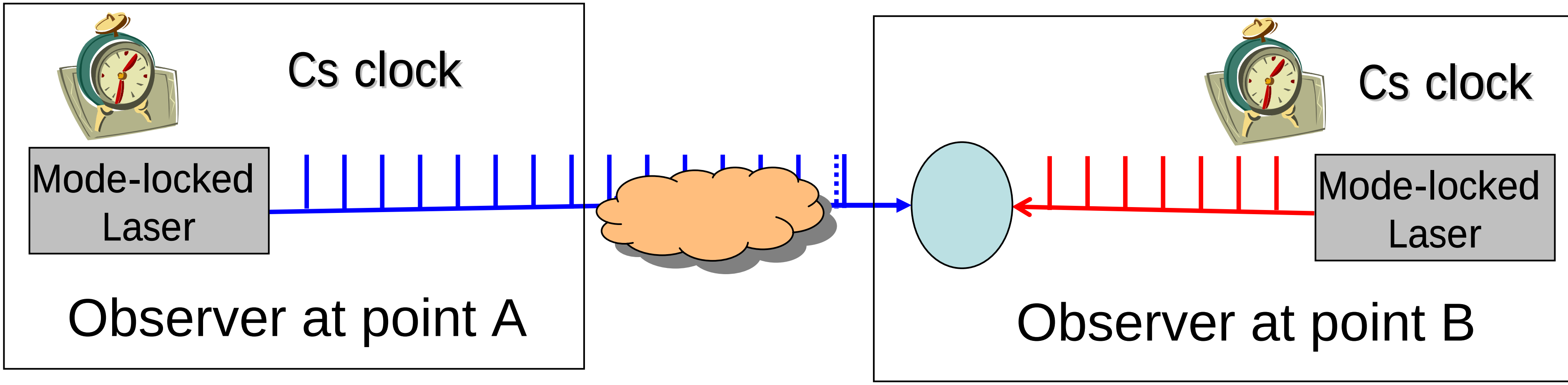
« Frequency combs »



Can it be used to improve information processing ?

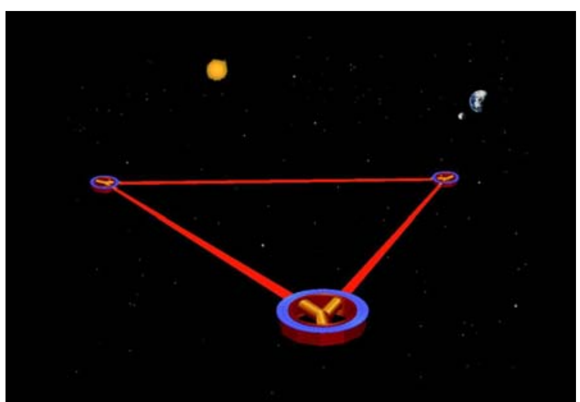
How to generate
entanglement and reduced quantum fluctuations
in such systems ?

temporal positioning of a train of pulses



time transfer problem

Implementation of Einstein's protocol for clock synchronization



Space-time positioning of satellites flying in formation

Quantum limit to the accuracy of time transfer

B. Lamine, C. Fabre, N. Treps, "*Quantum improvement of time transfer between remote clocks*"
To be published in Phys. Rev. Letters (2008)

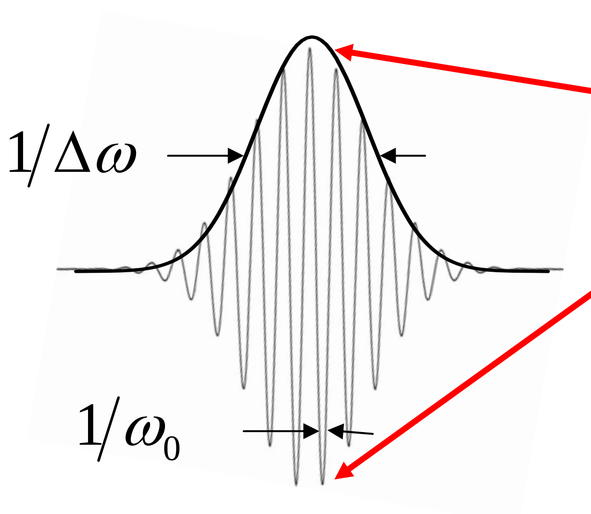
It is given by the **Cramer Rao Bound**
in the case of a Gaussian coherent pulse:

$$(\Delta t)_{S-CRb} = \frac{1}{\sqrt{N}} \frac{1}{2\sqrt{\omega_0^2 + \Delta\omega^2}}$$

