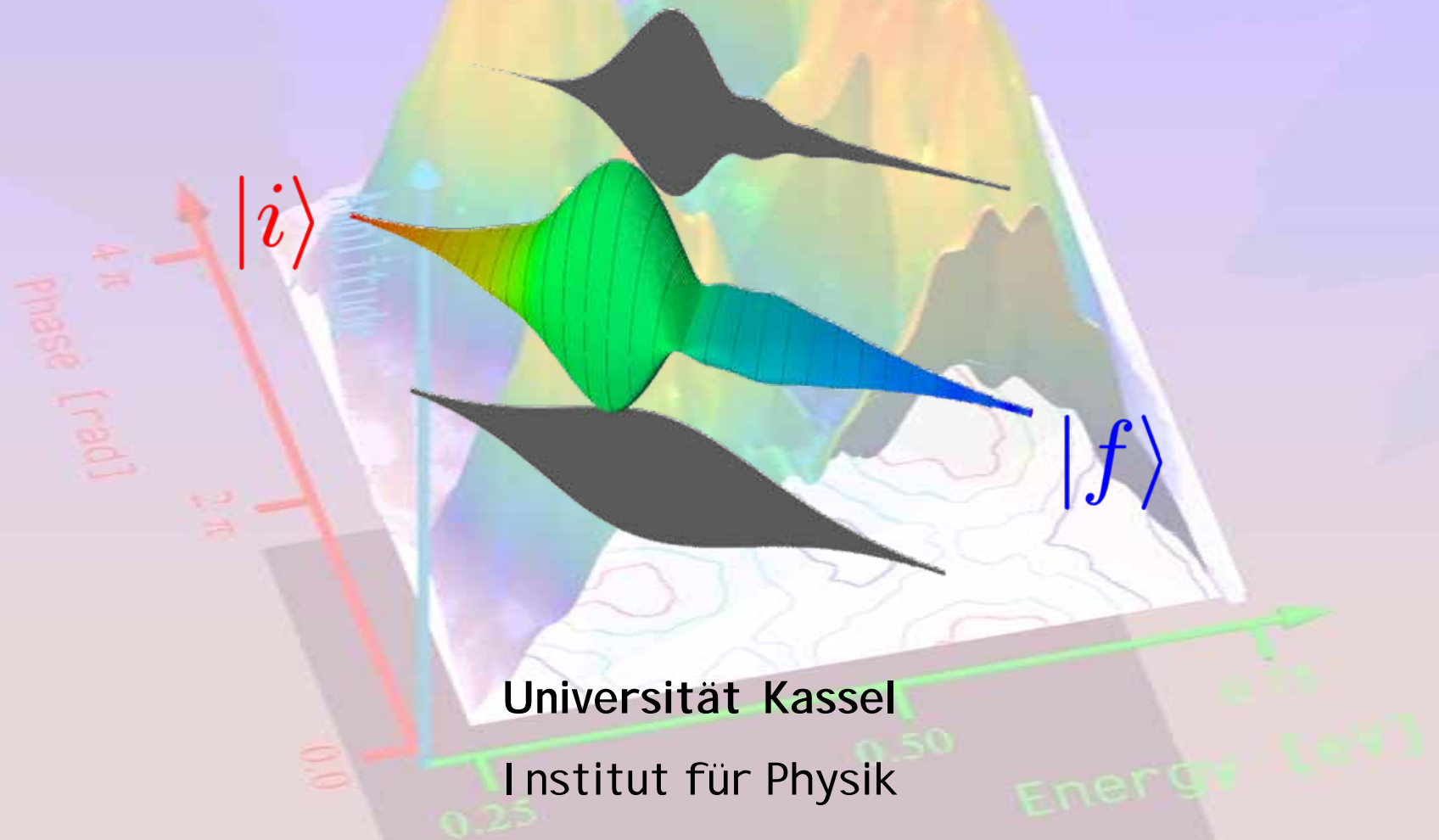


Creation, shaping and characterization of femtosecond laser pulses

M. Wollenhaupt and T. Baumert



Universität Kassel

Institut für Physik



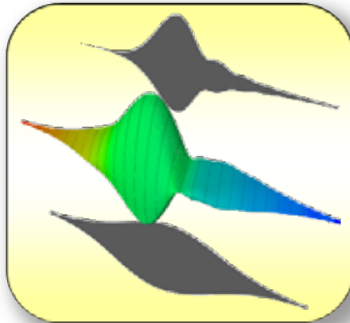
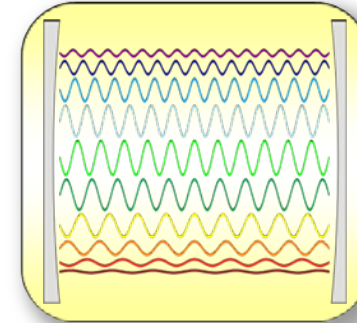
Outline

Generation

Laser modes

Mode locking

Pulse amplification



Manipulation

Dispersion in optical systems

Mathematical description

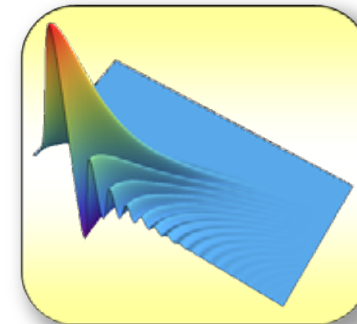
Pulse shaping

Measurement

Time domain

Frequency domain

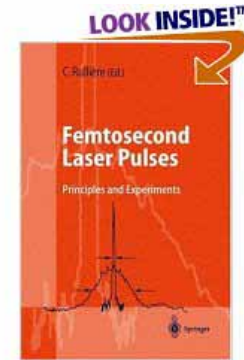
Joint time frequency domain





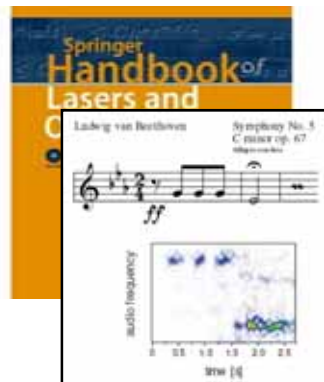
Literature

Femtosecond Laser Pulses (second edition)
Claude Rulliere, Springer 2004



Ultrashort Laser Pulse Phenomena (second edition)
Jean-Claude Diels and Wolfgang Rudolph, Academic Press 2006

Frequency-Resolved Optical Gating
Rick Trebino, Kluwer 2002

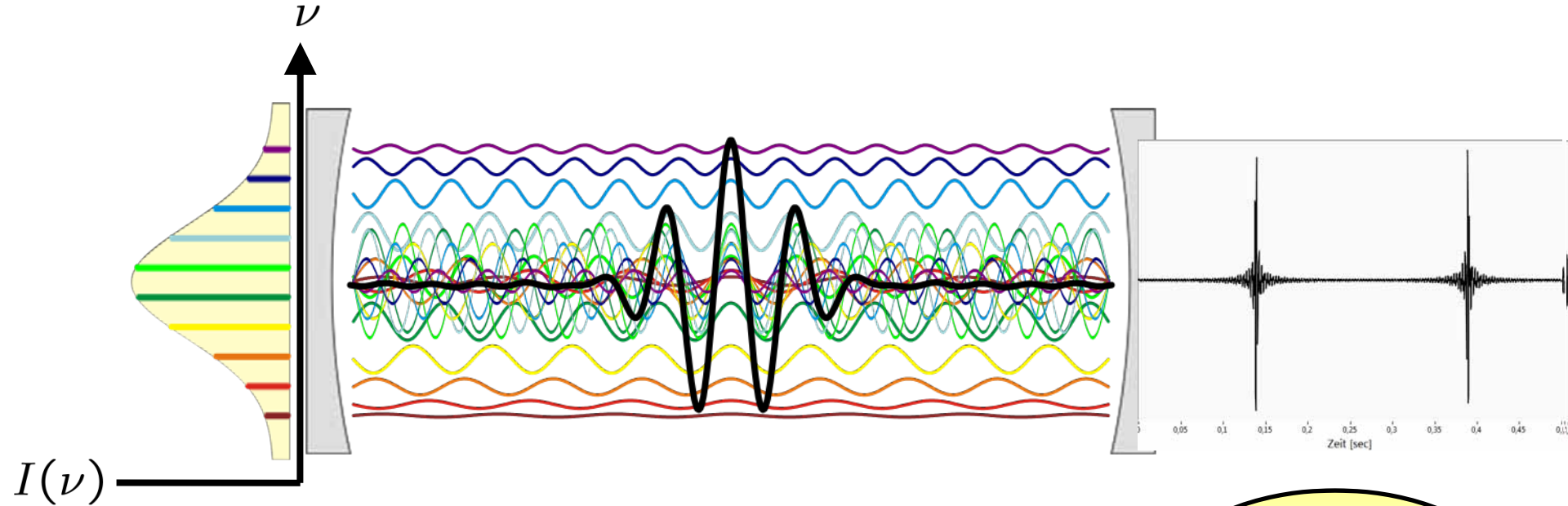


Springer Handbook of Laser and Optics, Chap. 12
Femtosecond Laser Pulses: Linear Properties, Manipulation, Generation and Measurement,
M. Wollenhaupt, A. Assion and T. Baumert, Springer 2007

(available from our homepage
physik.uni-kassel.de/index.php?id=exp3*)*



Generation of ultrashort laser pulses by mode locking



Longitudinal laser modes

$$L = n \frac{\lambda_n}{2}$$

$$\nu_n = n \frac{c}{2L}$$



Theodor W. Hänsch



John L. Hall

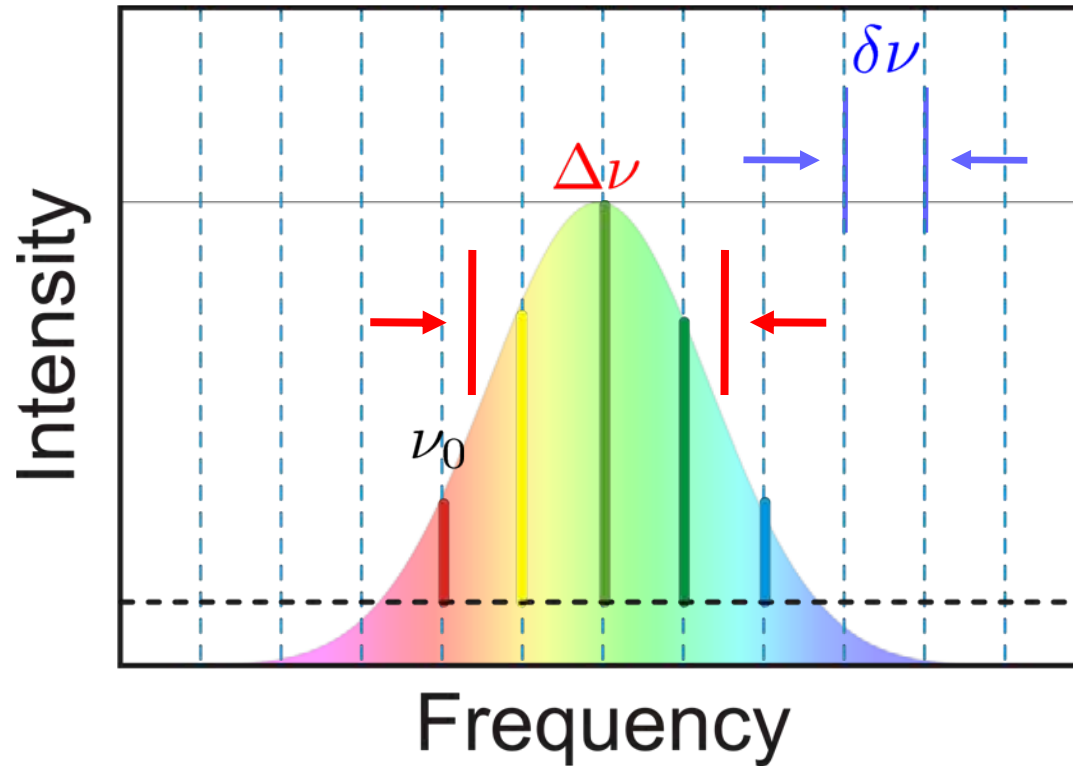
Frequency comb



Longitudinal laser modes in frequency domain

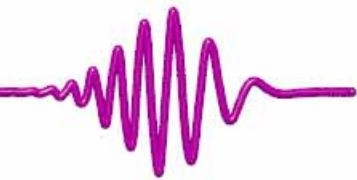
$$\nu_n = n \frac{c}{2L}$$

$$\delta\nu = \frac{c}{2L}$$



$$\Delta\nu \text{ gain bandwidth} \approx N \cdot \delta\nu$$

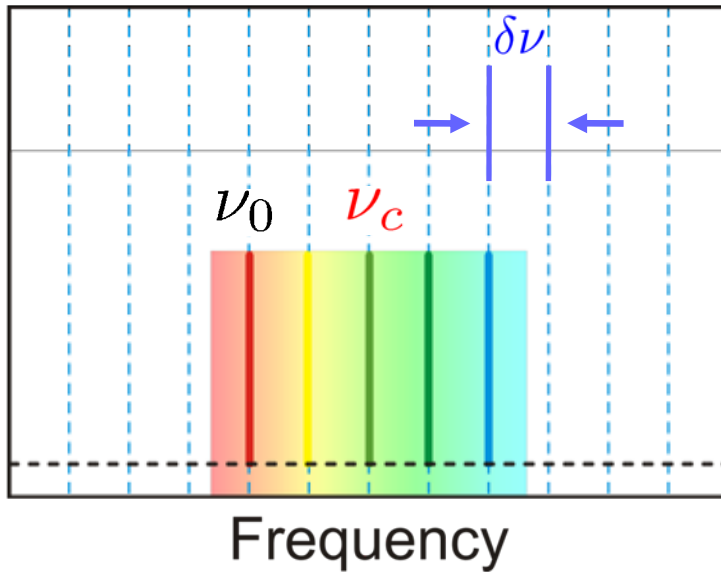
$$E_n(t) = A_n \sin \{ 2\pi [\nu_0 + n \delta\nu] t + \phi_n(t) \}$$



Mode locking quantitatively

Single mode: $E_n(t) = E_0 \sin \{2\pi [\nu_0 + n \delta\nu] t\}$

Simple case:



$$E(t) = \sum_{n=0}^{N-1} E_n(t)$$

$$A_n = E_0$$

$$\phi_n = 0$$

$$E(t) = E_0 \sum_{n=0}^{N-1} \sin (2\pi [\nu_0 + n \delta\nu] t)$$

$E(t) =$	$E(t) = E_0 \sin (2\pi \nu_c t) \cdot \frac{\sin(\pi N \delta\nu t)}{\sin(\pi \delta\nu t)} \frac{N \delta\nu t}{\pi \delta\nu t}$
----------	--

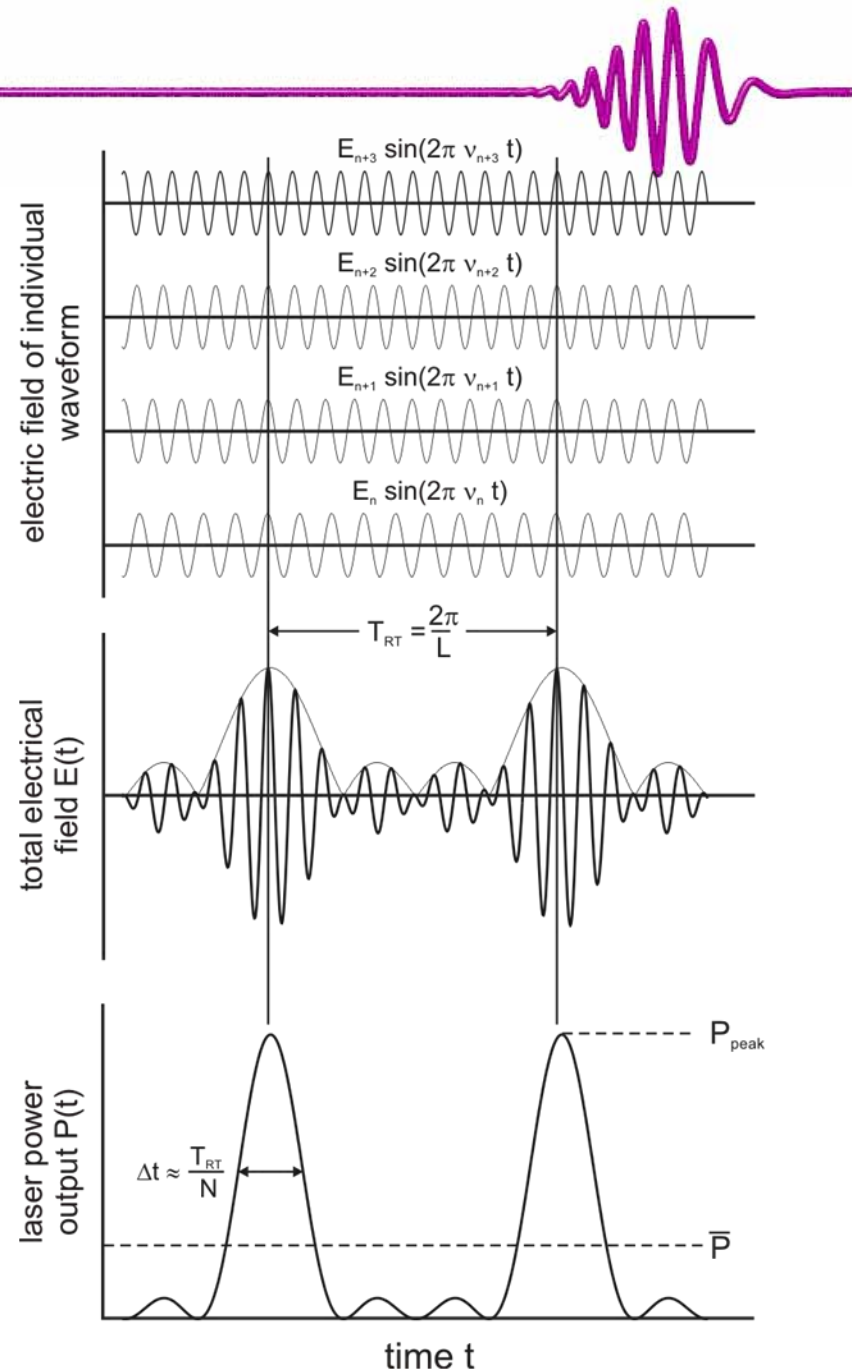
Example: four modes

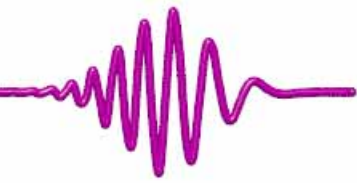
$$I(t) \propto \frac{\sin^2(\pi N \delta \nu t)}{\sin^2(\pi \delta \nu t)}$$

$$P_{Peak} = N^2 P_0$$

$$\langle P \rangle = N P_0$$

$$\Delta t \approx \frac{T_{RT}}{N} = \frac{1}{N \delta \nu} = \frac{1}{\Delta \nu}$$

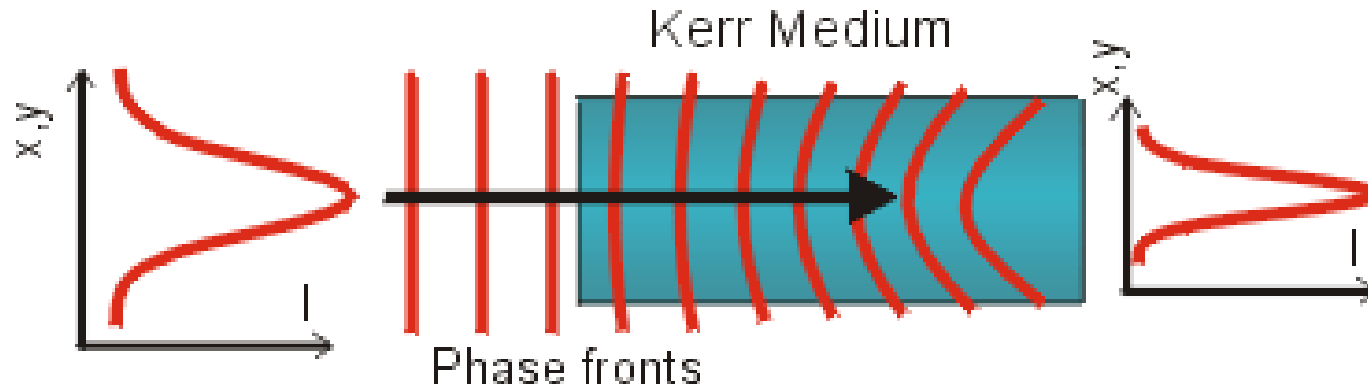




Passive mode locking with Kerr lens

At high intensities, the index of refraction is getting intensity dependent

$$n(x, y, t) = n_0 + n_2 I(x, y, t)$$



The Kerr lens generates self focussing

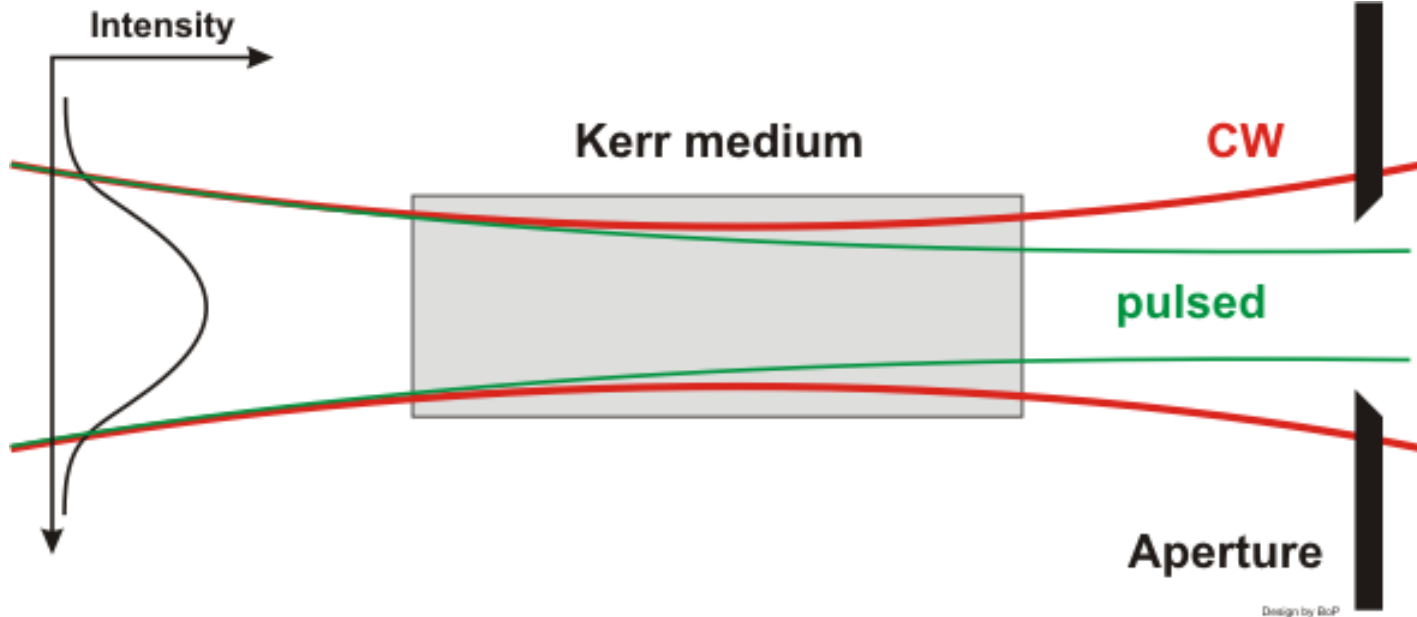
The **spatial** intensity distribution of the laser beam together with a Kerr nonlinearity establishes a KERR LENS



