Laser physics

M. A. Bouchene

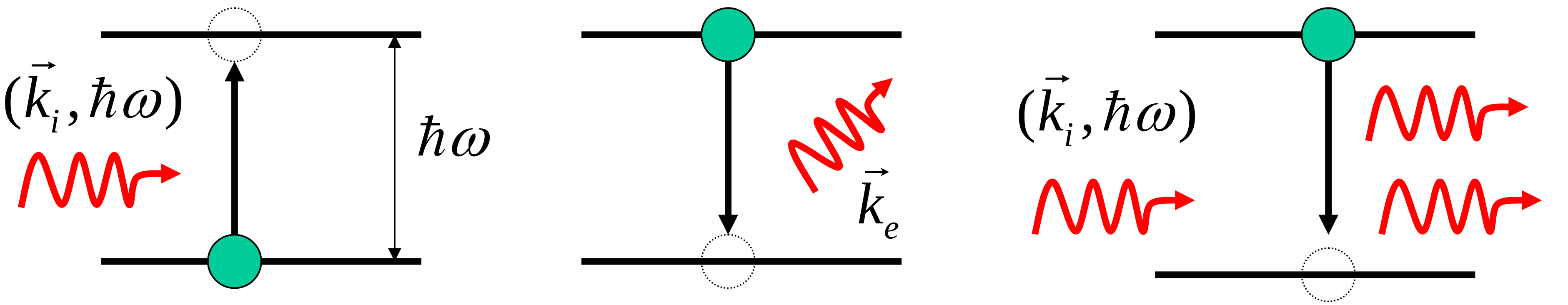
Laboratoire « Collisions, Agrégats, Réactivité »,
Université Paul Sabatier, Toulouse, France

Outlook

- 1- Basic introduction
- 2- Optical cavity
- 3- Energetic model of the interaction
- 4- Laser oscillation (CW laser)
- 5- Laser frequency
- 6- Pulsed regime (summary)

1- Basic introduction

Basic Ideas



$\vec{k}_e \neq \vec{k}_i$

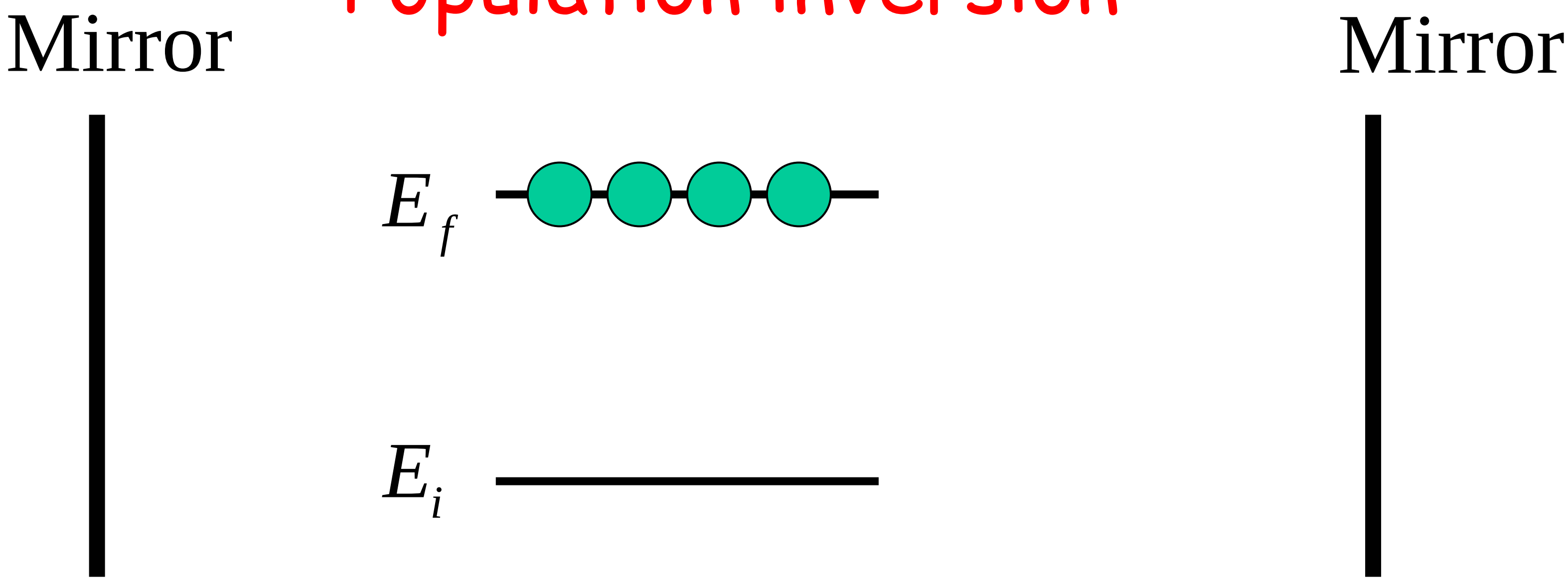
$\vec{k}_e = \vec{k}_i, \omega_e = \omega_i$