N : total number of photons

ω_0 : mean frequency

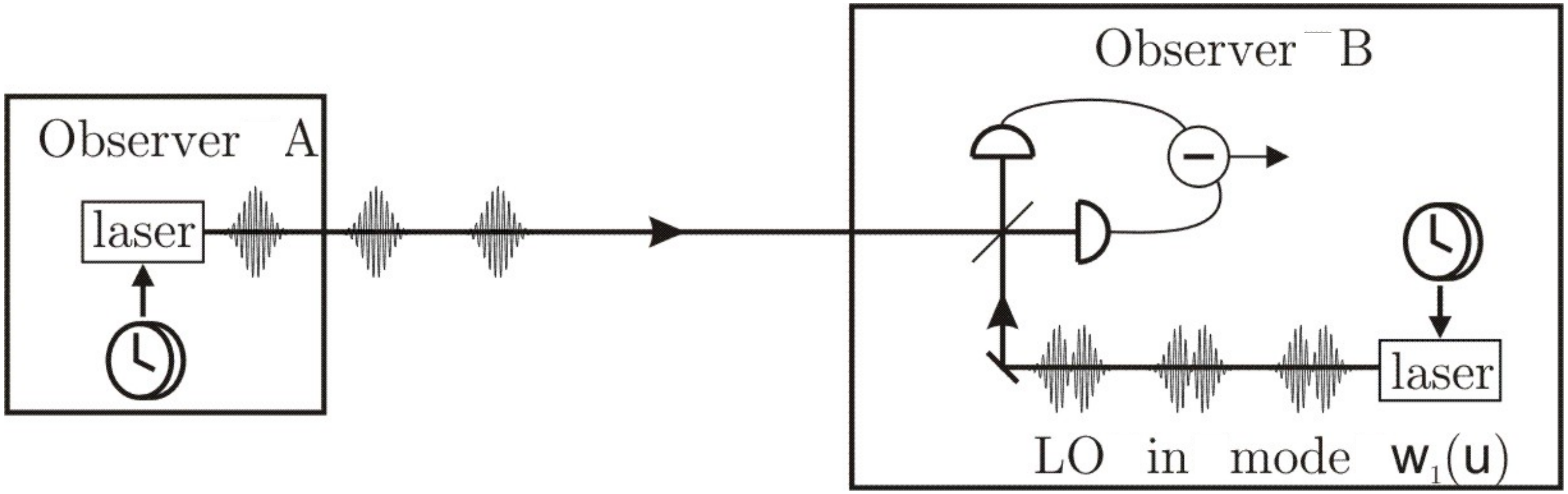
$\Delta\omega$: frequency spread



shift of pulse enveloppe maximum

shift of oscillation at optical frequency

Optimal balanced homodyne measurement



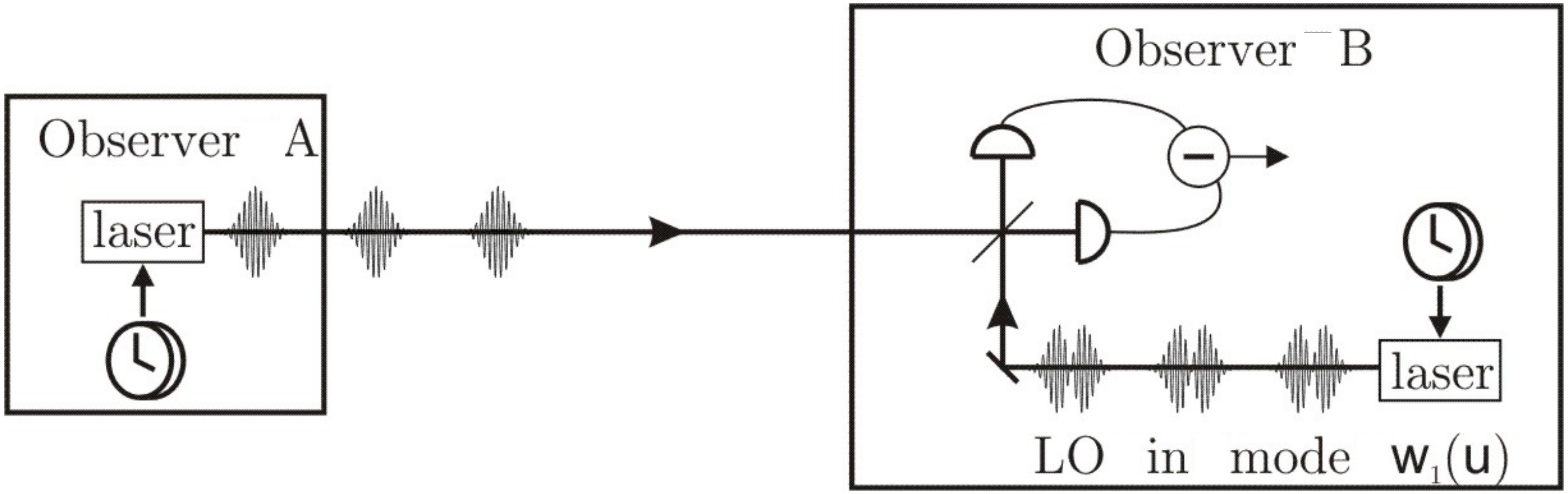
Optimal Local Oscillator temporal shape to be found

w_1 : combination of a  quadrature and of 

Cramer Rao Bound reached:
no other measurement can do better on a shot noise limited pulse

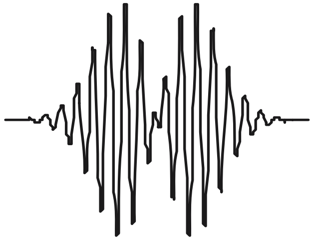
Ultimate sensitivity of 20 yoctoseconds (10 fs, 1s integration time)

Simplified version of homodyne measurement



Simplified version of LO: the « temporal TEM_{01} pulse »

w_1 :