Passive mode locking with Kerr lens

For low intensity cw beams: No Lens effect

For high intensity pulsed radiation: Kerr Lens



An aperture yields **high losses for cw** and **low losses for pulsed** operation



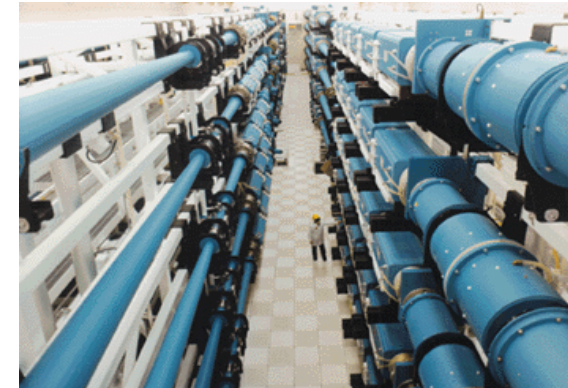
Pulse amplification

Problem: High **intensities!** (self focusing followed by destruction)

$$Power = \frac{Energy}{Pulse\ duration}$$

$$Intensity = \frac{Energy}{Pulse\ duration \cdot Area}$$

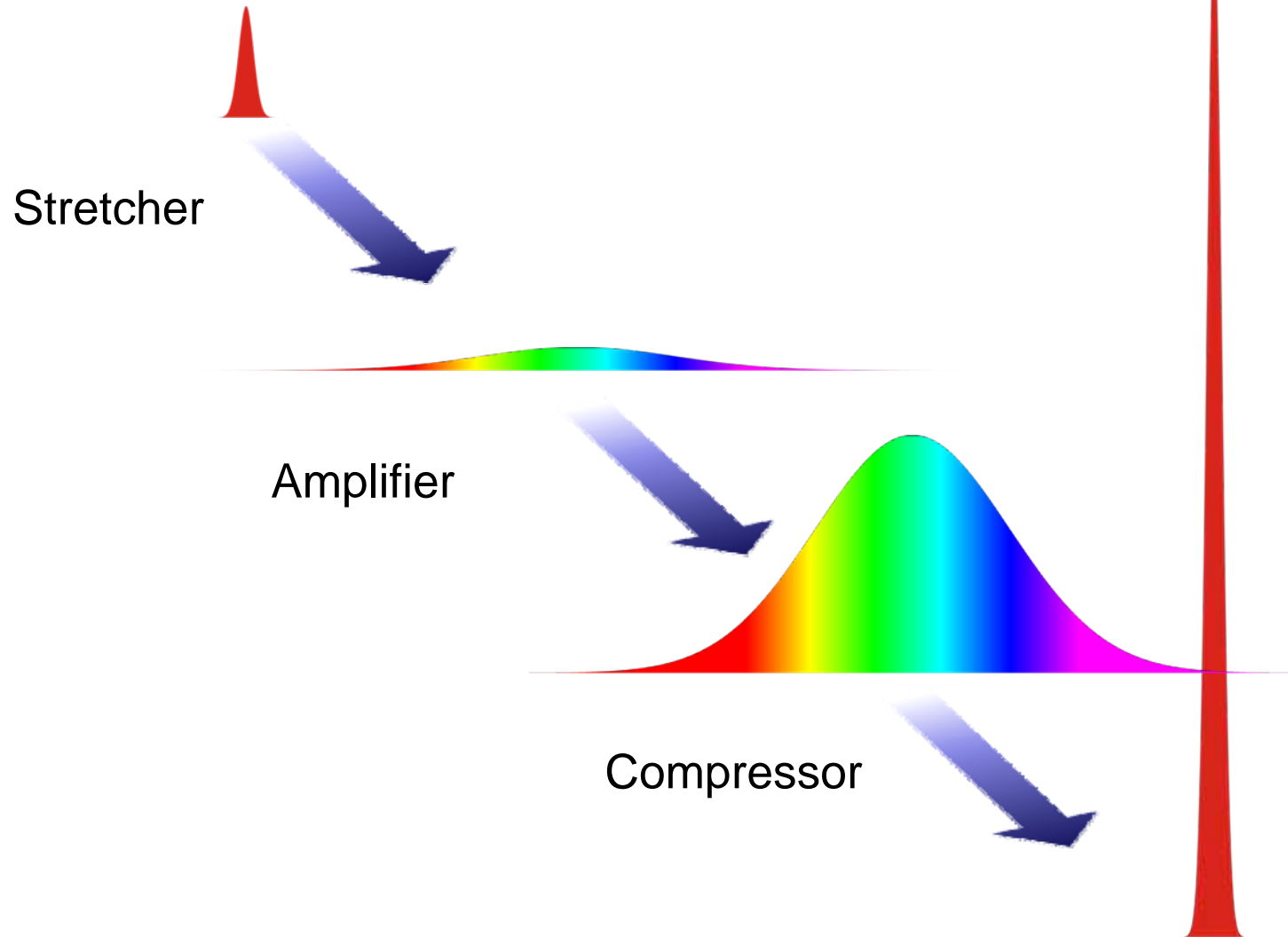
BIG lasers $Area \uparrow \Rightarrow Intensity \downarrow$

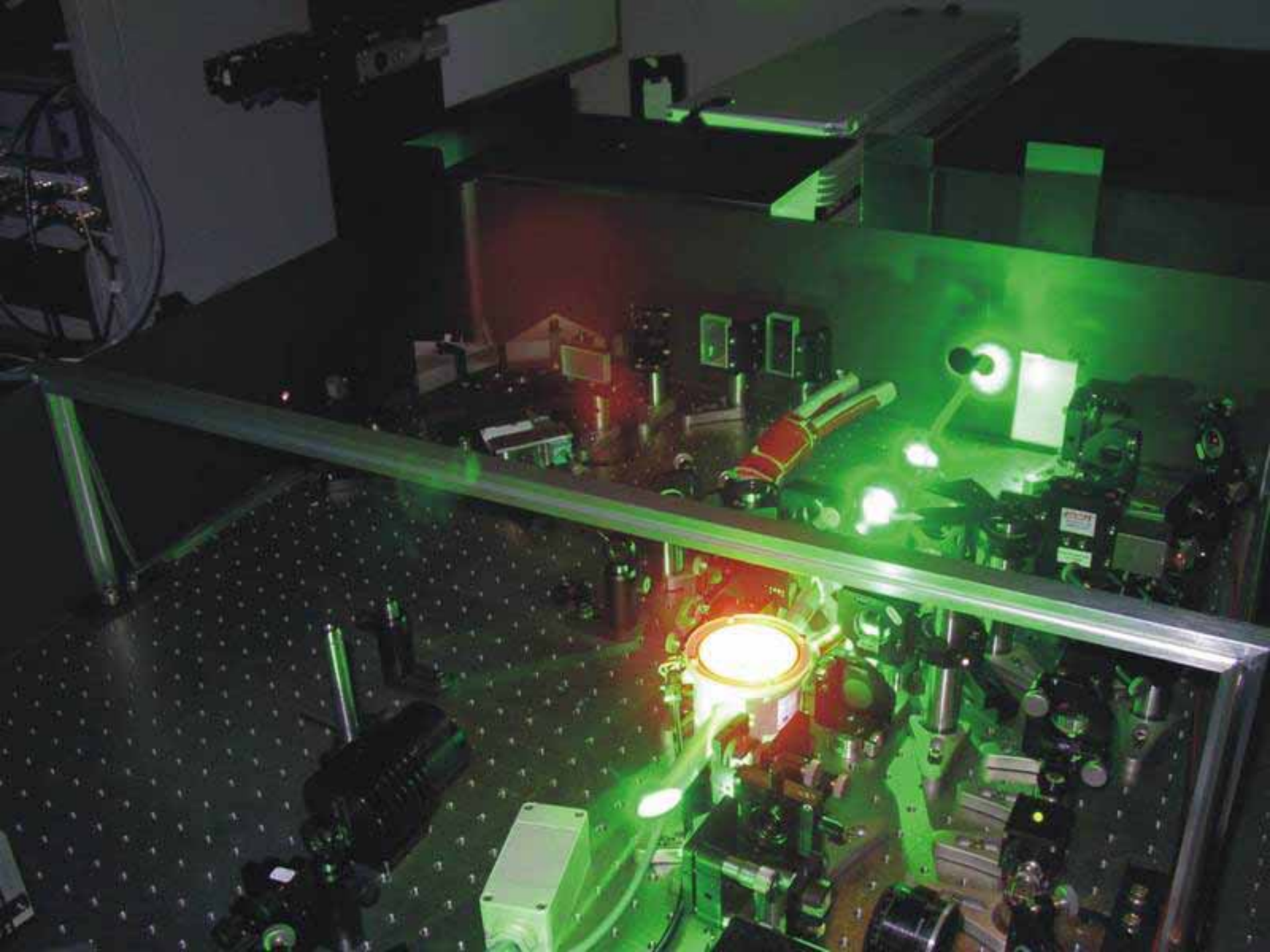


Chirped Pulse Amplification (CPA) $Pulse\ duration \uparrow \Rightarrow Intensity \downarrow$



Chirped pulse amplification (CPA)





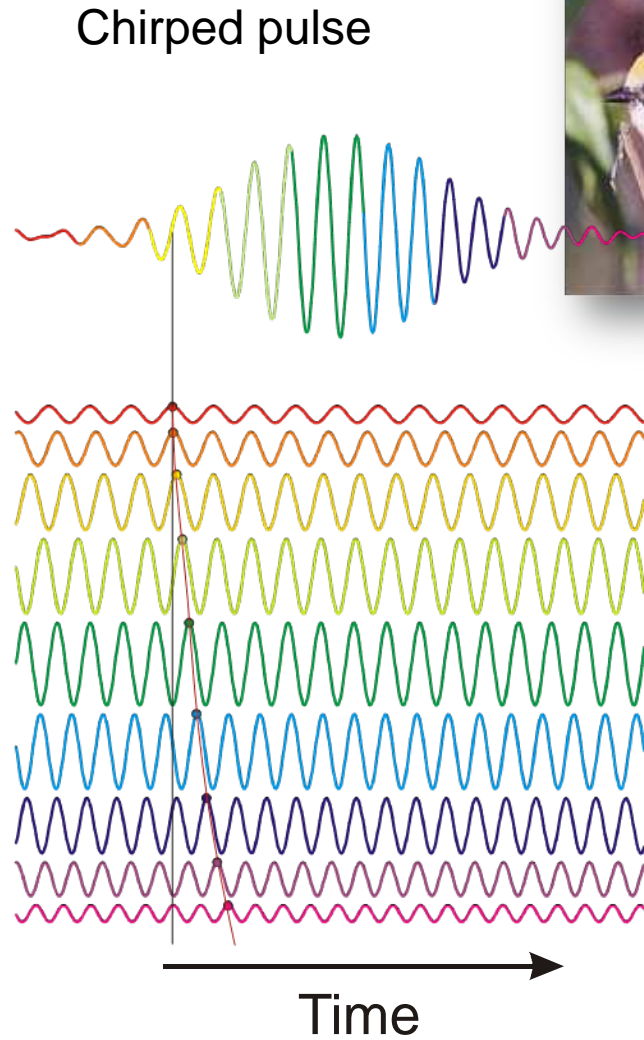
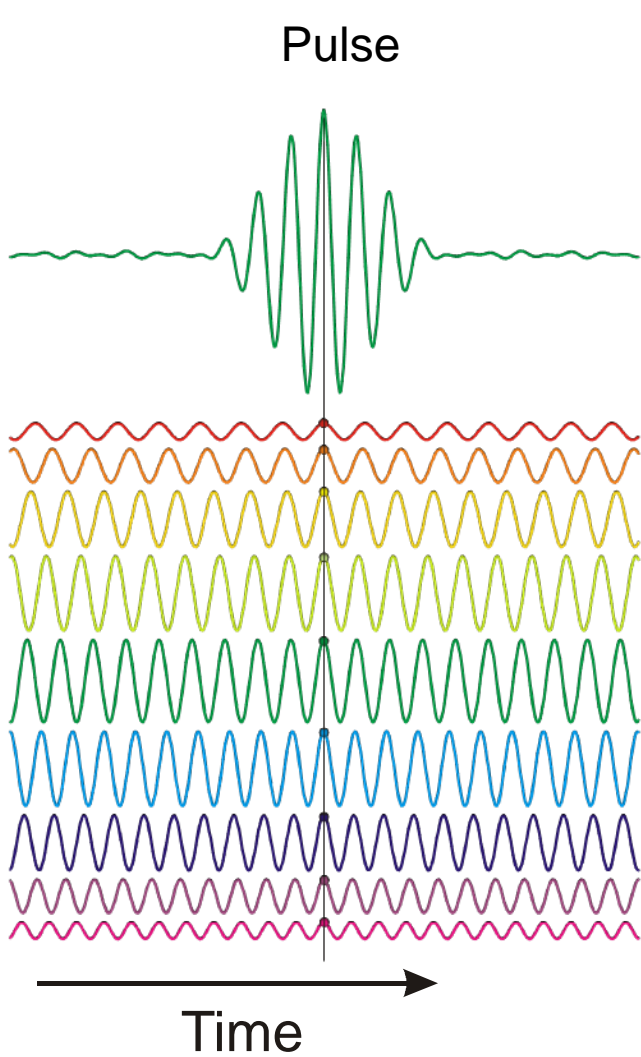


Parameters of our laser system

- Pulse duration: 25 fs
- Repetition rate: 1 kHz
- Spectral width: 750 nm – 840 nm
- Energy per pulse: 1 mJ (can heat 1 g H₂O by 1/4000 °C)
- Number of photons per pulse: 10¹⁵
- Peak power: 40 GW (nuclear power plant typical 1-2 GW)
- Intensity in 10 μm focus: 50 PW/cm² (Solar constant 0.14 W/cm²)
- Tunability with nonlinear optics: 200 nm – 2300 nm
- Pulse shaping in phase, amplitude and polarization

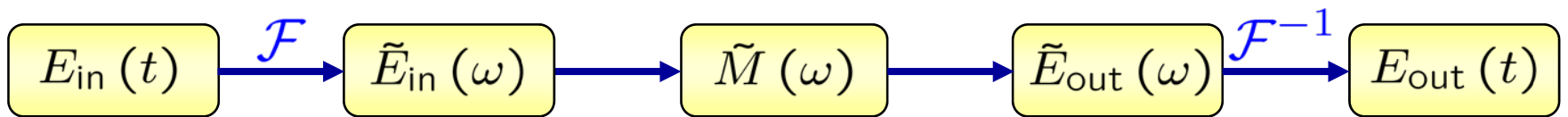


Dispersion of broadband light

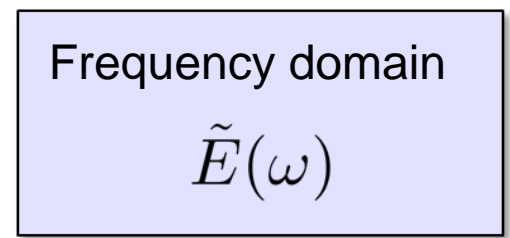
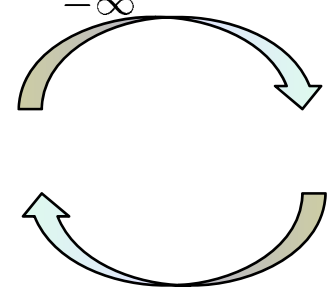
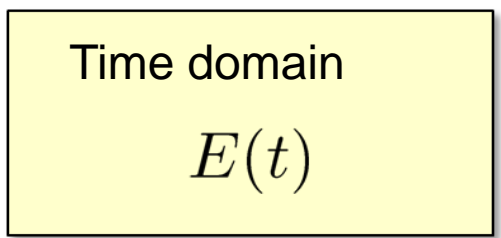




Pulse shaping: mathematical description



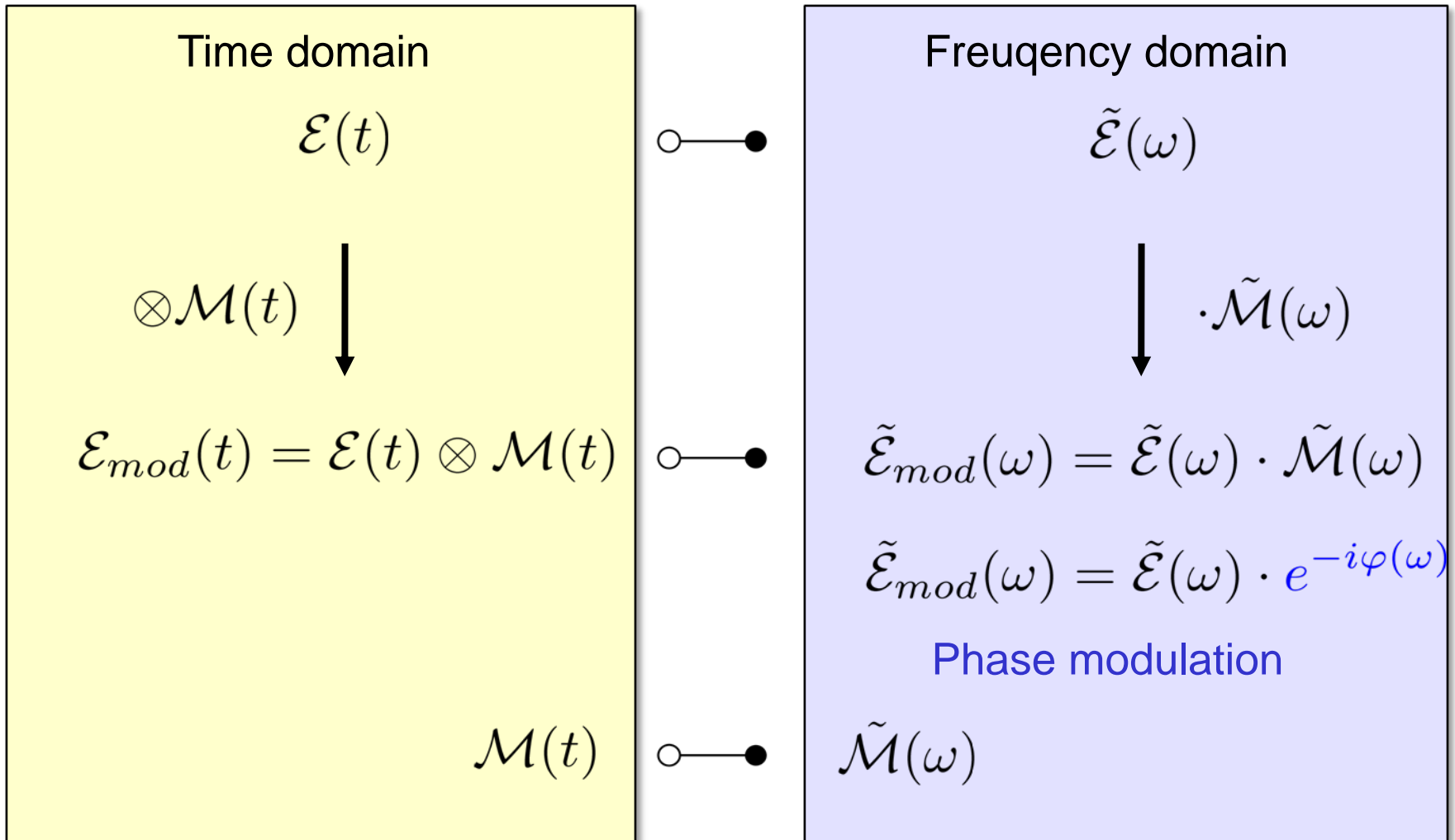
$$\mathcal{F} \quad \tilde{E}(\omega) := \int_{-\infty}^{\infty} E(t) e^{-i\omega t} dt$$

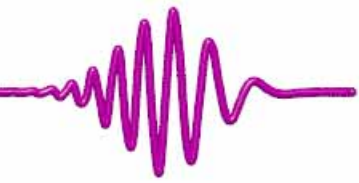


$$\mathcal{F}^{-1} \quad E(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{E}(\omega) e^{i\omega t} d\omega$$



Mathematical description





Taylor expansion of the phase function

$$\varphi(\omega) = \sum_{n=0} \frac{1}{n!} \left. \frac{\partial^n \varphi}{\partial \omega^n} \right|_0 \cdot \omega^n \quad \phi_n = \left. \frac{\partial^n \varphi}{\partial \omega^n} \right|_0$$

- Absolute phase ϕ_0
- Linear phase ϕ_1
- Quadratic phase ϕ_2 (GDD, chirp)
- Cubic phase ϕ_3 (TOD)

- Sinusoidal modulation



Absolute phase $\tilde{\mathcal{E}}(\omega) \cdot e^{-i\phi_0}$

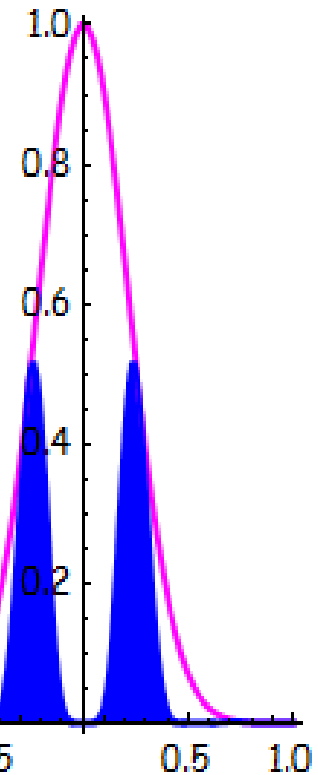
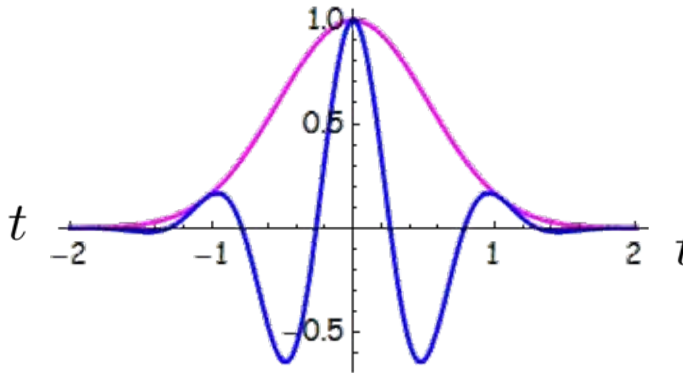
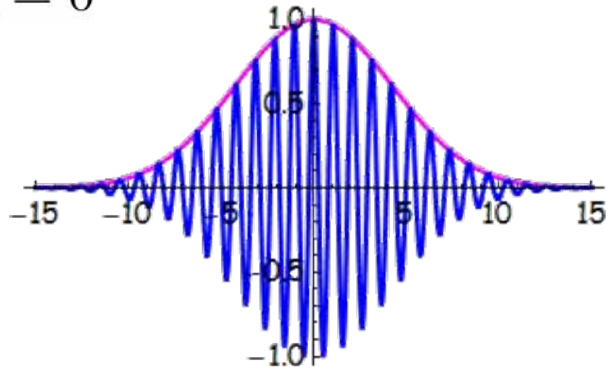
Is it physically relevant?