Absorption

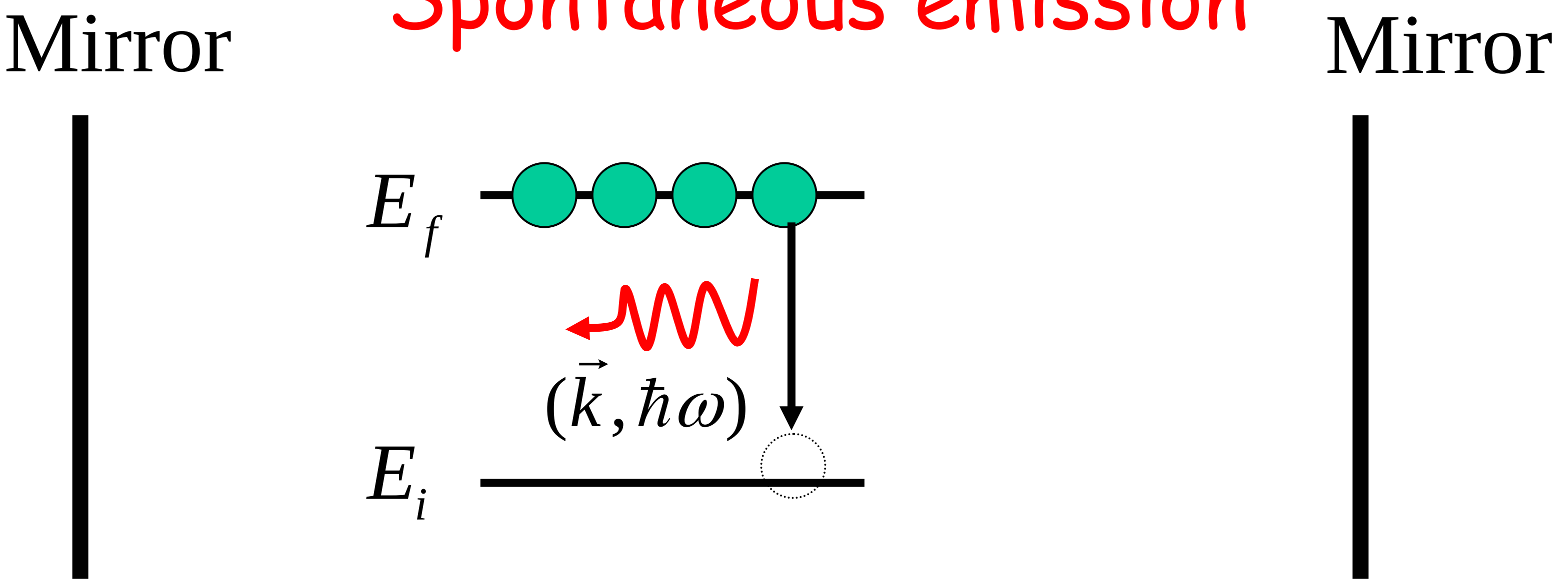
Spontaneous emission

Stimulated emission