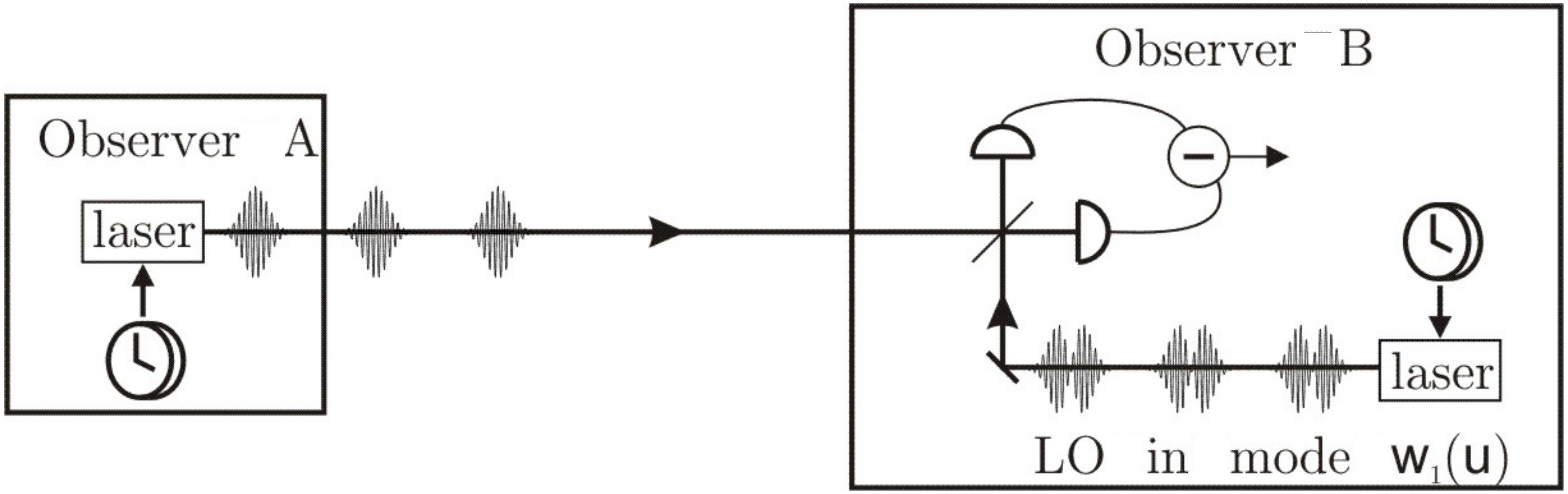


$$\left(\Delta t\right)_{S-CRb} = \frac{1}{\sqrt{N}} \frac{1}{2\Delta\omega}$$

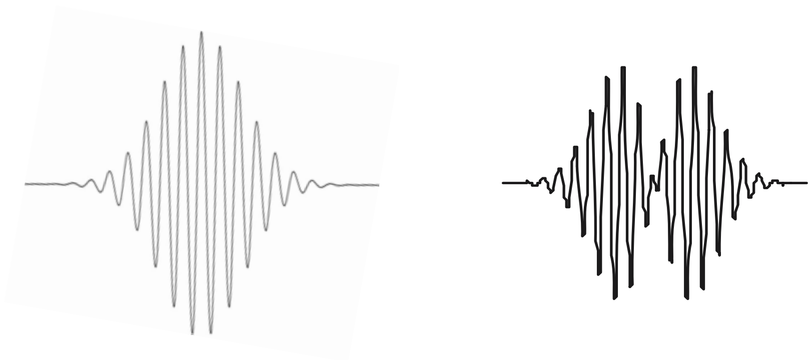
optimizes only the measurement of the pulse maximum