NO!

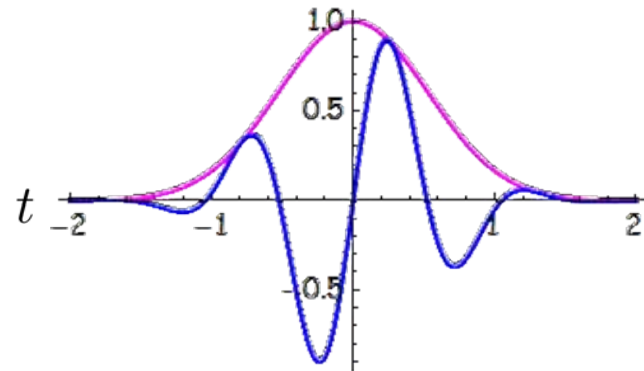
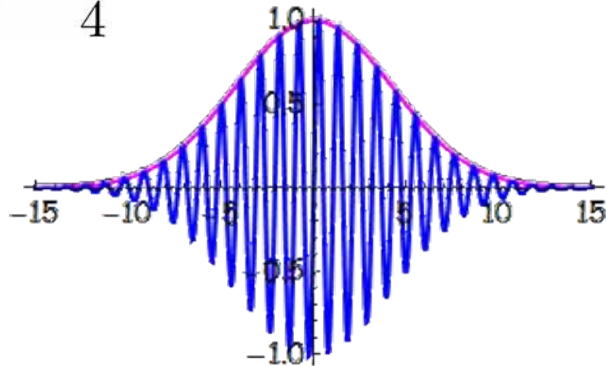
Yes!

...indeed, for non-linear processes

$\phi_0 = 0$



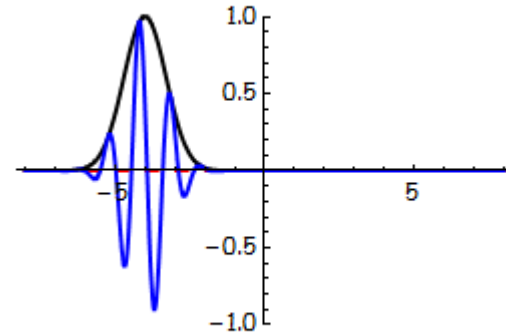
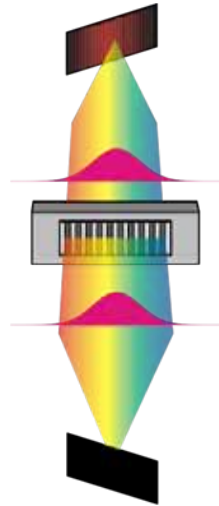
$\phi_0 = \frac{\pi}{4}$



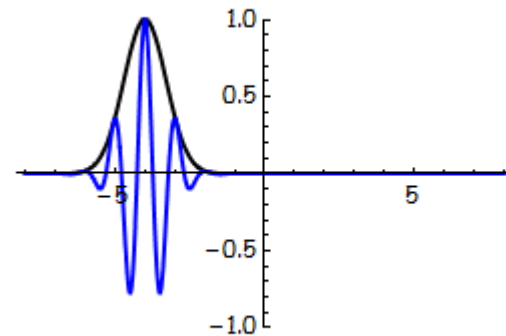
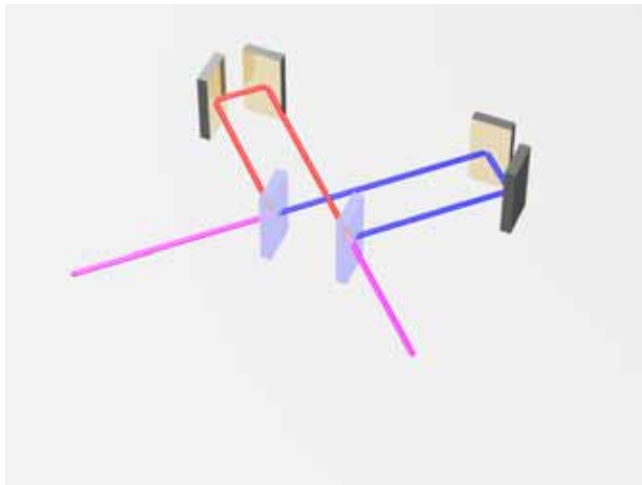


Linear phase $\tilde{\mathcal{E}}(\omega) \cdot e^{-i\phi_1 \cdot \omega}$

Phasenmodulation



Interferometer





Quadratic spectral phase $\tilde{\mathcal{E}}(\omega) \cdot e^{-i\frac{\phi_2}{2} \cdot \omega^2}$

$$\mathcal{E}(t) = \frac{\mathcal{E}_0}{2} e^{-2 \ln(2) \left(\frac{t}{\Delta t}\right)^2}$$

FWHM = Δt

$$\otimes \frac{e^{\frac{it^2}{2\phi_2}}}{\sqrt{2\pi i \phi_2}}$$



$$\mathcal{E}_{mod}(t)$$

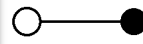
$$= \frac{\mathcal{E}_0 \Delta t}{2} \frac{e^{-\frac{2t^2 \ln(2)}{\Delta t^2 + 4i\phi_2 \ln(2)}}}{\sqrt{\Delta t^2 + 4i\phi_2 \ln(2)}}$$



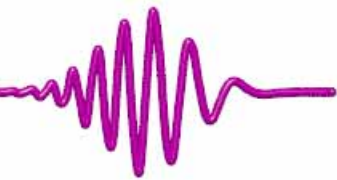
$$\tilde{\mathcal{E}}(\omega) = \frac{\mathcal{E}_0 \Delta t}{2} \sqrt{\frac{\pi}{2 \ln(2)}} e^{-\frac{(\omega \Delta t)^2}{8 \ln(2)}}$$



$$\cdot e^{-i\frac{\phi_2}{2!} \cdot \omega^2}$$



$$\tilde{\mathcal{E}}_{mod}(\omega) = \tilde{\mathcal{E}}(\omega) \cdot e^{-i\frac{\phi_2}{2!} \cdot \omega^2}$$



Quadratic spectral phase: chirp

$$\phi_2 \quad [\text{fs}^2]$$

$$\mathcal{E}_{mod}(t) = \frac{\mathcal{E}_0 \Delta t}{2} \frac{e^{-\frac{2t^2 \ln(2)}{\Delta t^2 + 4i\phi_2 \ln(2)}}}{\sqrt{\Delta t^2 + 4i\phi_2 \ln(2)}} \Rightarrow$$

$$\beta = \frac{\Delta t^2}{8 \ln(2)} \quad [\text{fs}^2]$$

$$\gamma = 1 + \frac{\phi_2^2}{4\beta^2}$$

$$\mathcal{E}_{mod}(t) = \frac{\mathcal{E}_0}{2\gamma^{\frac{1}{4}}} e^{-\frac{t^2}{4\beta\gamma}} e^{i(\alpha t^2 - \varepsilon)}$$

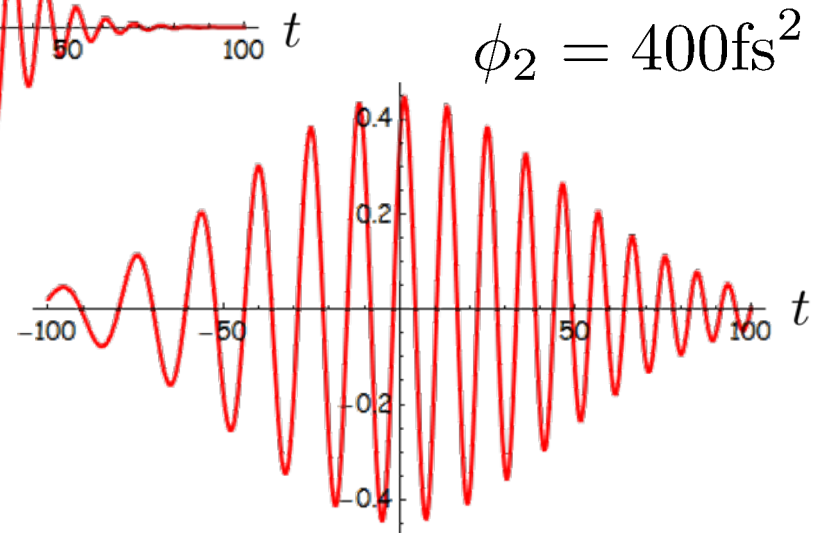
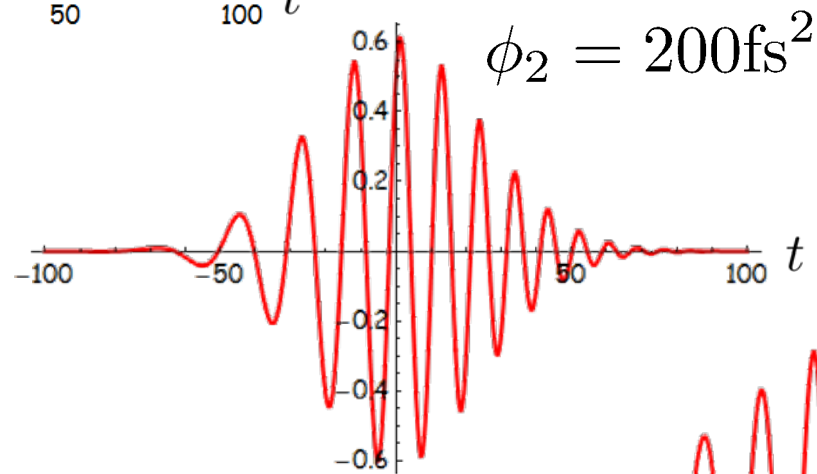
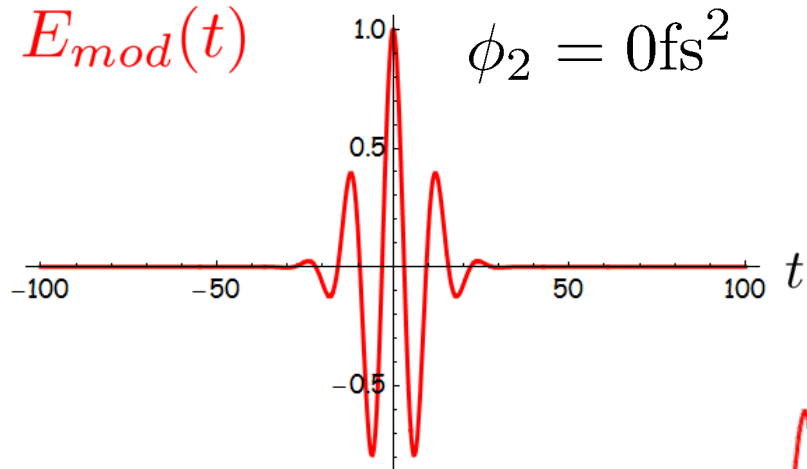
$$\alpha = \frac{\phi_2}{8\beta^2\gamma} = \left\{ 2\phi_2 + \frac{\Delta t^4}{8\phi_2[\ln(2)]^2} \right\}^{-1} \quad [\text{fs}^{-2}]$$

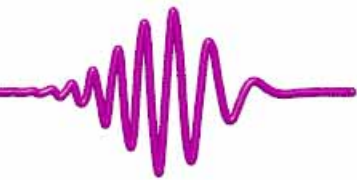
$$\varepsilon = \arctan \left[\frac{\phi_2}{2\beta} \right]$$



Controlling the chirp by ϕ_2

$E_{mod}(t)$



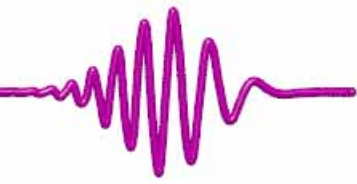


Duration of a chirped pulse

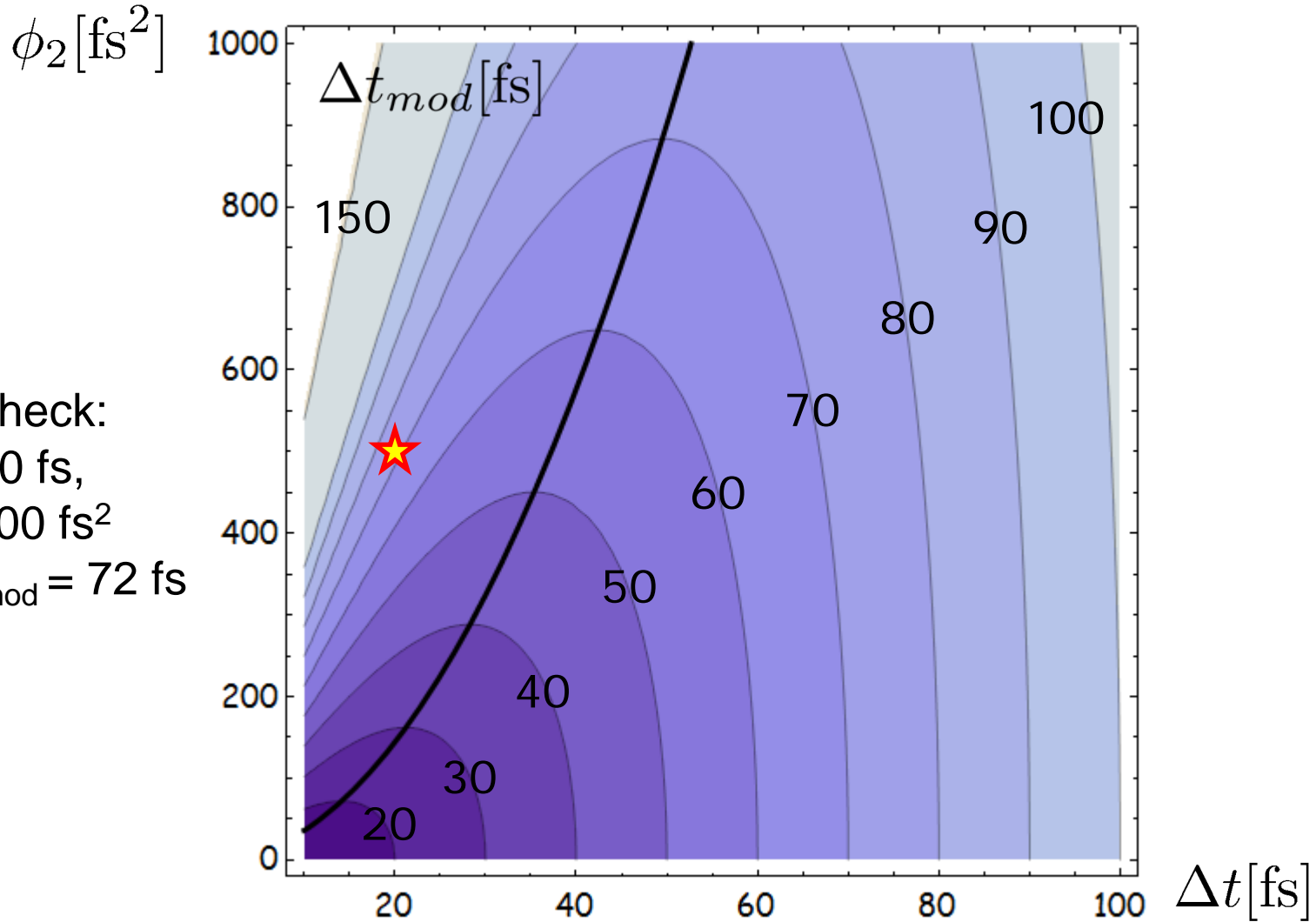
$$\mathcal{E}_{mod}(t) = \frac{\mathcal{E}_0}{2\gamma^{\frac{1}{4}}} e^{-\frac{t^2}{4\beta\gamma}} e^{i(\alpha t^2 - \varepsilon)}$$

$$\Delta t_{mod} = \sqrt{\Delta t^2 + [\ln(16)]^2 \left(\frac{\phi_2}{\Delta t}\right)^2}$$

Δt	ϕ_2	100 fs ²	200 fs ²	500 fs ²	1000 fs ²	2000 fs ²	5000 fs ²
10 fs		29.5	56.3	139	277.4	554.6	1386.3
20 fs		24.3	34.2	72.1	140.1	278	693.4
30 fs		31.4	35.2	55.1	97.2	187.3	463.1
100 fs		100	100.2	101	103.8	114.3	170.9



Duration of a chirped pulse



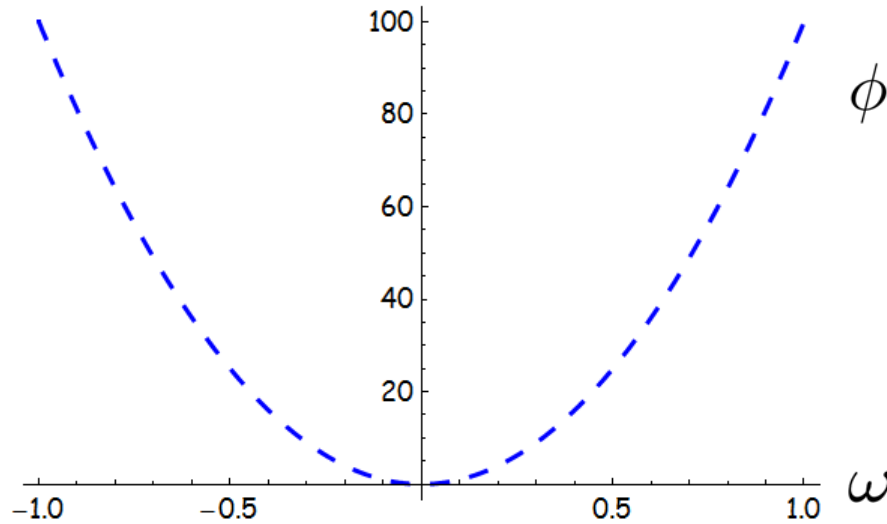
★ Check:
 $\Delta t = 20$ fs,
 $\phi_2 = 500$ fs²
 $\Rightarrow \Delta t_{mod} = 72$ fs



Spectral physical picture: group delay

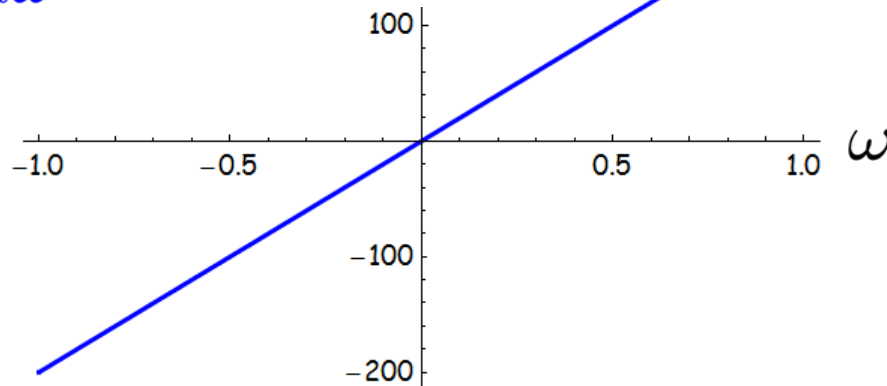
$$\varphi(\omega) = \frac{\phi_2}{2!} \cdot \omega^2$$

Spectral phase



$$T_g(\omega) = GD(\omega) = \frac{d}{d\omega} \varphi(\omega) = \phi_2 \cdot \omega$$