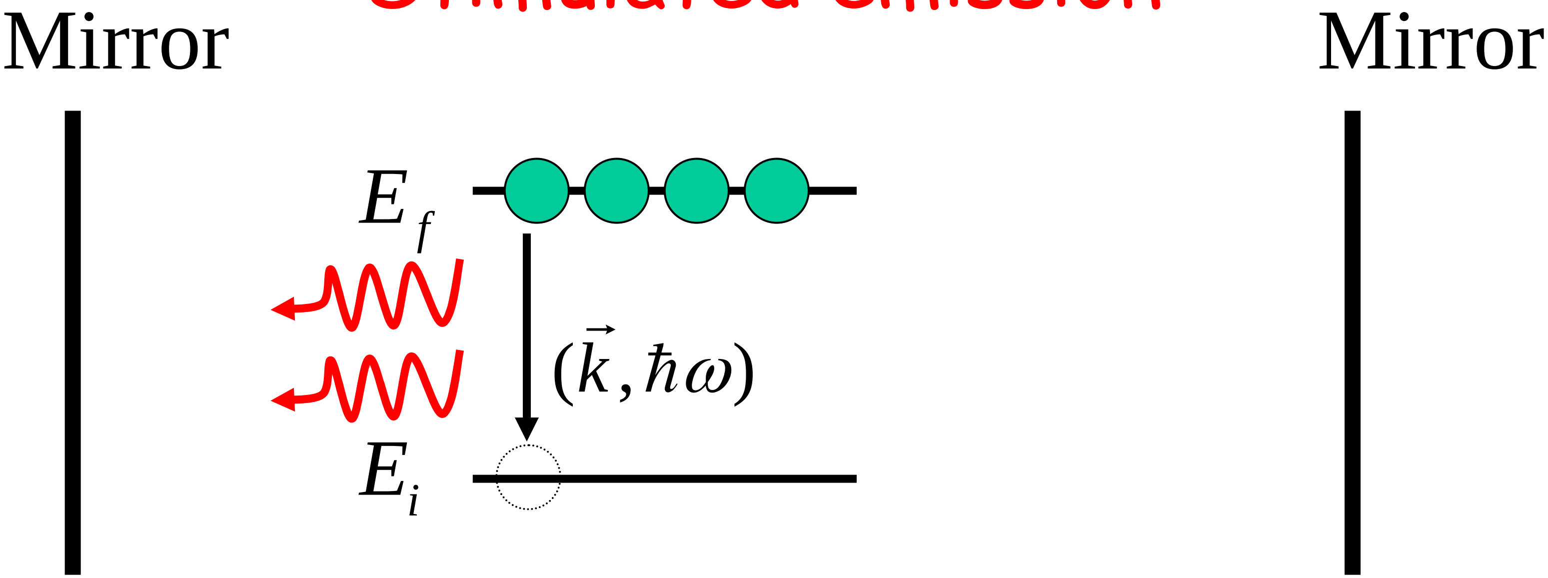
Population inversion



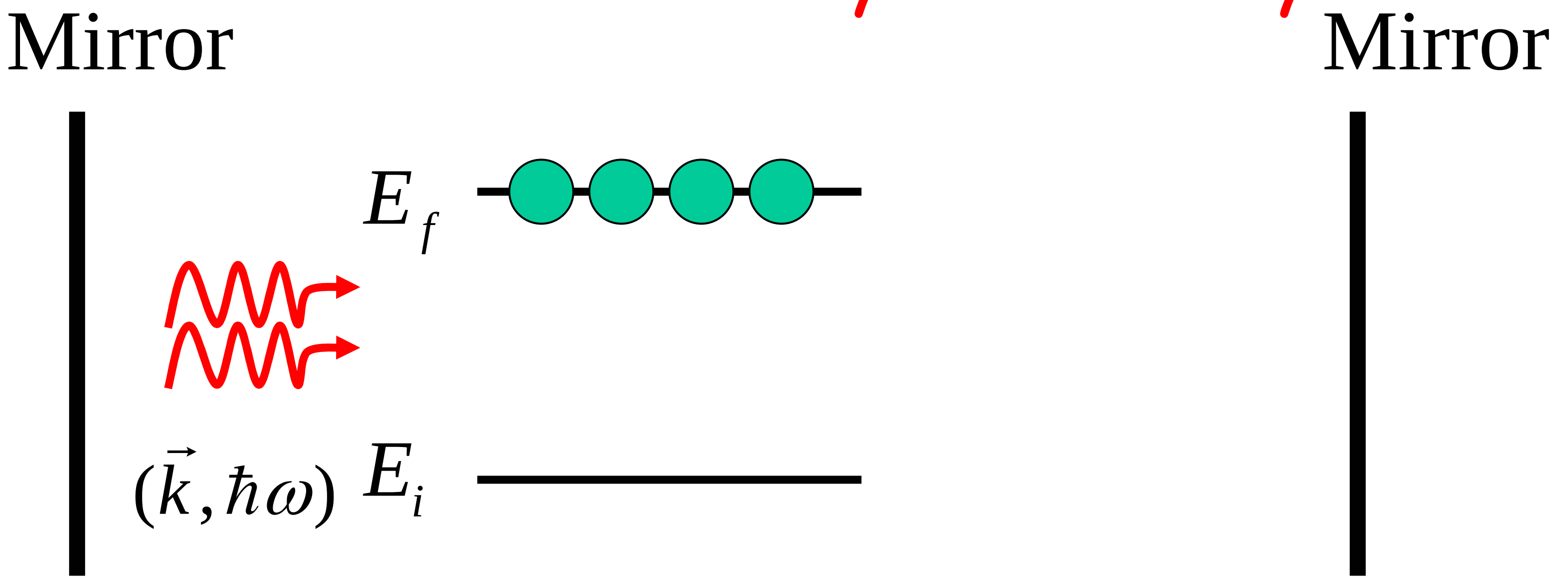