Sensitivity of 70 yoctoseconds

Beyond the standard quantum limit in optimal time transfer



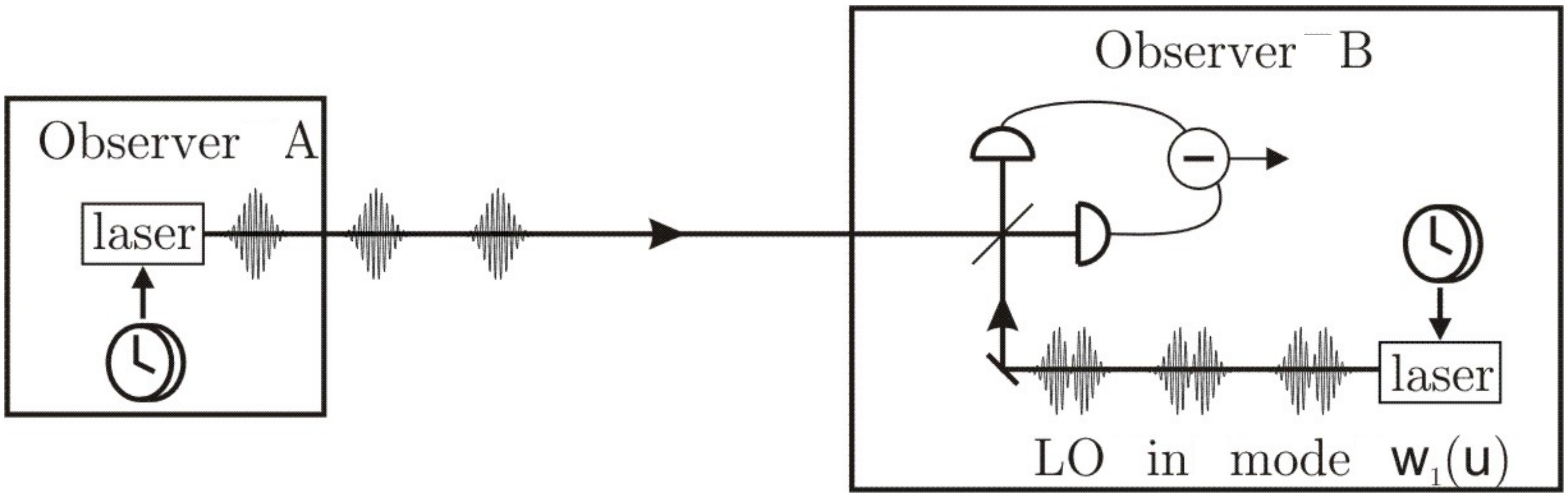
observer A sends a superposition of two modes:



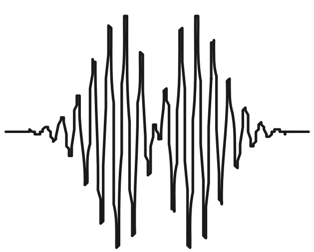
Phase squeezed state ⊗ Squeezed vacuum

Better sensitivity than by sharing entangled light between A and B

Beyond the standard quantum limit in simplified configuration



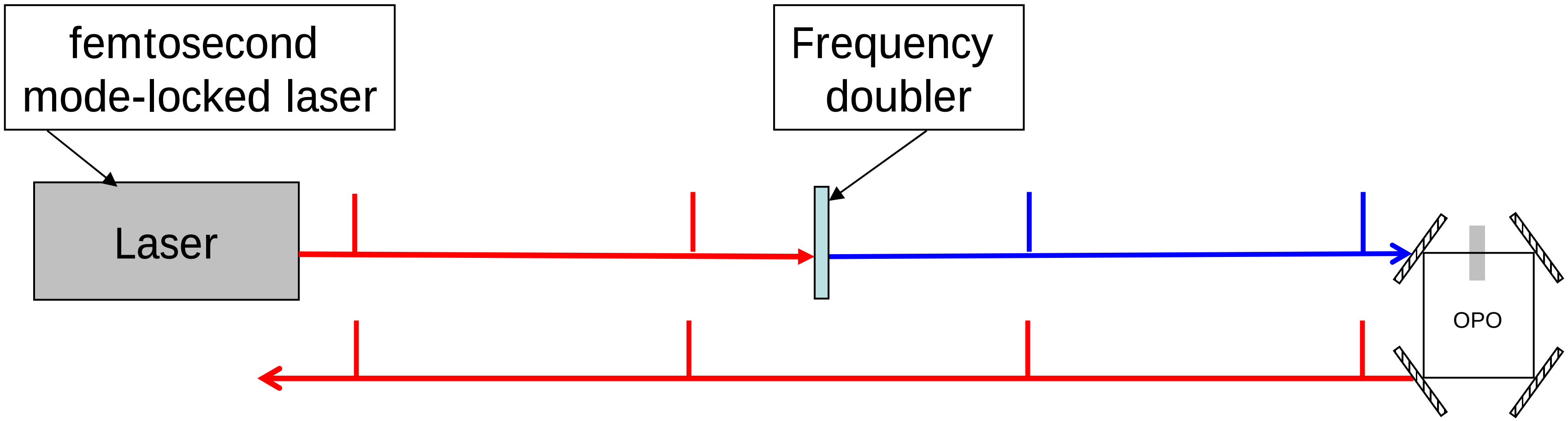
observer A sends:



Coherent state ⊗ Squeezed vacuum

How to produce non-classical combs ?

Synchronously Pumped OPOs (« SPOPOs »)