Group delay



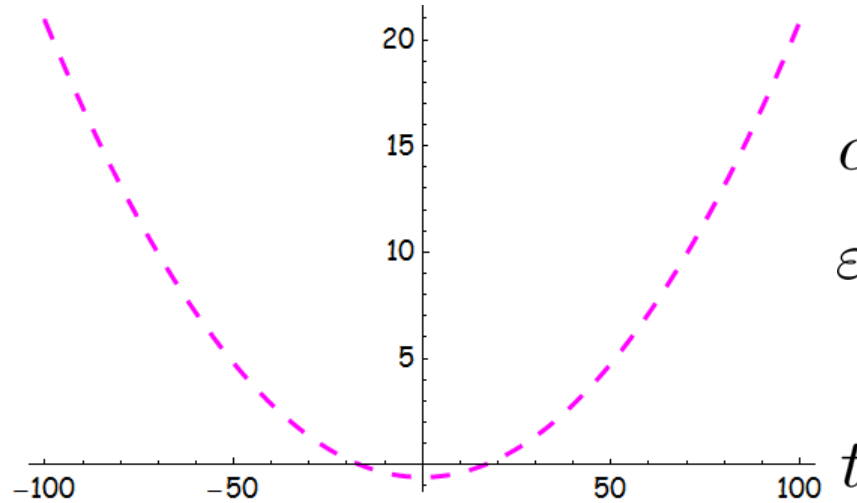
$$T_g(\omega) = GD(\omega) \text{ [fs]}$$



Temporal physical picture: instantaneous frequency

$$\zeta(t) = \alpha \cdot t^2 - \varepsilon$$

Temporal phase

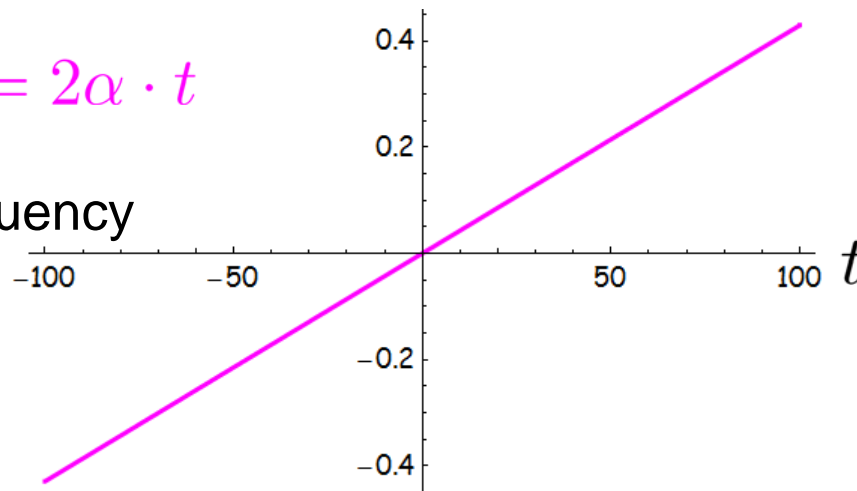


$$\alpha = 2.14 \cdot 10^{-3} \text{ fs}^{-2}$$

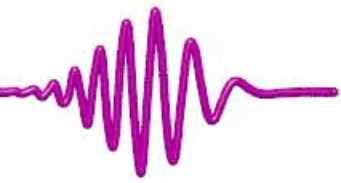
$$\varepsilon = 0.59 \text{ rad}$$

$$\Delta\omega(t) = \frac{d}{dt}\zeta(t) = 2\alpha \cdot t$$

Instantaneous frequency



$$\Delta\omega(t) [\text{fs}^{-1}]$$



Summary chirp

- The linear chirp is generated by a **quadratic spectral phase function** $\phi(\omega)$
- ...corresponding to a **linear** $T_g(\omega)$ (Group Delay Dispersion, GDD).
- A Gaussian envelope pulse is stretched symmetrically and stays Gaussian reducing the intensity
- Short pulses will be stretched more than long pulses (for a given ϕ_2) .
- The sign of ϕ_2 controls the „direction“ of the chirp: > 0 up-chirp, < 0 down-chirp
- The **temporal envelope** is complex valued characterized by a **quadratic temporal phase** $\zeta(t)$
- ...leading to a **linear increase / decrease of the instantaneous frequency** $\Delta\omega(t)$
- There is a maximum (temporal) chirp rate α for a given pulse duration



Cubic spectral phase (TOD) $\tilde{\mathcal{E}}(\omega) \cdot e^{-i \frac{\phi_3}{3!} \cdot \omega^3}$

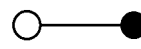
$\mathcal{E}(t)$

↓

$\otimes \Omega A i [-\sigma(\phi_3) \Omega t]$

↓

$\mathcal{E}_{mod}(t)$
 $= \mathcal{E}(t) \otimes \Omega A i [-\sigma(\phi_3) \Omega t]$



$\tilde{\mathcal{E}}(\omega)$

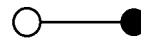
↓

$\cdot e^{-i \frac{\phi_3}{3!} \cdot \omega^3}$

$\phi(\omega)$

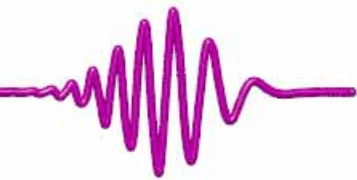
↓

$\tilde{\mathcal{E}}_{mod}(\omega) = \tilde{\mathcal{E}}(\omega) e^{-i \frac{\phi_3}{3!} \cdot \omega^3}$



$$\Omega = \sqrt[3]{\frac{2}{|\phi_3|}}$$

ϕ_3 [rad fs³]



TOD: ϕ_3 variation

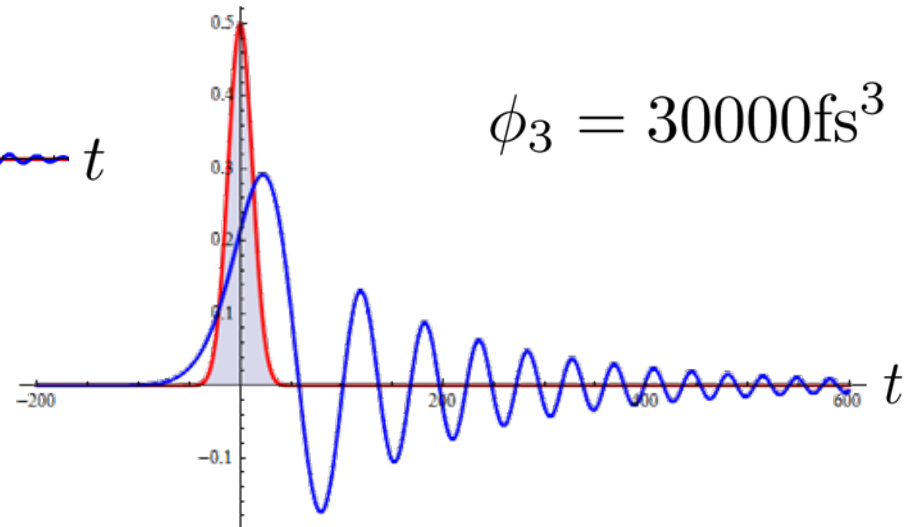
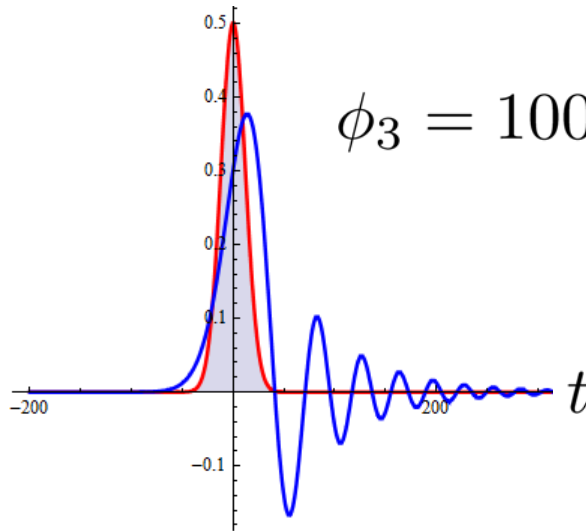
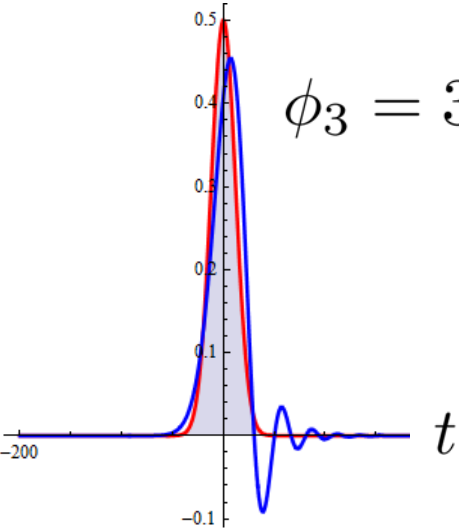
$\Delta t = 20[\text{fs}]$

$$\mathcal{E}_{mod}(t) = f \cdot \frac{\mathcal{E}_0 \Delta t}{2\phi} e^{\frac{\ln(2)}{2} \frac{2}{3} \frac{\tau-t}{\tau_{12}}} Ai \left[\frac{\tau-t}{\Delta t} \right]$$

$\phi_3 = 3000\text{fs}^3$

$\phi_3 = 10000\text{fs}^3$

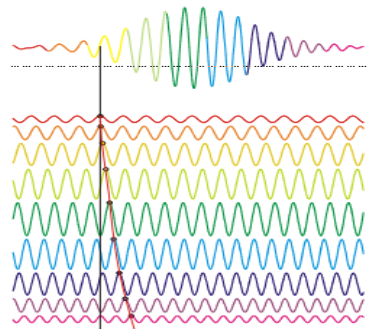
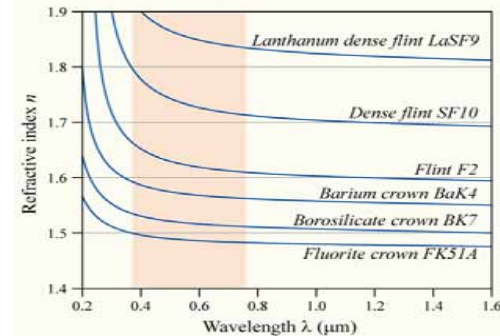
$\phi_3 = 30000\text{fs}^3$



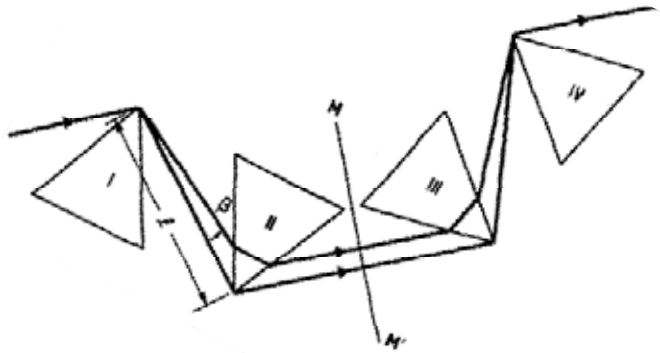


Counteracting dispersion in an optical system

In (most) transparent media **red** frequency components travel faster than **blue** ones leading to up-chirped pulses

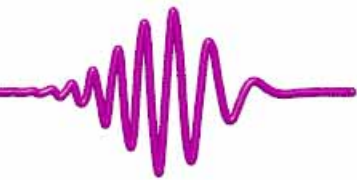


Find optical system where **blue** components travel faster (shorter optical length) than **red** ones introducing negative dispersion

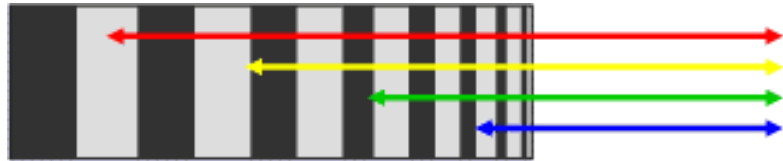


Several realizations:

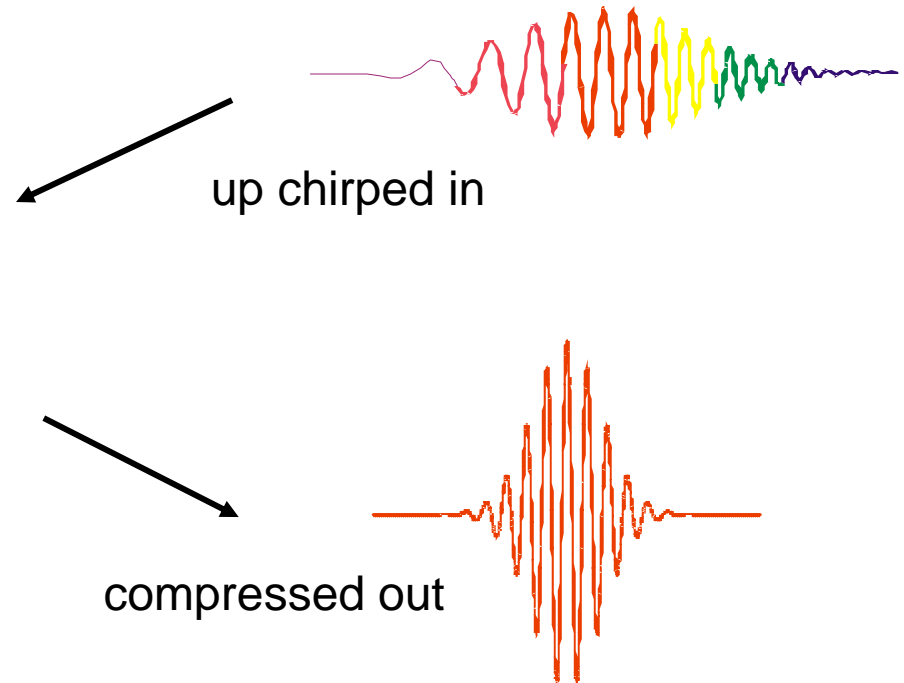
- chirped mirrors
- angular dispersion (grating and prism arrangements)
- programmable pulse shapers



Chirped mirrors

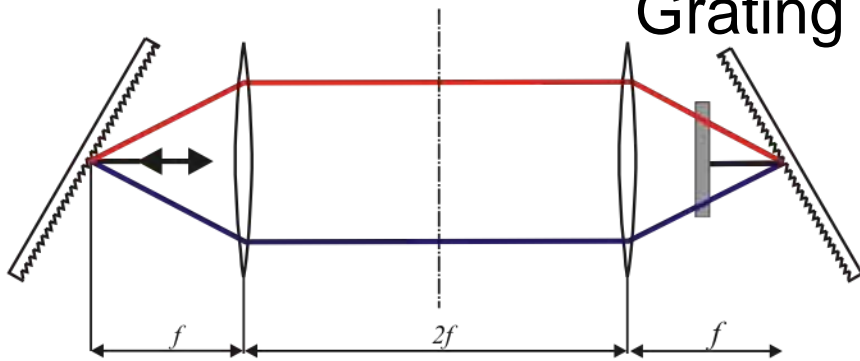


Long wavelengths penetrate deeper into mirror and experience larger group delay (anomalous chromatic dispersion)

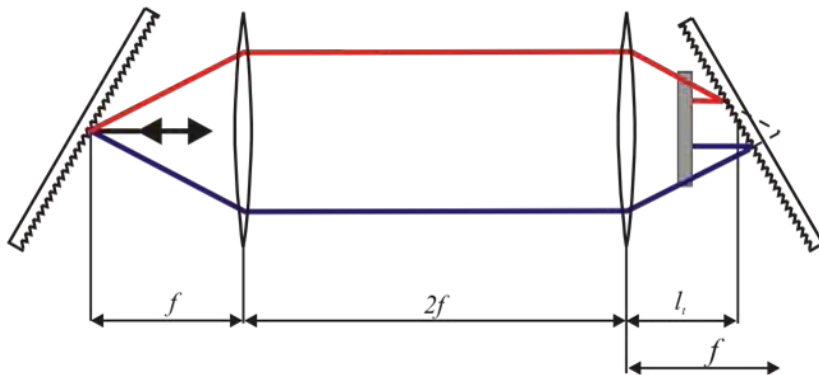




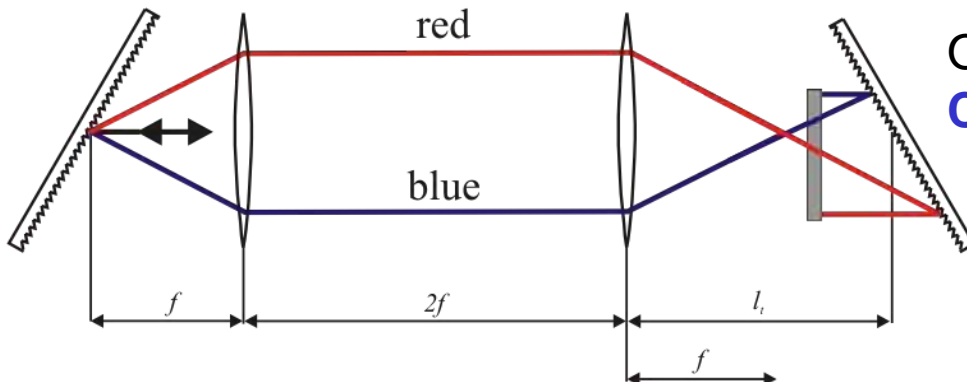
Grating arrangements



Optical path "red" equals optical path "blue"
Zero Dispersion Compressor
often used in pulse shaping devices



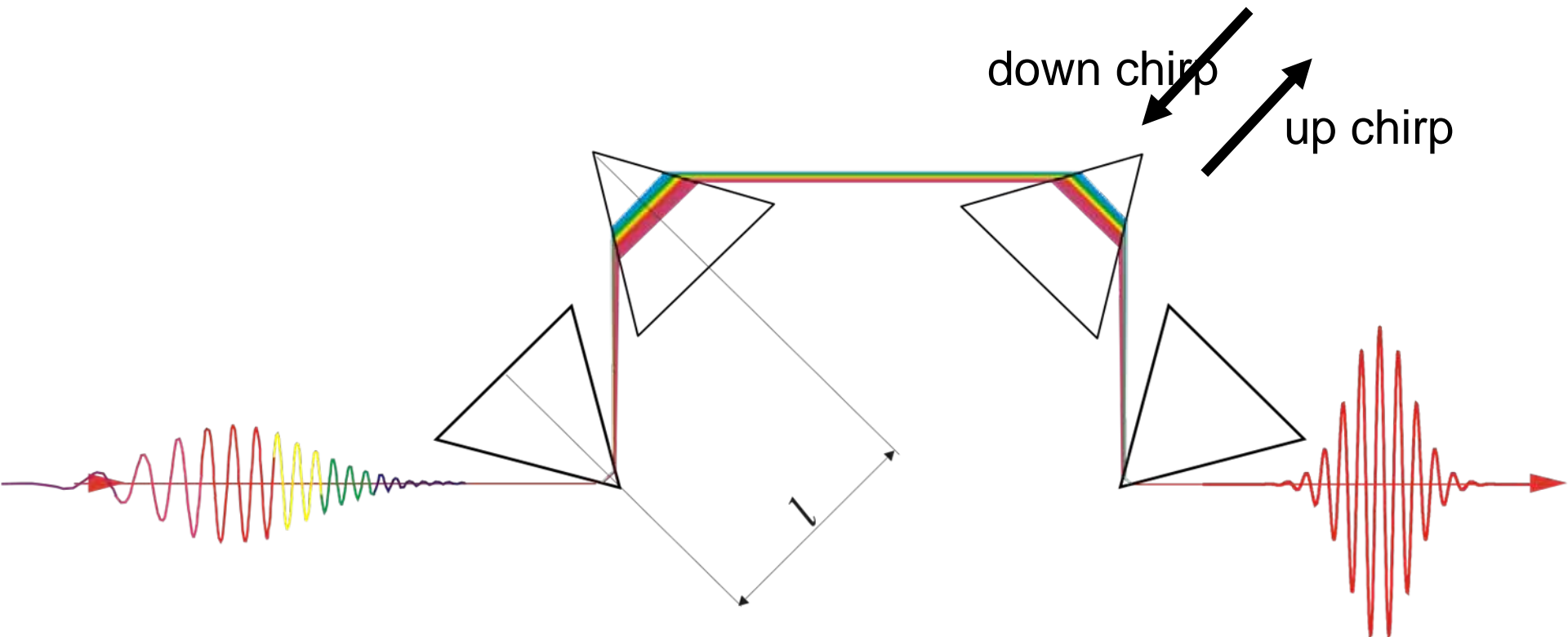
Optical path "red" smaller optical path "blue"
Stretcher