Spontaneous emission



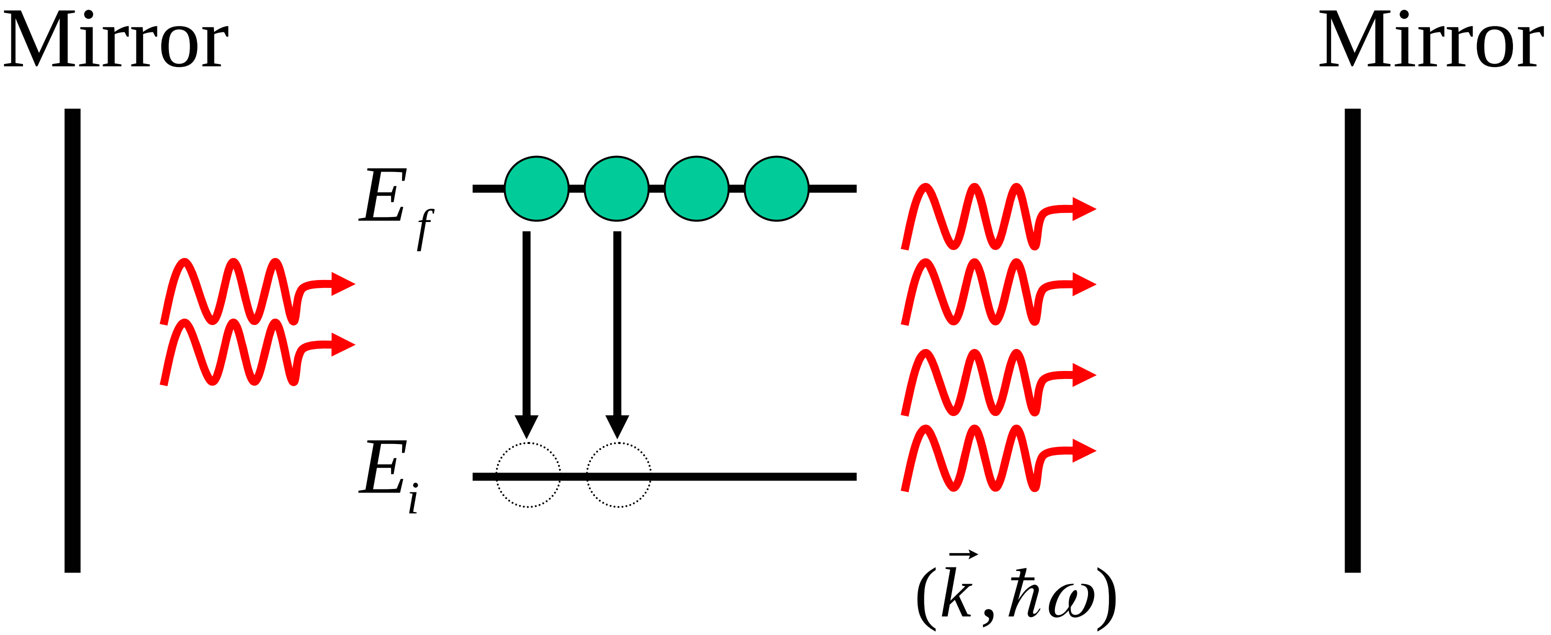
Stimulated emission