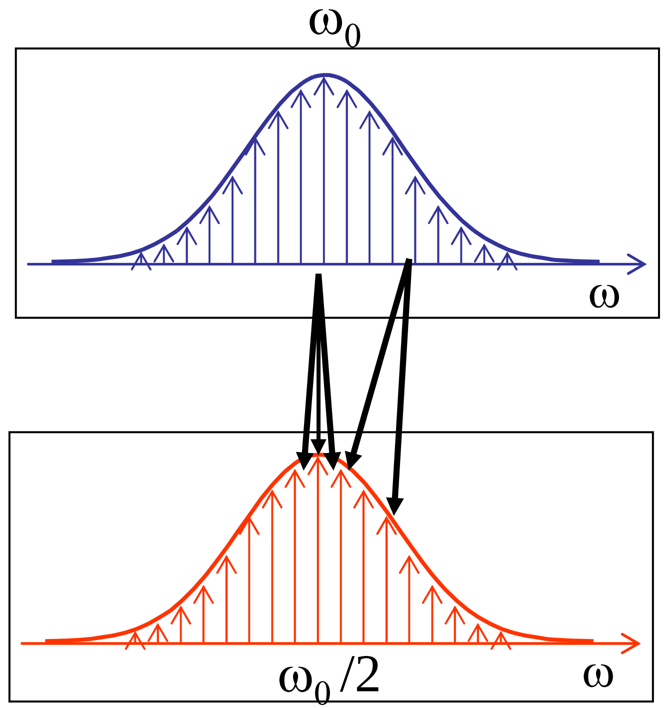


Twin photon generation

Phase matching conditions for degenerate operation

frequency comb
for pump

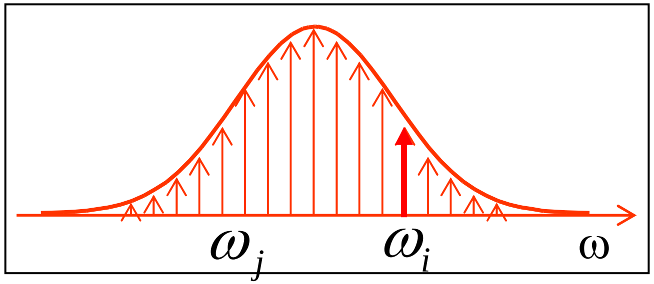
frequency comb
for signal/idler



Multiple fathers for the twins !

Every pair of photons can find a common father

Evolution of modes in the parametric crystal



$$\frac{d}{dz}\hat{a}_{\omega_i} = \sum_j \chi\left(\omega_i, \omega_j\right) A_{pump}(\omega_i + \omega_j)\, \hat{a}^+{}_{\omega_j}$$

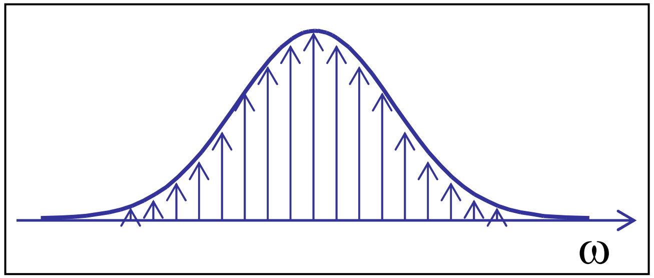
Coupling between the modes :
linear, symmetrical

Diagonalization by a set of specific combinations of modes or frequency combs: « supermodes » b_j

$$\frac{d}{dz}\hat{b}_j = \Lambda_j\, \hat{b}^+{}_j$$

Squeezing transformation

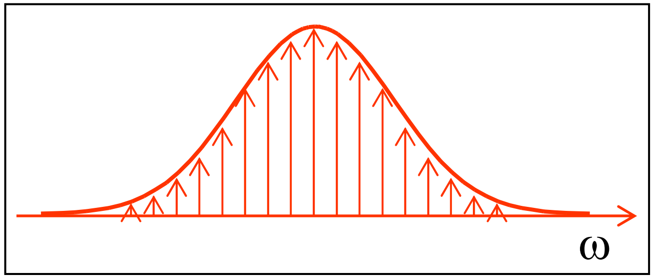
Generation of squeezed frequency combs



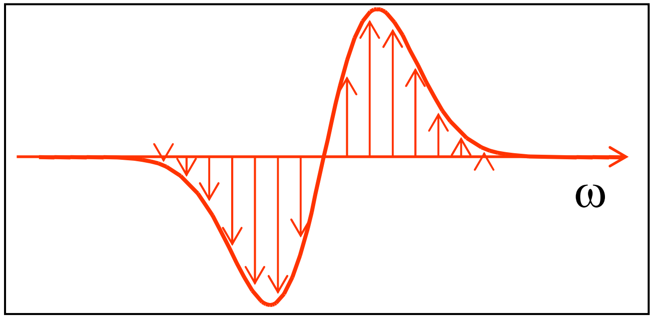
Gaussian variation
of pump*phase matching

G. De Valcarcel et al,
Phys. Rev A**74** 061801R (2006)

eigen « super modes»



mode 1



mode 2

⋮

Below the oscillation threshold, all modes are squeezed
especially the mode 1, with maximum $|\Lambda|$

Above threshold, mode 1 oscillates, the others are squeezed

C.F.
Nicolas Treps,
German de Valcarcel
Vincent Delaubert
Giuseppe Patera
Benoit Chalopin
Olivier Pinel
Jean-François Morizur

Strong and efficient collaboration with
Hans Bachor group at ANU Canberra Australia

Post doc position available immediately !!