Optical path "red" larger optical path "blue"
Compressor



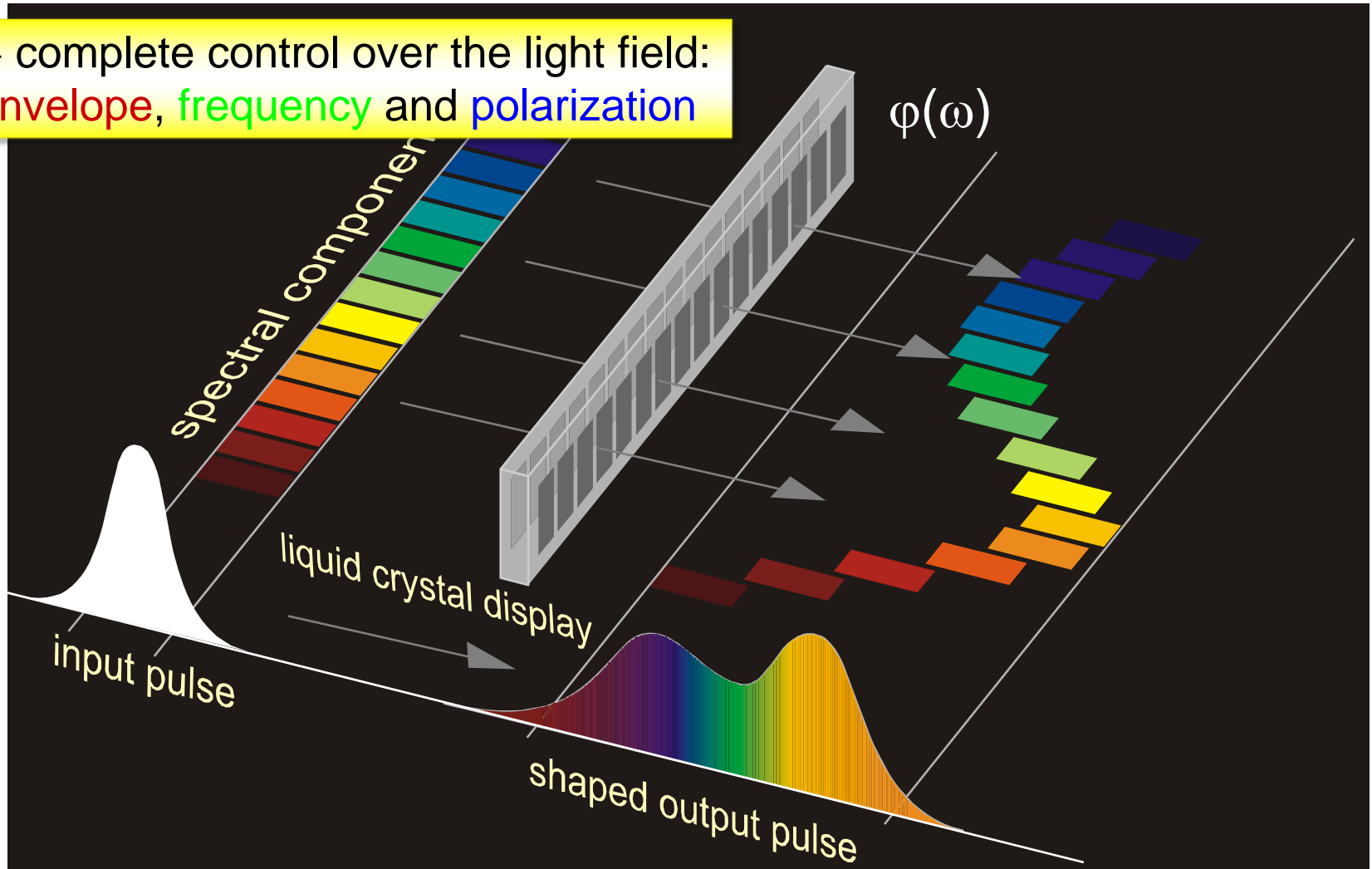
Prism compressors

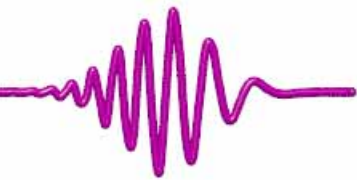




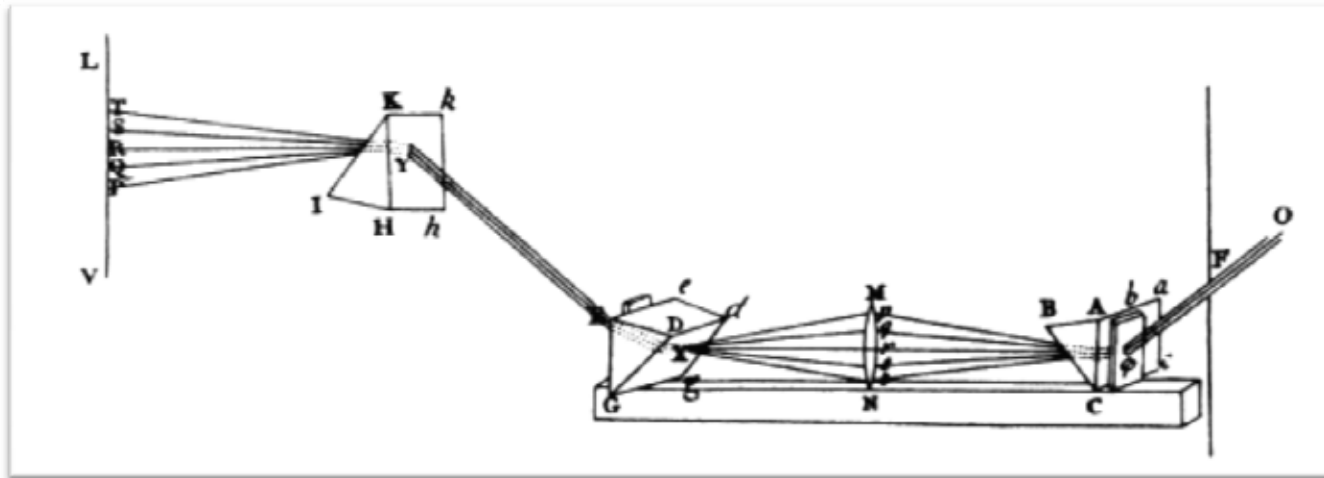
Shaping ultrashort light pulses by Fourier synthesis

⇒ complete control over the light field:
envelope, frequency and polarization





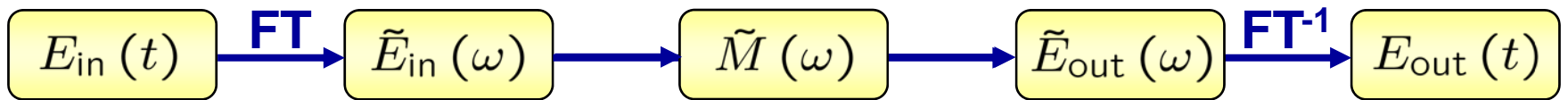
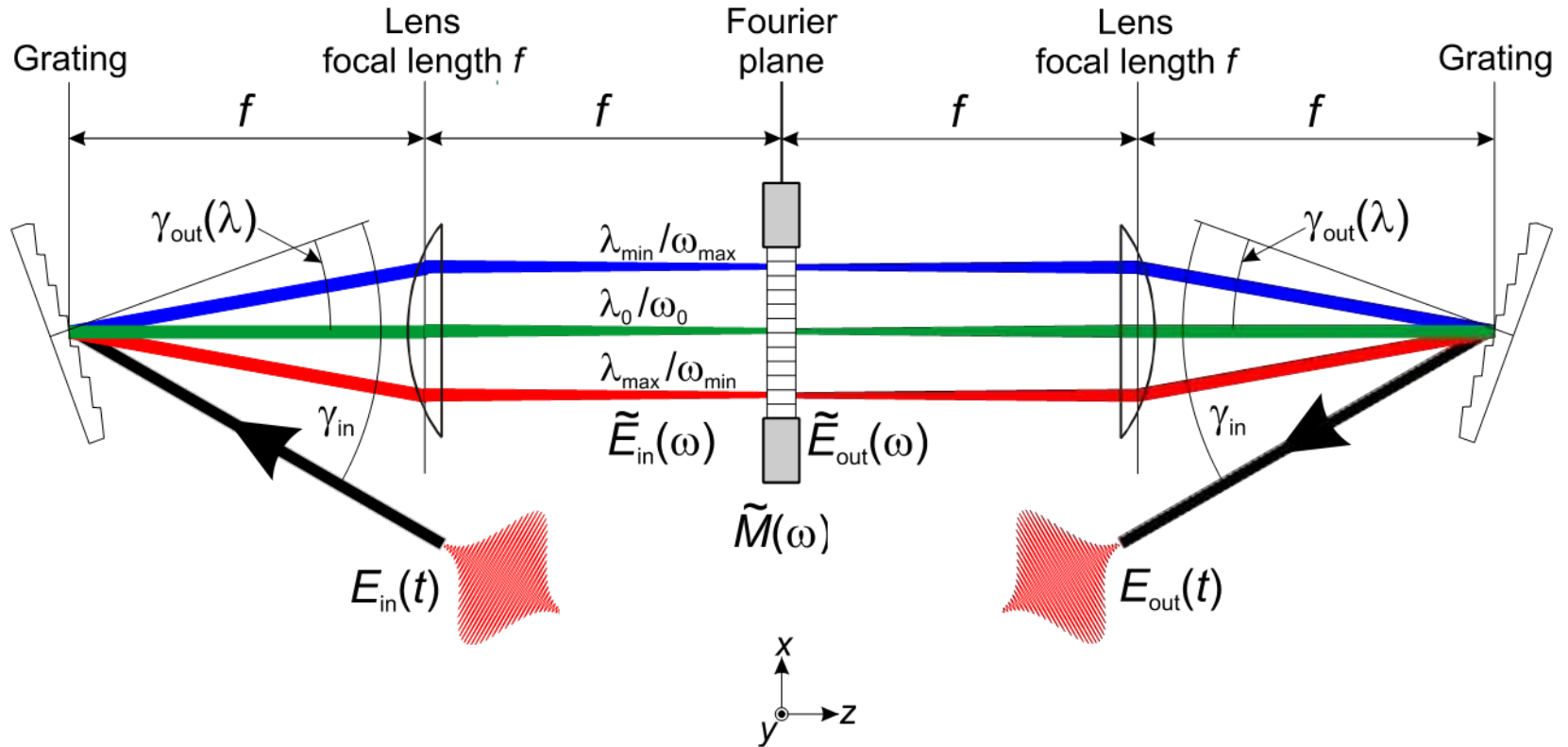
Shaping light: an early experimental layout



Sir Isaac Newton
Opticks (1721 edn),
book I, part II, fig.16.



Fourier transform pulse shaper



Martínez: IEEE J. Quantum Electron 24(12), 2530-2536 (1988), Weiner et al.: JOSA B 5(8), 1563-1572 (1988)



Compact pulse shaper

$E_{out}(t)$

$E_{in}(t)$

$\tilde{E}_{out}(\omega) =$

$\tilde{E}_{in}(\omega) \cdot e^{-i\varphi(\omega)}$

$\tilde{E}_{in}(\omega)$



Example: sinusoidal phase modulation $\tilde{\mathcal{E}}(\omega) \cdot e^{-iA \sin(\omega \cdot T + \phi)}$

$$\mathcal{E}(t)$$

$$\otimes \sum_n J_n(A) e^{-in\phi} \delta(t - nT)$$

$$\mathcal{E}_{mod}(t)$$

$$= \sum_n J_n(A) e^{-in\phi} \mathcal{E}(t - nT)$$

A [rad] T [fs] ϕ [rad]

$$\tilde{\mathcal{E}}(\omega)$$

$$\cdot e^{-iA \sin(\omega \cdot T + \phi)}$$

$$\tilde{\mathcal{E}}_{mod}(\omega)$$

$$= \tilde{\mathcal{E}}(\omega) \cdot e^{-iA \sin(\omega \cdot T + \phi)}$$



Sinusoidal phase modulation $\tilde{\mathcal{E}}(\omega) \cdot e^{-iA \sin(\omega \cdot T + \phi)}$

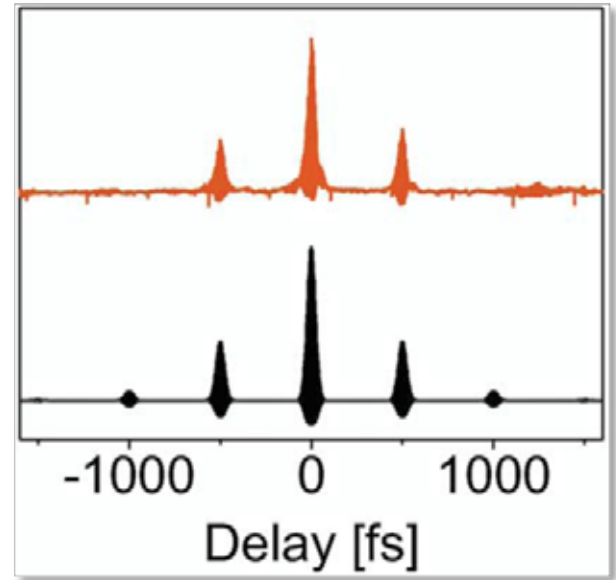
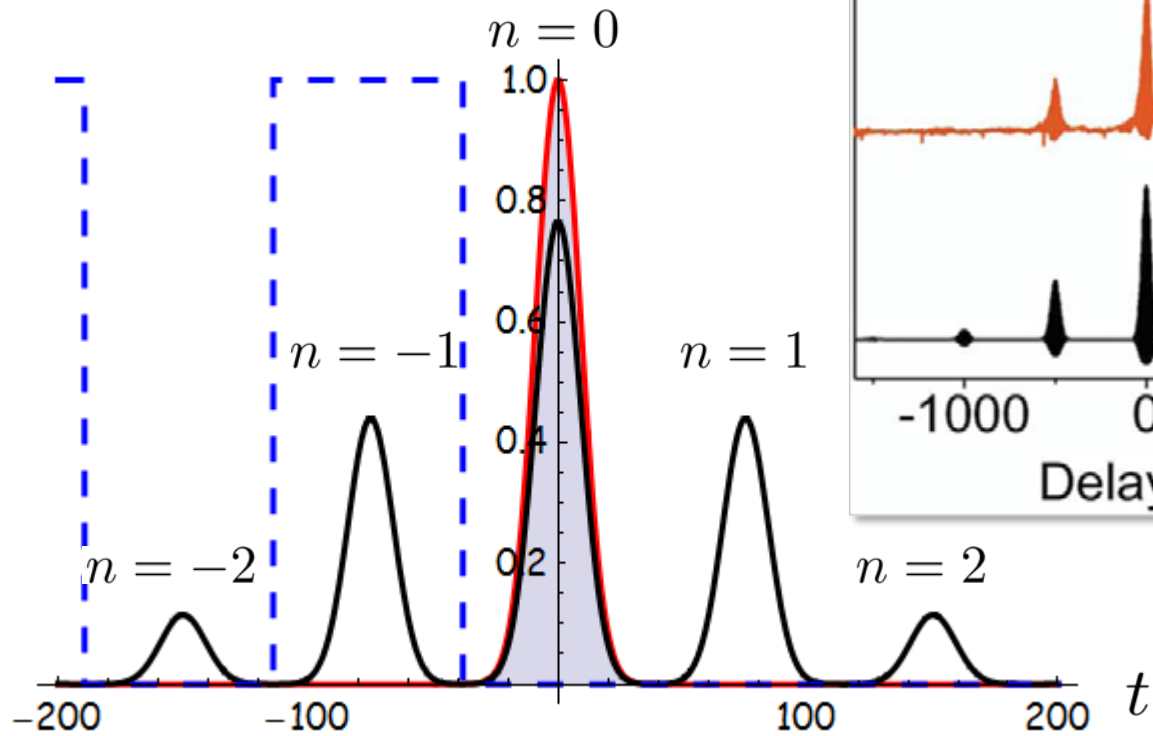
$$\mathcal{E}_{mod}(t) = \sum_n J_n(A) e^{-in\phi} \mathcal{E}(t - nT)$$

$A = 1.0$

$T = 75\text{fs}$

$\phi = 0$

$\Delta t = 15\text{fs}$

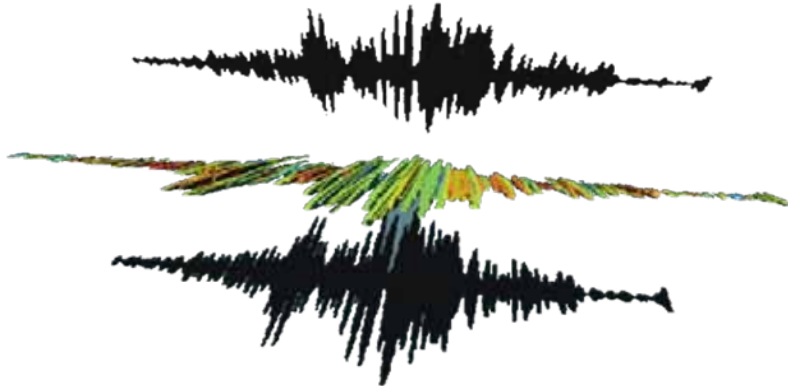


Sinusoidal phase modulation produces pulse trains



New twist in pulse shaping: polarization shaping

Scalar



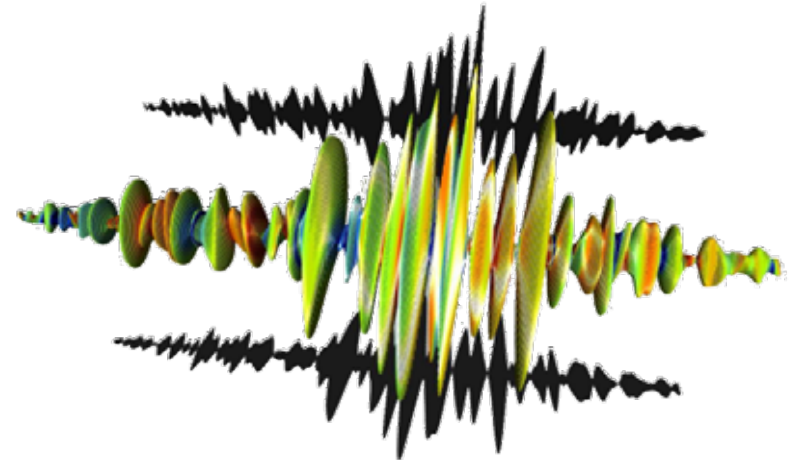
- Modulation of intensity and phase
- Linearly polarized of E-field

Review:

Weiner: *Rev. Sci. Instrum.* **71**, 1929, (2000)

Rev. Sci. Instr., **74**, 4950, (2003)

Vectorial

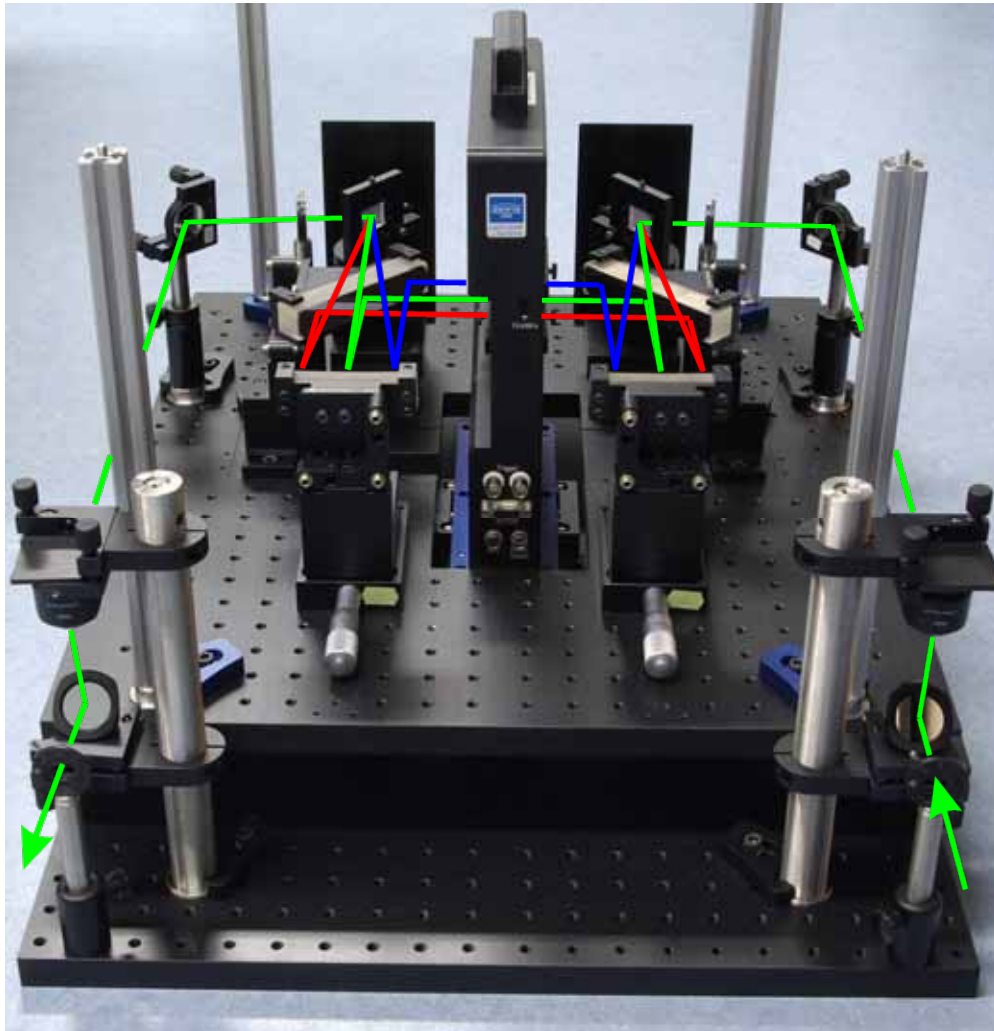


- Controlling intensity, phase and polarization
- Polarization state varies within a single pulse

Brixner et al.: *Opt. Lett.* **26**, 557, (2001)



A compact set-up for polarization shaping



Key features:

- Polarization shaping
- Phase & amplitude shaping
- 2x 640 pixel modulator



High spectral resolution

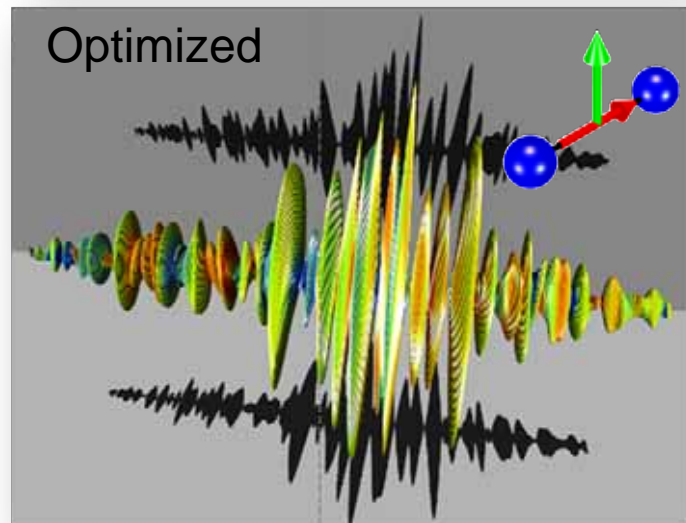
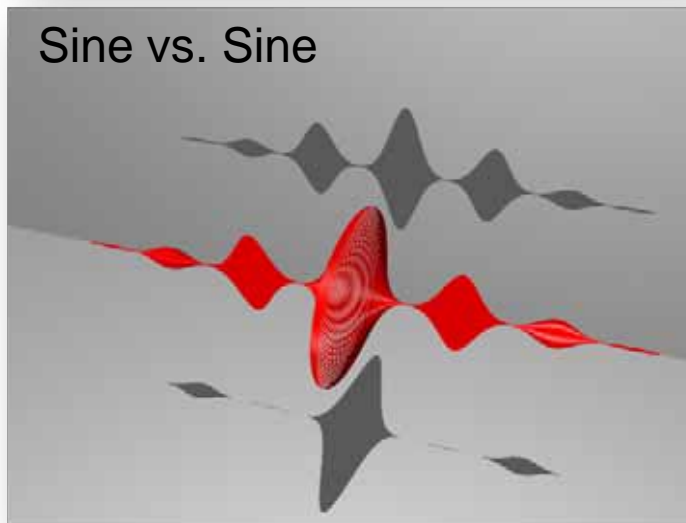
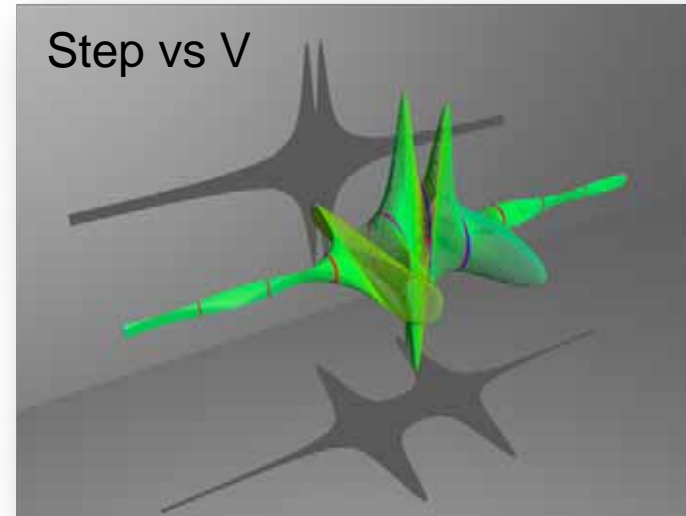
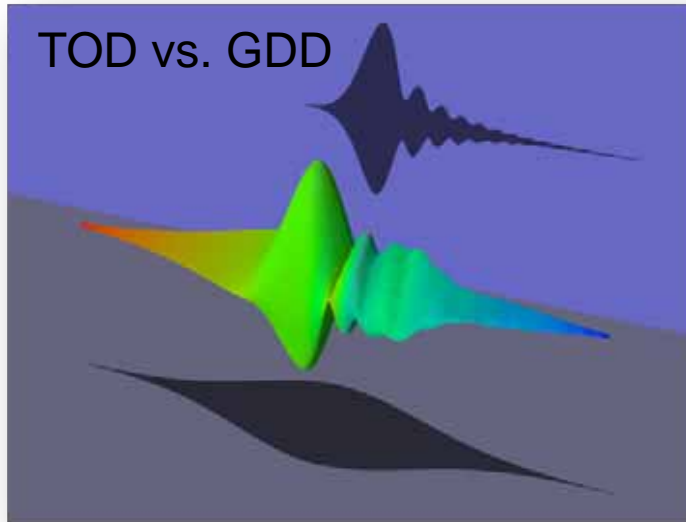
0.16 nm @ 800 nm



Large temporal window
> 10 ps



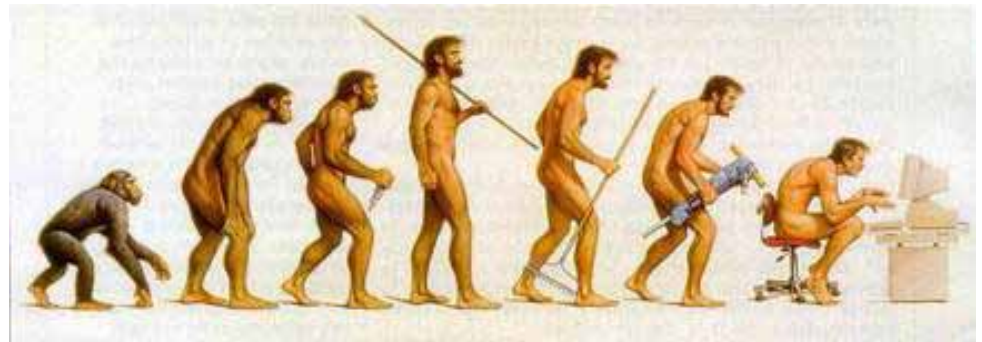
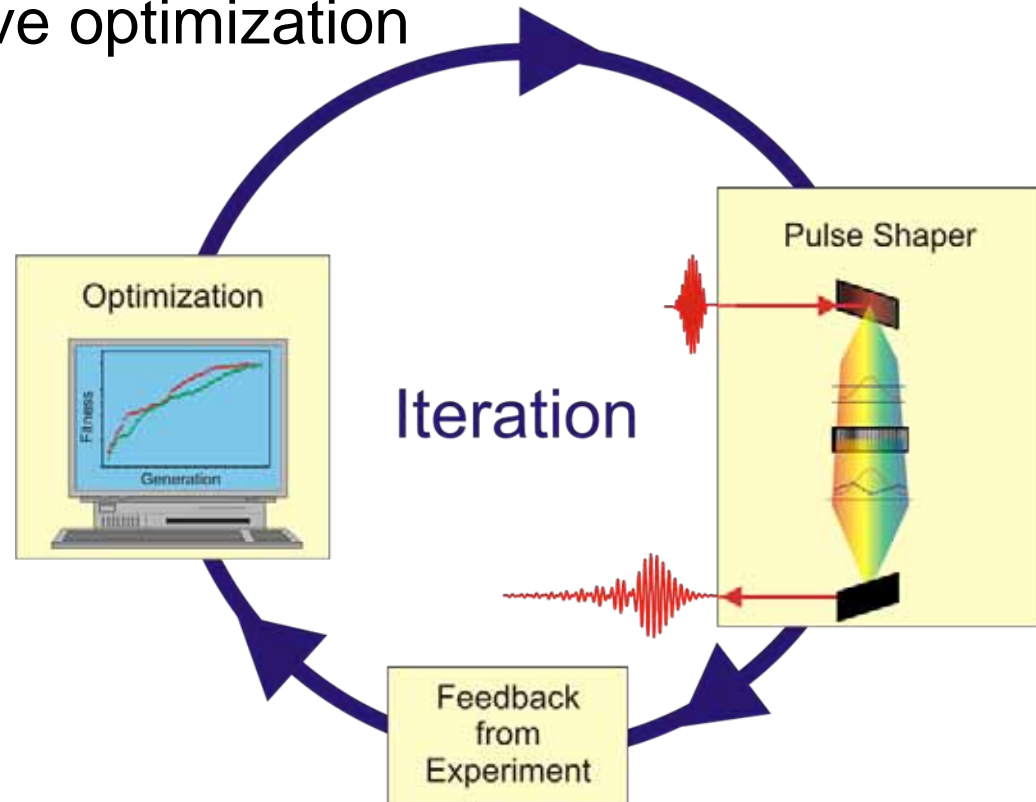
Pulse gallery





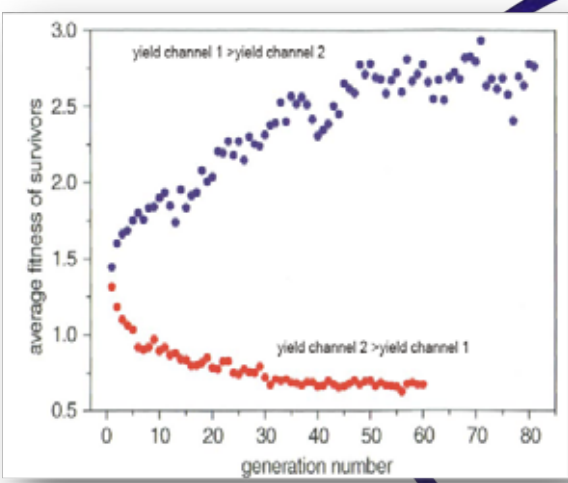
Adaptive optimization

- Complex systems
- No *a priori* knowledge required
- Feedback by experiment
- Genetic optimization algorithm
- Iterative

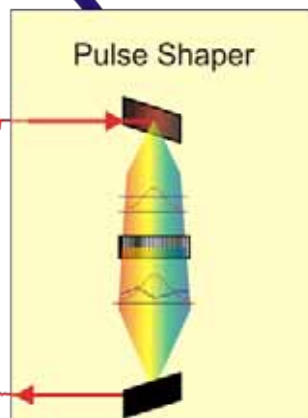




Laser control by adaptive optimization



Iteration

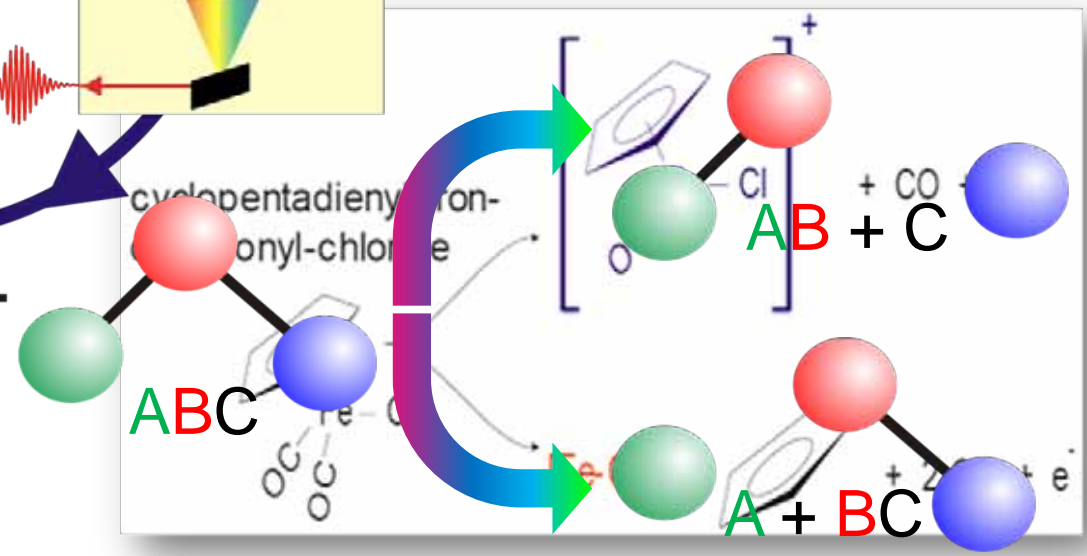


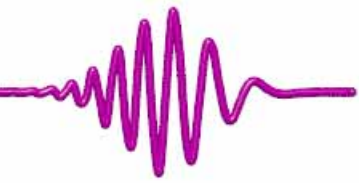
Control of Chemical Reactions by Feedback-Optimized Phase-Shaped Femtosecond Laser Pulses

A. Assion, T. Baumert,* M. Bergt, T. Brixner, B. Kiefer, V. Seyfried, M. Strehle, G. Gerber

SCIENCE VOL 282 30 OCTOBER 1998

Feedback from Experiment

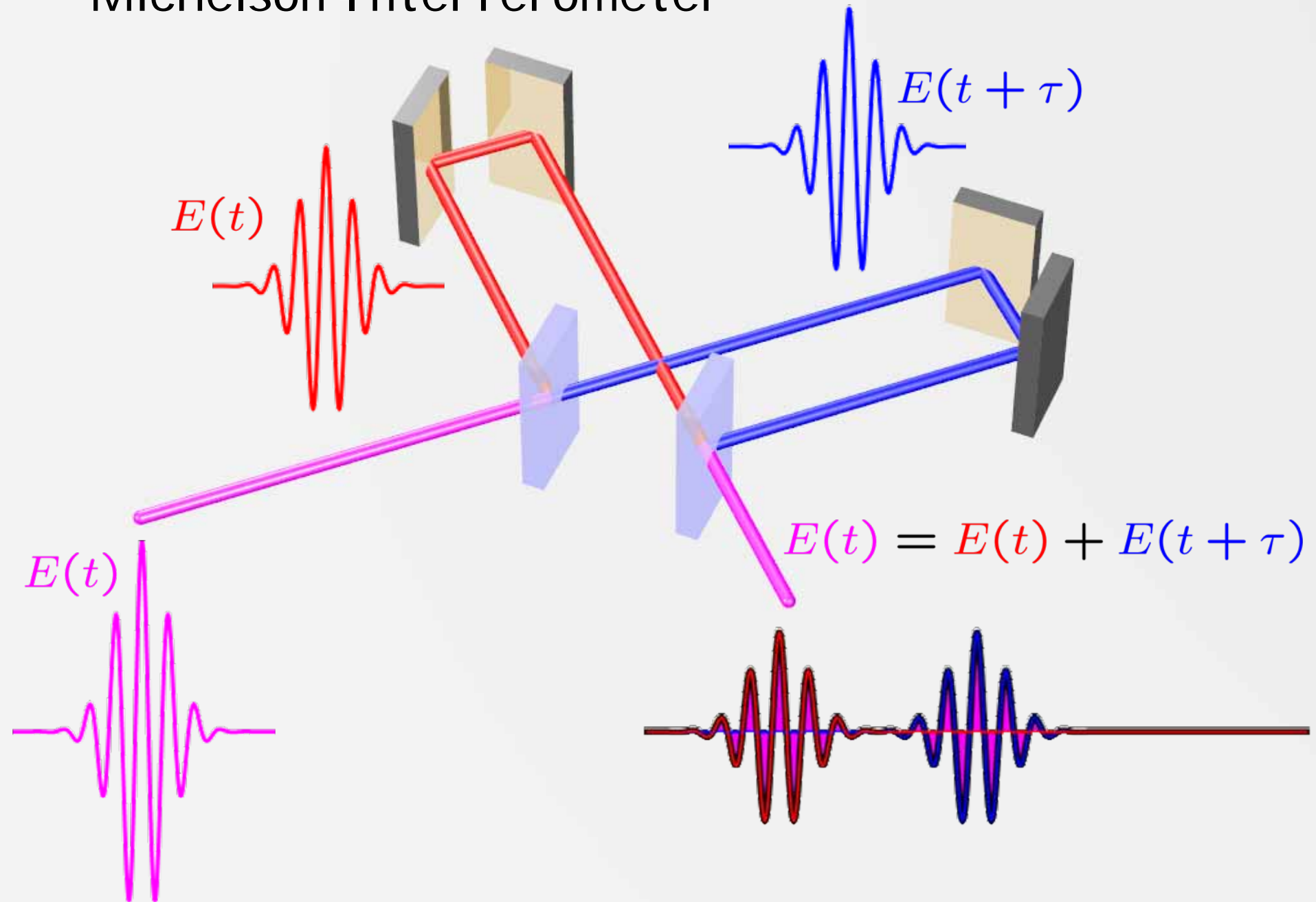


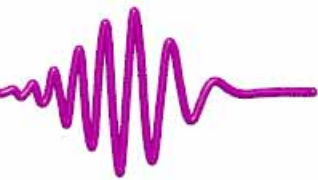


Pulse characterization

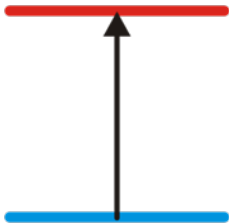
- Time domain
- Frequency domain
- Joint time frequency domain

Michelson Interferometer





Signals



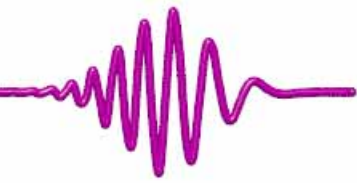
One photon

$$S_{linear}(\tau) = \int_{-\infty}^{\infty} \{E(t) + E(t + \tau)\}^2 dt$$



Two photons

$$S_{quad}(\tau) = \int_{-\infty}^{\infty} \{E(t) + E(t + \tau)\}^4 dt$$

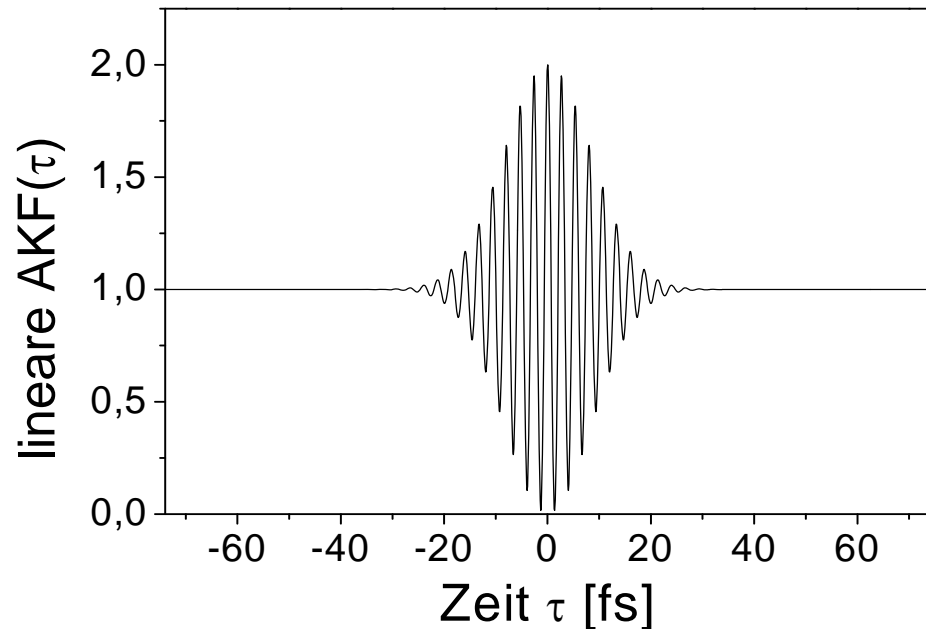


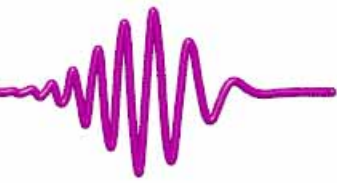
Linear autocorrelation

$$S_{linear}(\tau) = \int_{-\infty}^{\infty} \{E(t) + E(t + \tau)\}^2 dt$$

$$= 2 \int_{-\infty}^{\infty} E^2(t) dt + 2 \int_{-\infty}^{\infty} E(t)E(t + \tau) dt$$

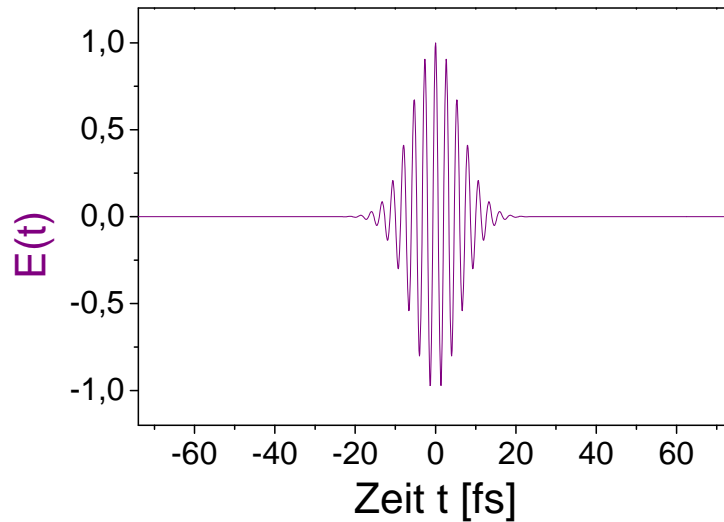
$$\frac{S_{linear}(0)}{S_{linear}(\infty)} = \frac{4I_0}{2I_0} = 2$$



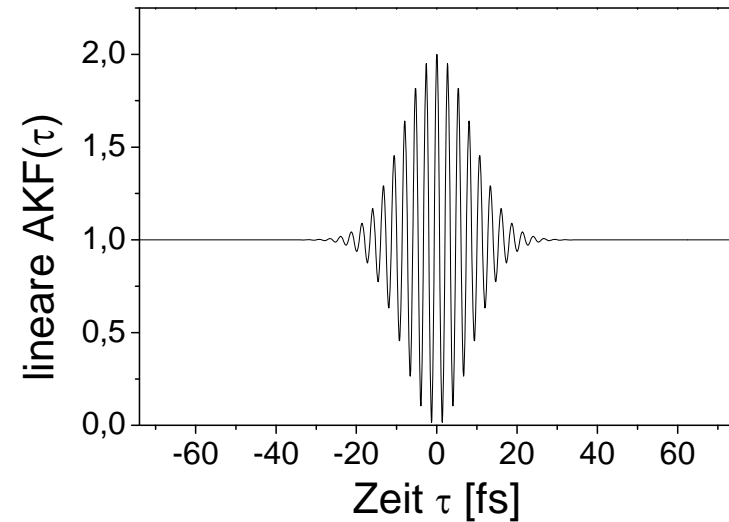


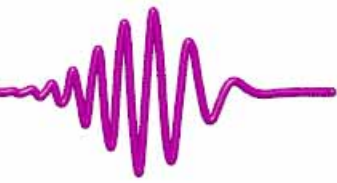
Linear autocorrelation of a bandwidth limited pulse

10 fs bandwidth limited



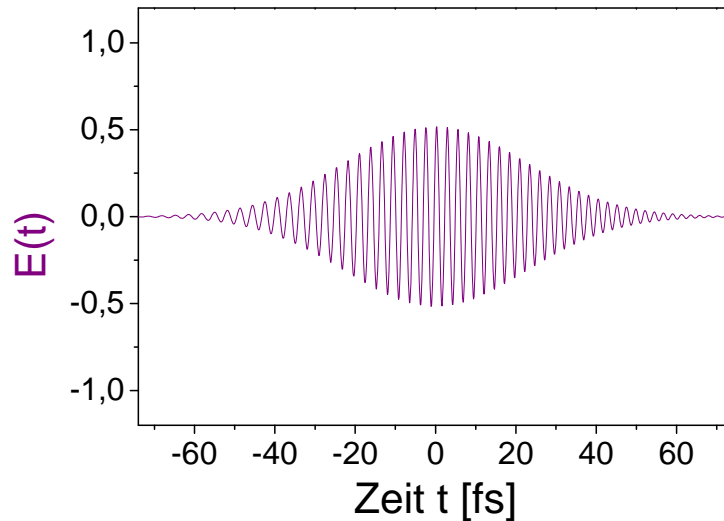
Linear autocorrelation



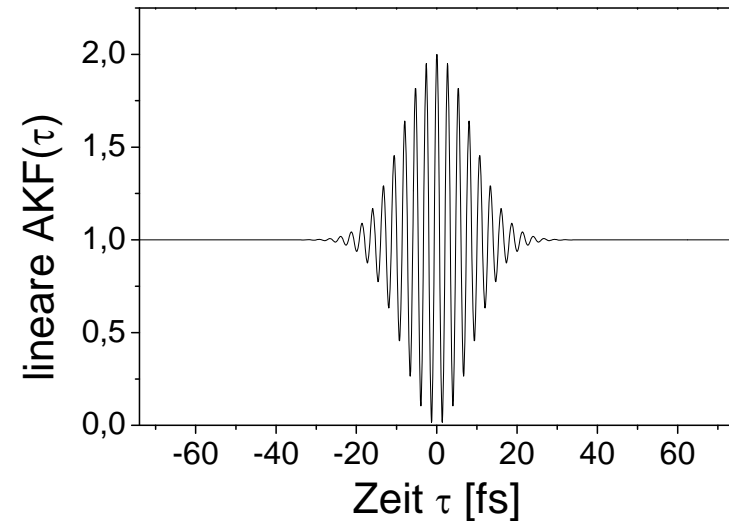


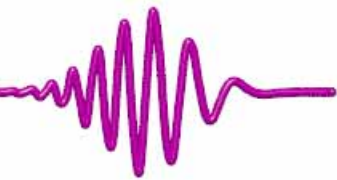
Linear autocorrelation of a chirped pulse

Linear chirp (130 fs^2)



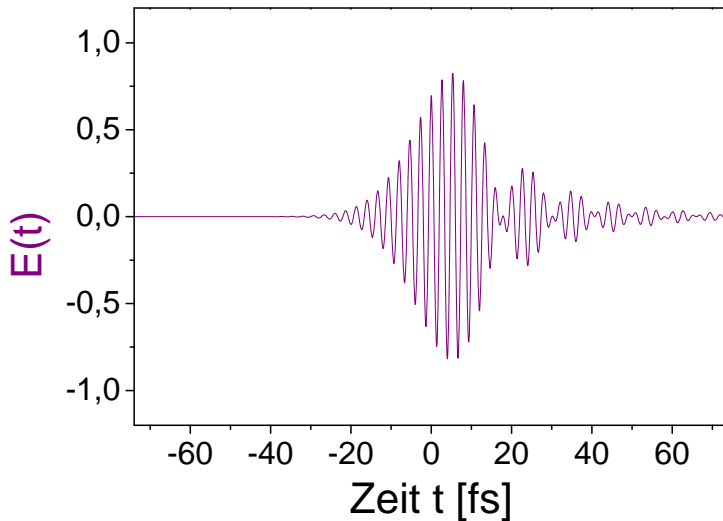
Linear autocorrelation



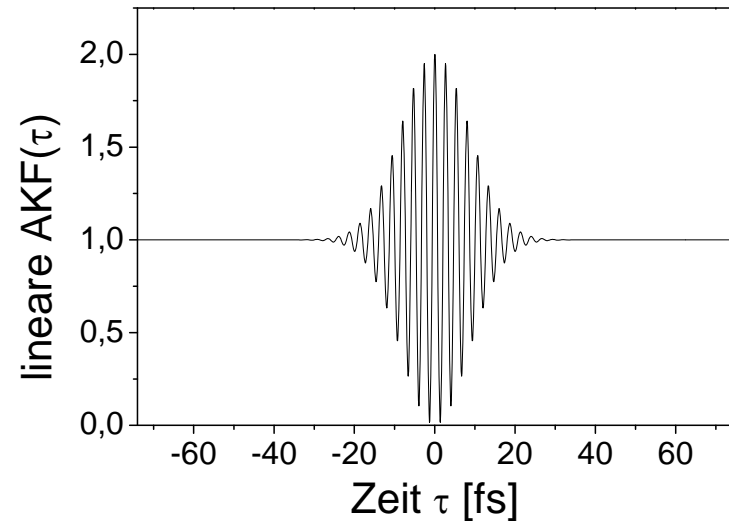


Linear autocorrelation of a TOD pulse

TOD (750 fs³)



Linear autocorrelation

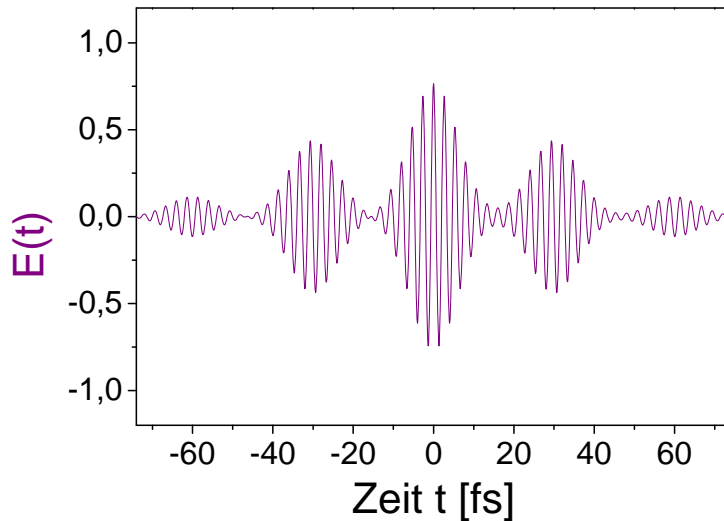


The linear autocorrelation function is always *gerade*!

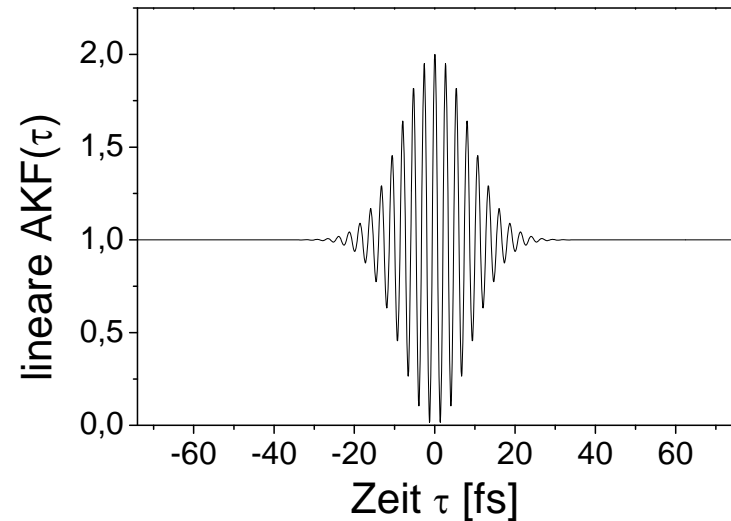


Linear autocorrelation of a sinusoidally modulated pulse

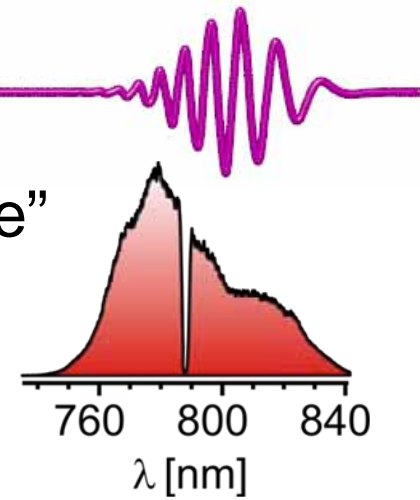
Sinusoidal modulation ($A = 1$)



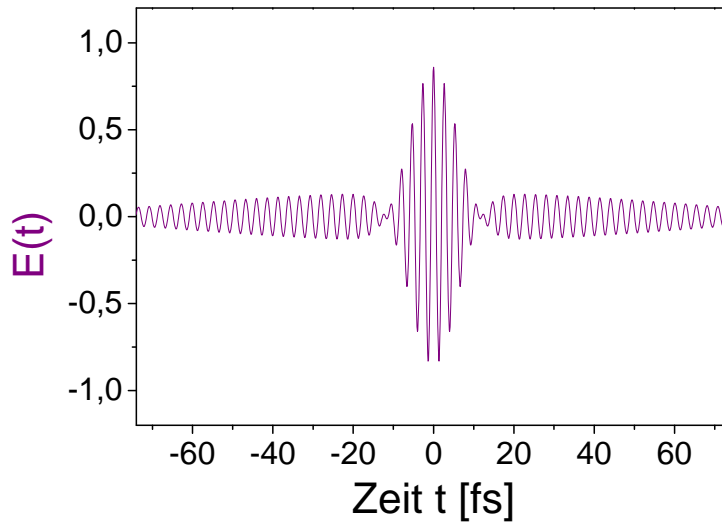
Linear autocorrelation



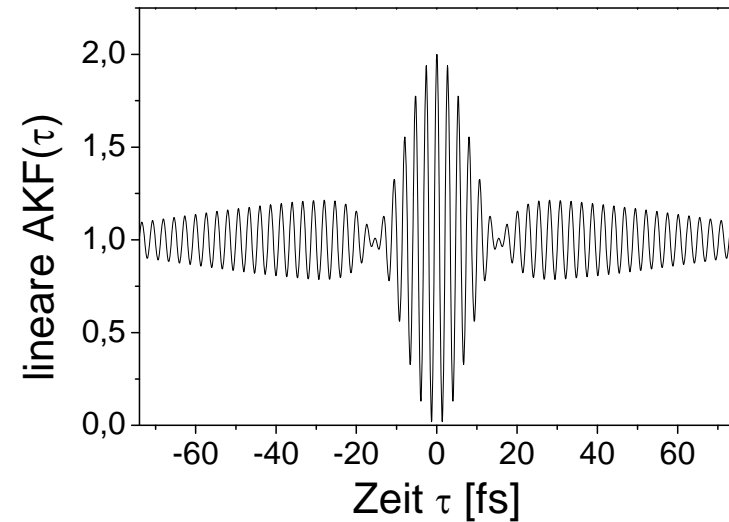
Linear autocorrelation of a “spectral hole”



„Spectral hole“



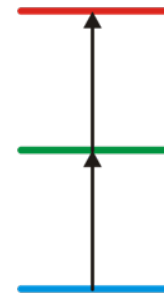
Linear autocorrelation



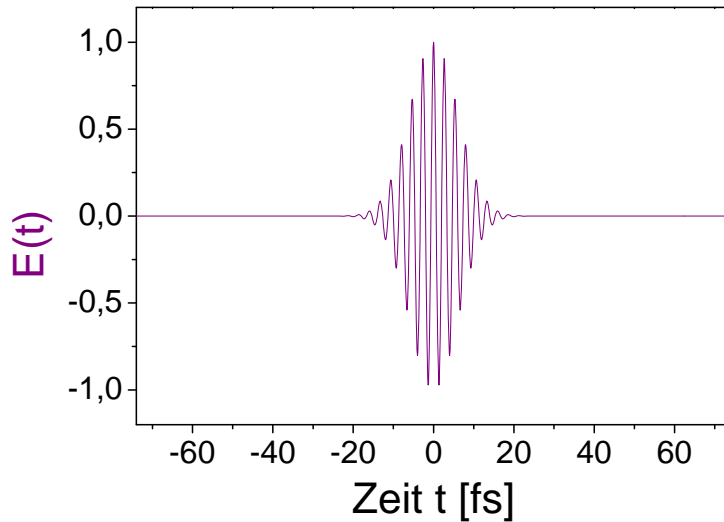


2nd order autocorrelation of a bandwidth limited pulse

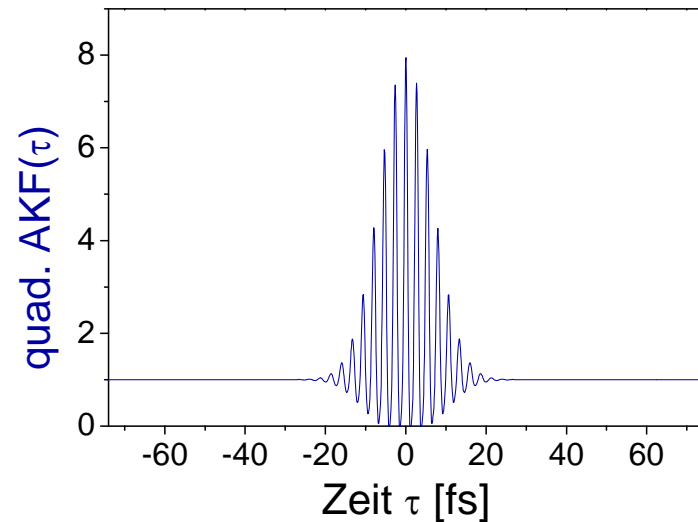
$$S_{quad}(\tau) = \int_{-\infty}^{\infty} \{E(t) + E(t + \tau)\}^4 dt$$

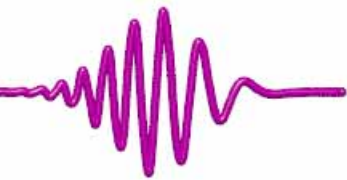


10 fs bandwidth limited



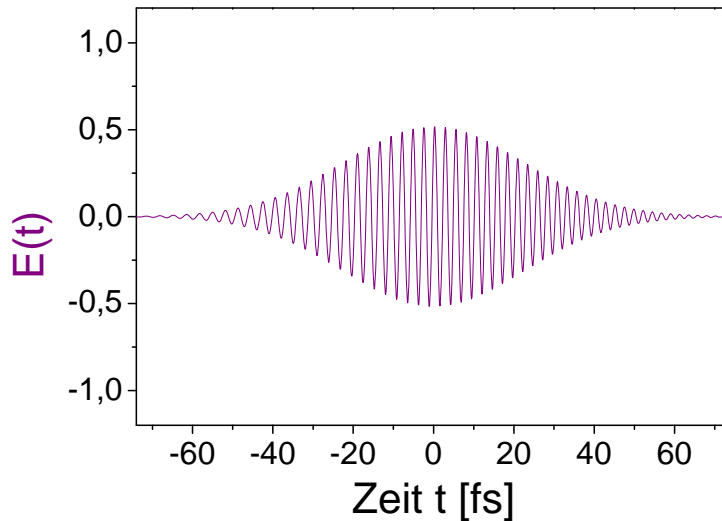
2nd order autocorrelation



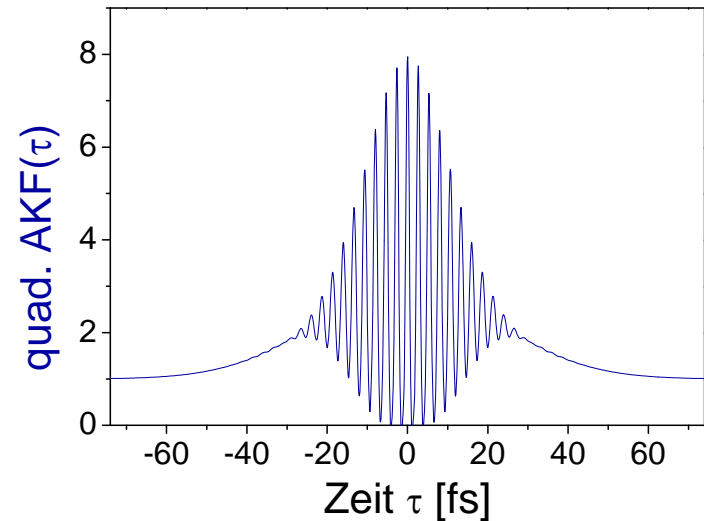


2nd order autocorrelation of a chirped pulse

Linear chirp (130 fs²)

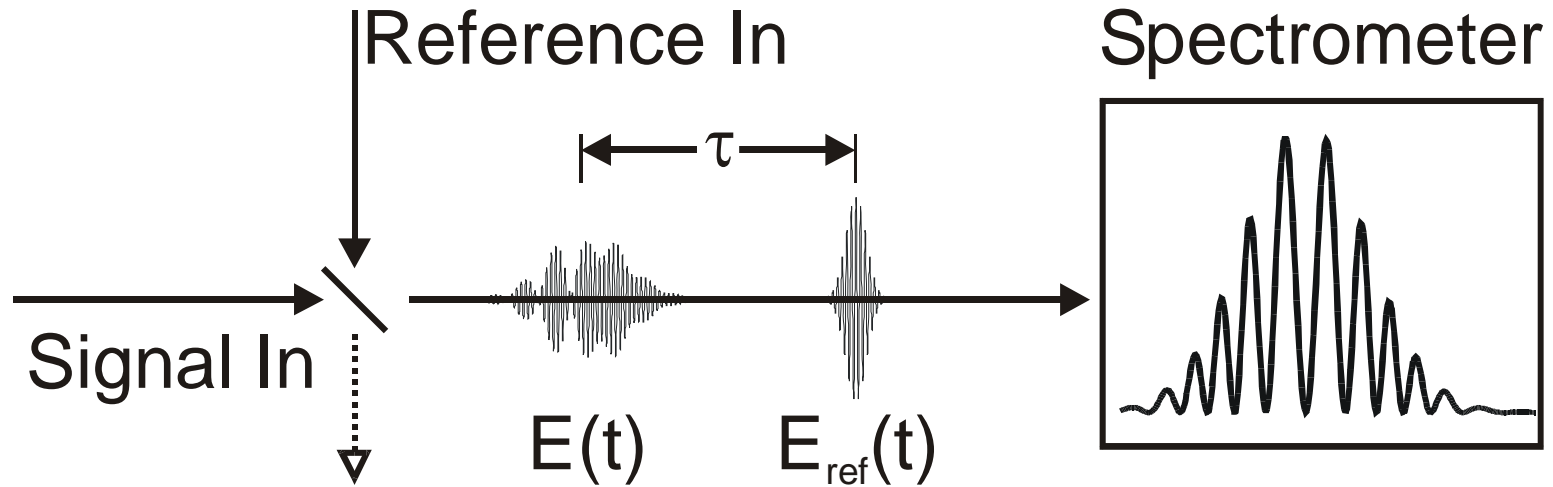


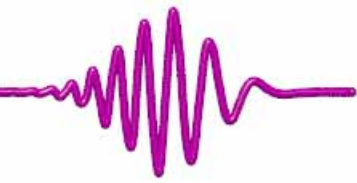
2nd order autocorrelation





Spectral interference





Spectral interference

$$E_2(t) = E_{mod}(t) + E(t + \tau)$$

$$\begin{aligned}\tilde{E}_2(\omega) &= \tilde{E}_{mod}(\omega) + \tilde{E}(\omega)e^{i\omega\tau} \\ &= \tilde{E}(\omega)e^{-i\varphi(\omega)} + \tilde{E}(\omega)e^{i\omega\tau}\end{aligned}$$

$$PSD_2(\omega) = 2 \{1 + \cos[\omega\tau + \varphi(\omega)]\} PSD(\omega)$$

Power Spectral Density $PSD = |\tilde{E}(\omega)|^2$



SI of an up-chirp

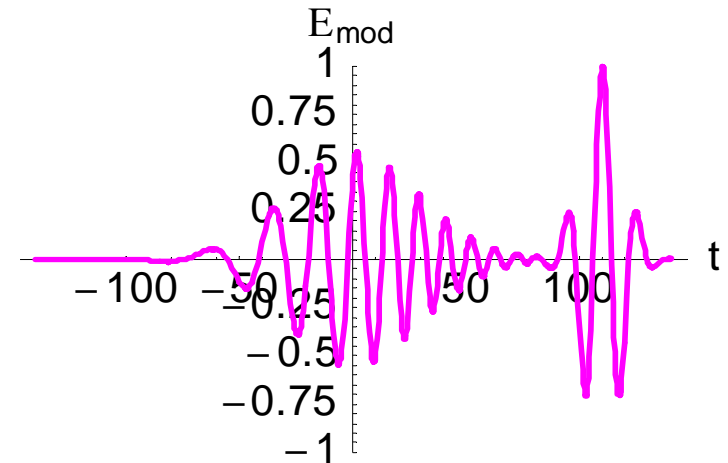
$$E_2(t) = E_{mod}(t) + E(t + \tau)$$

$$\varphi(\omega) = \frac{\varphi_2}{2} \omega^2$$

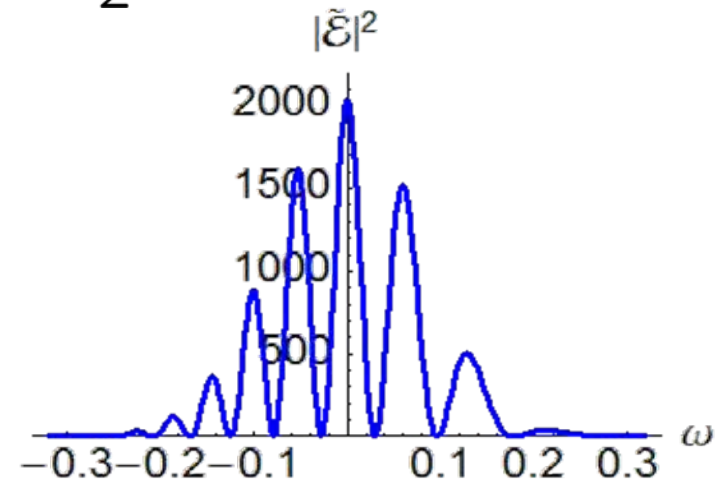
$$\varphi_2 = 250\text{fs}^2$$

$$\Delta t = 15\text{fs}$$

$$\tau = -110\text{fs}$$

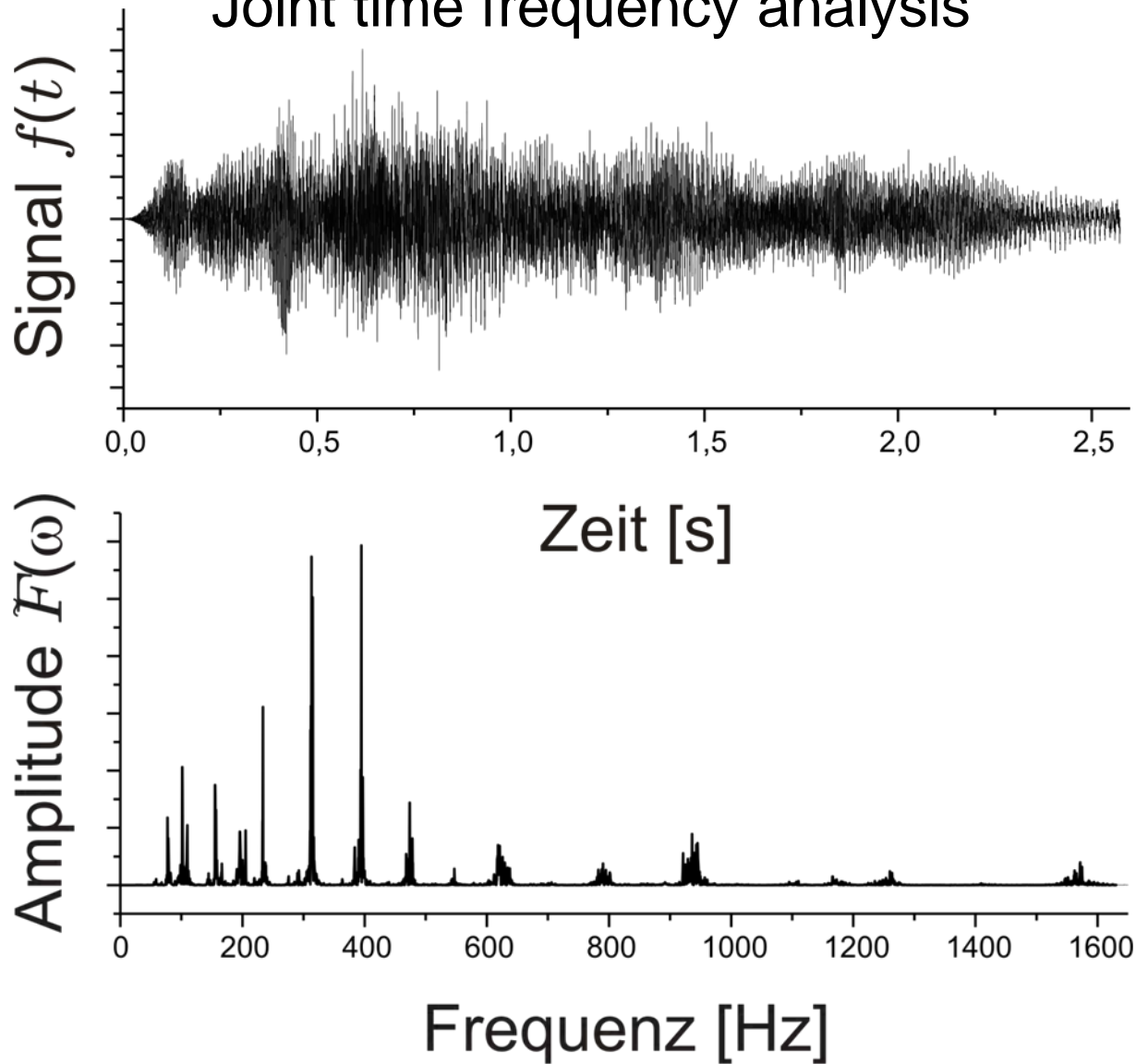


$$PSD_2(\omega) = 2PSD(\omega) \left\{ 1 + \cos\left[\omega\tau + \frac{\varphi_2}{2} \cdot \omega^2\right] \right\}$$





Joint time frequency analysis

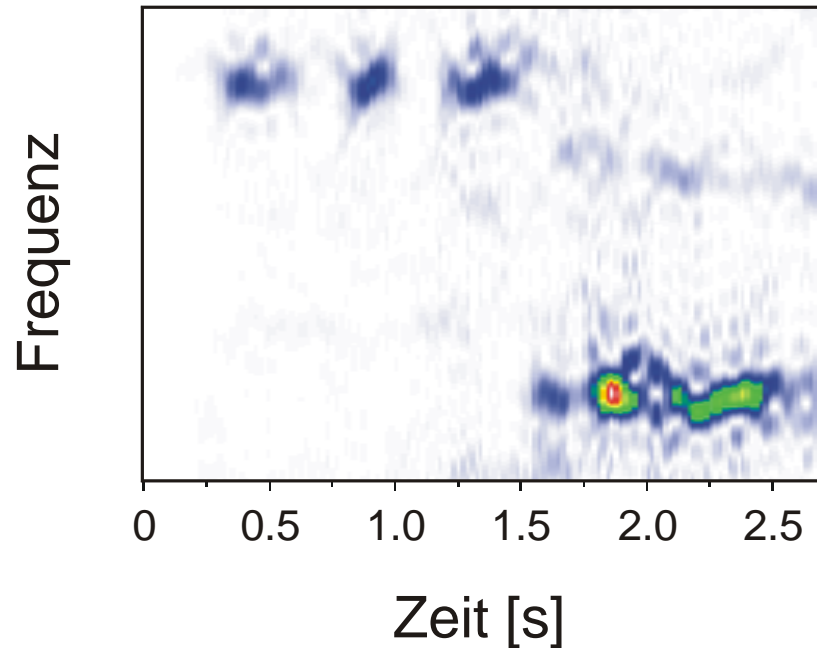


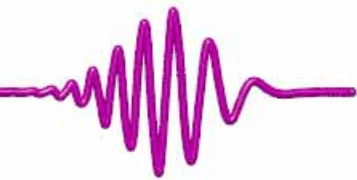


A spectrogram

Ludwig van Beethoven

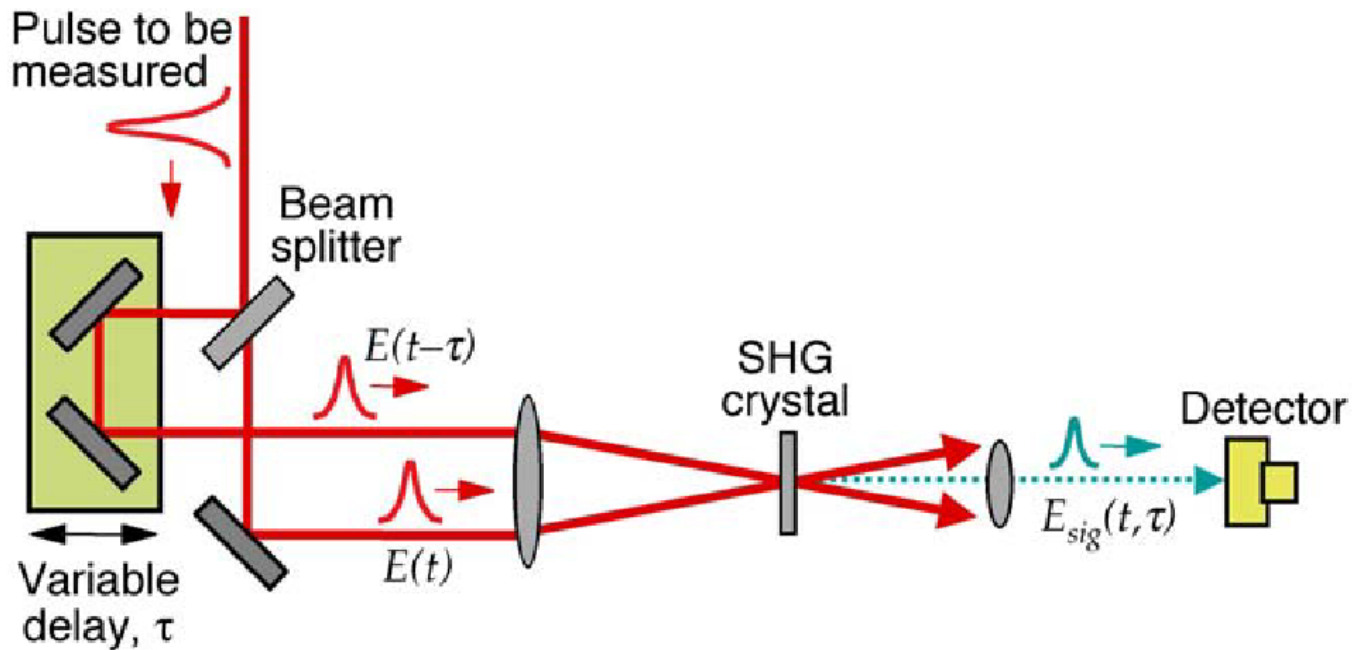
Symphony No. 5
C minor op. 67
Allegro con brio





JTFA in optics: FROG

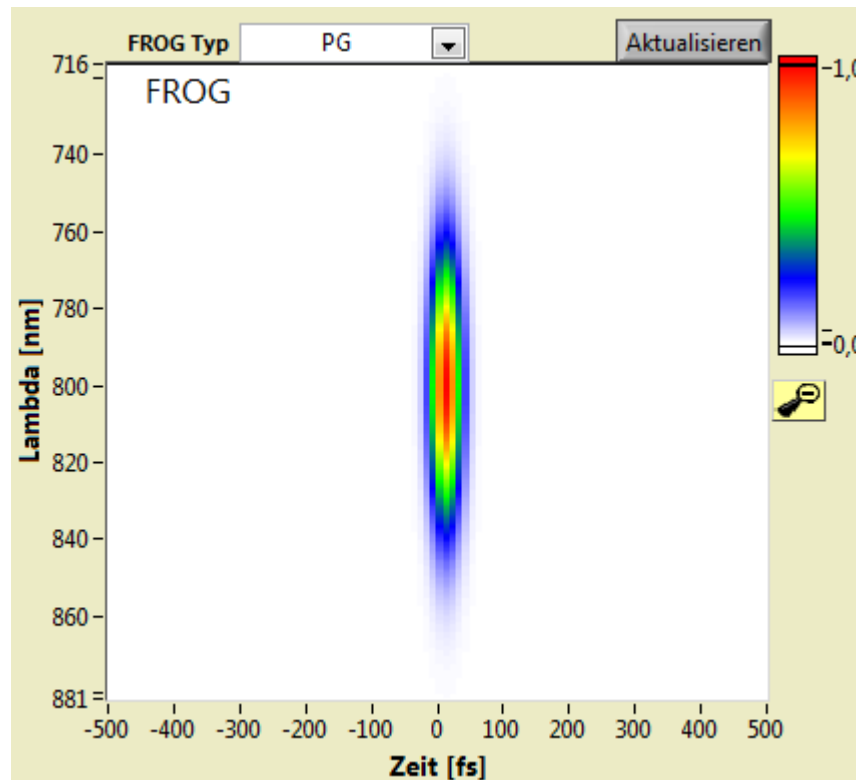
FROG = Frequency Resolved Optical Gating

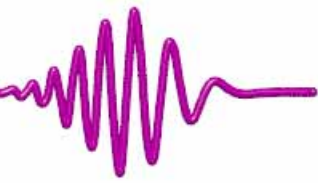




PG FROG of a bandwidth limited pulse

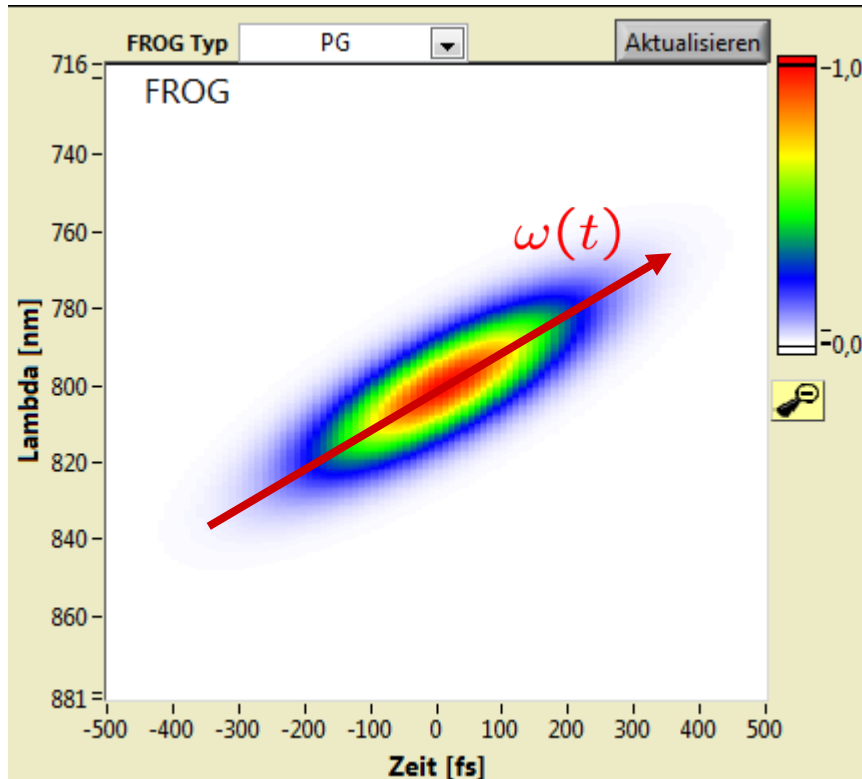
$$I_{Frog}^{PG}(\omega, \tau) = \left| \int_{-\infty}^{\infty} \mathcal{E}(t) |\mathcal{E}(t - \tau)|^2 e^{i\omega t} dt \right|^2$$



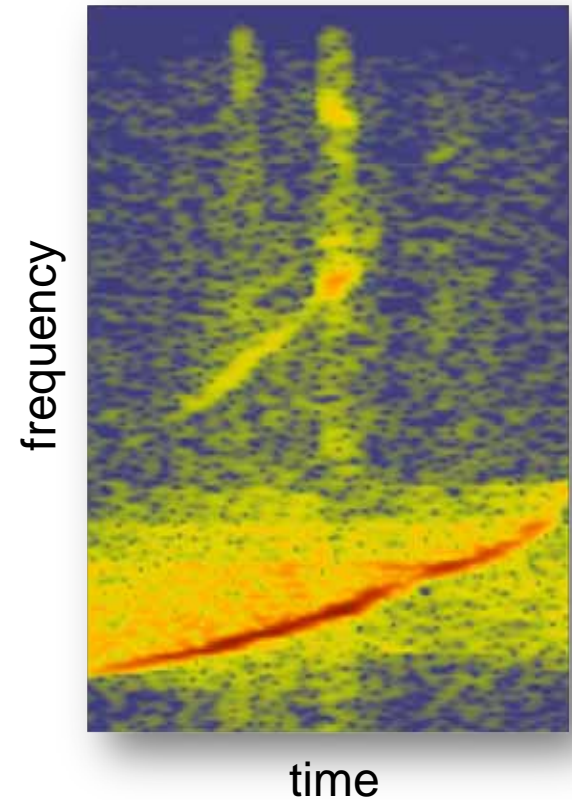


PG FROG of a chirped pulse

$$I_{Frog}^{PG}(\omega, \tau) = \left| \int_{-\infty}^{\infty} \mathcal{E}(t) |\mathcal{E}(t - \tau)|^2 e^{i\omega t} dt \right|^2$$



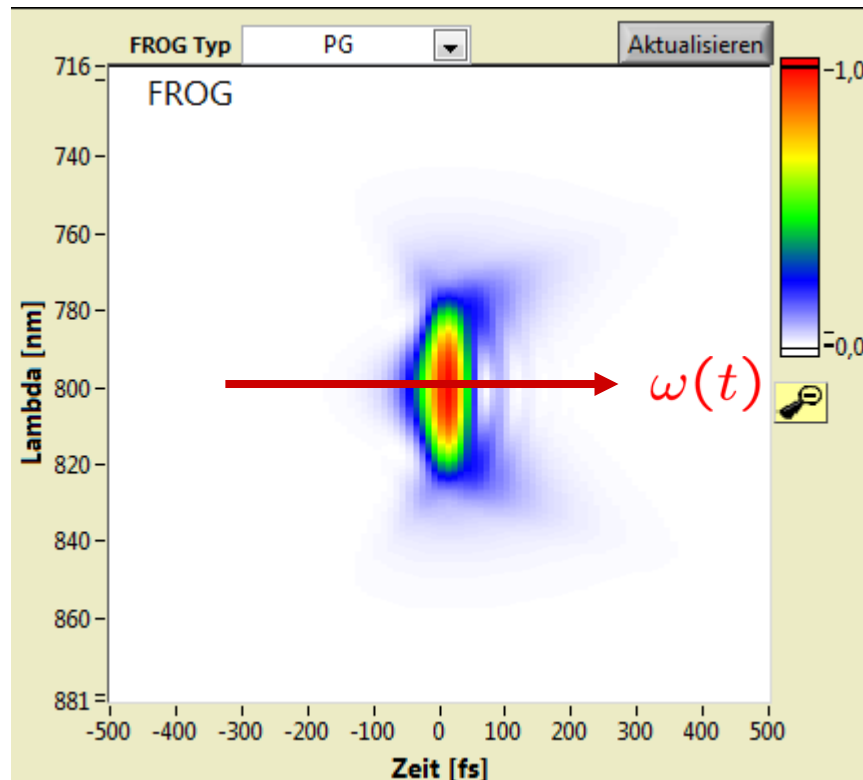
Spectrogram

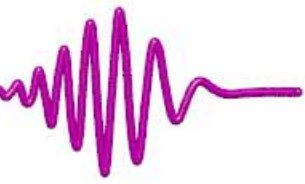




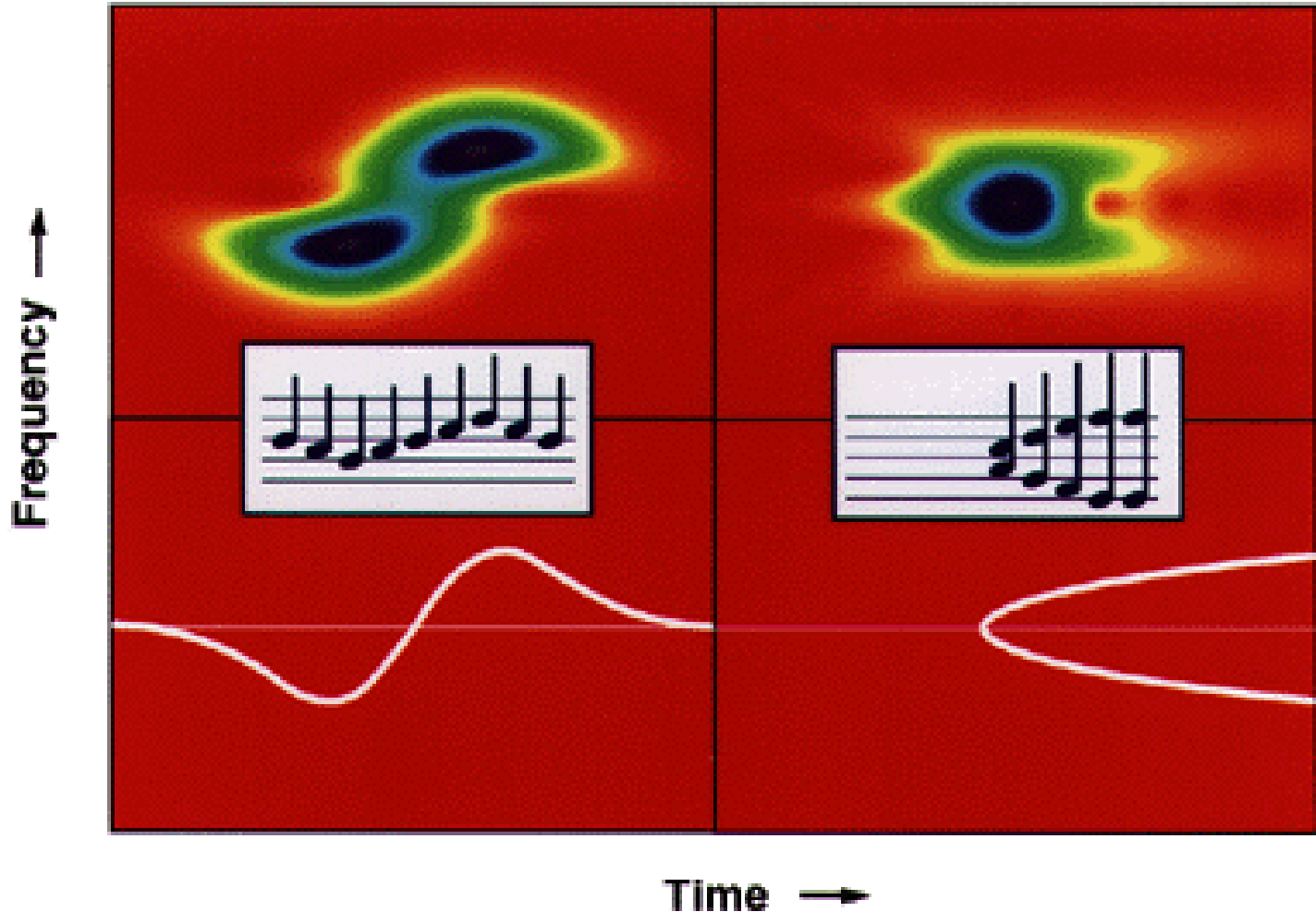
PG FROG of a TOD pulse

$$I_{Frog}^{PG}(\omega, \tau) = \left| \int_{-\infty}^{\infty} \mathcal{E}(t) |\mathcal{E}(t - \tau)|^2 e^{i\omega t} dt \right|^2$$





Analogy to music





Prof. Dr. T. Baumert



PD Dr. M. Wollenhaupt



T. Bayer



L. Englert



L. Haag



C. Horn



P. Kasper



U. Meier-Diedrich



J. Mildner



C. Sarpe



J. Schneider



M. Winter



PD Dr. A. Assion



R. Bäumner



Dr. D. Liese



Dr. A. Präkelt

PHD positions available