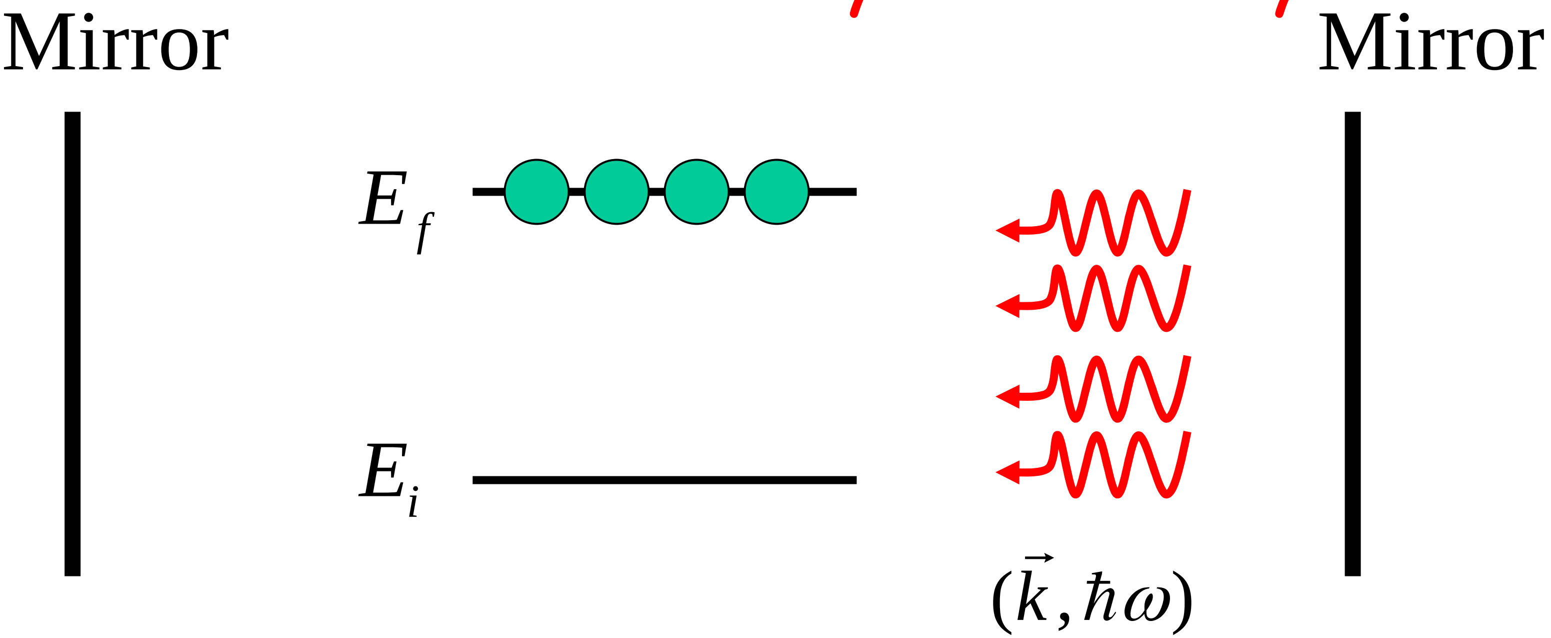
Feed-back by the cavity



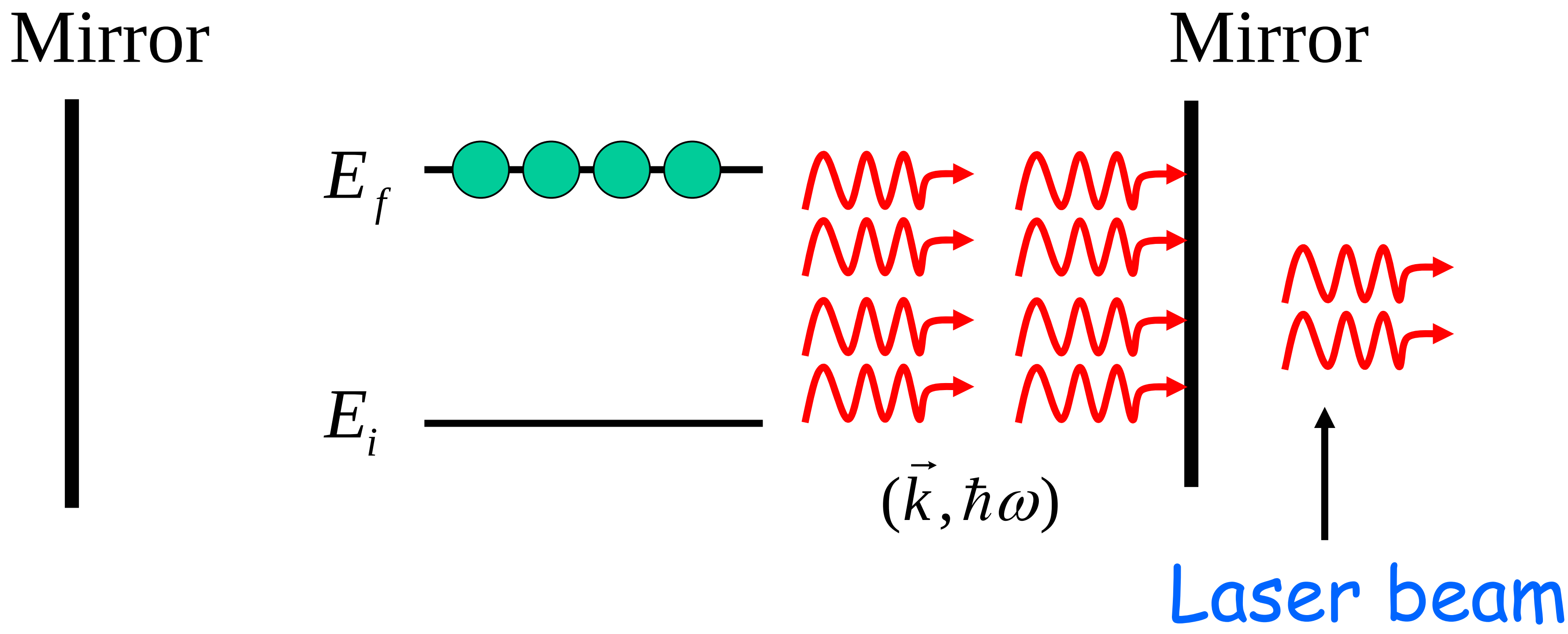
Stimulated emission



Feed-back by the cavity



After several round trips…

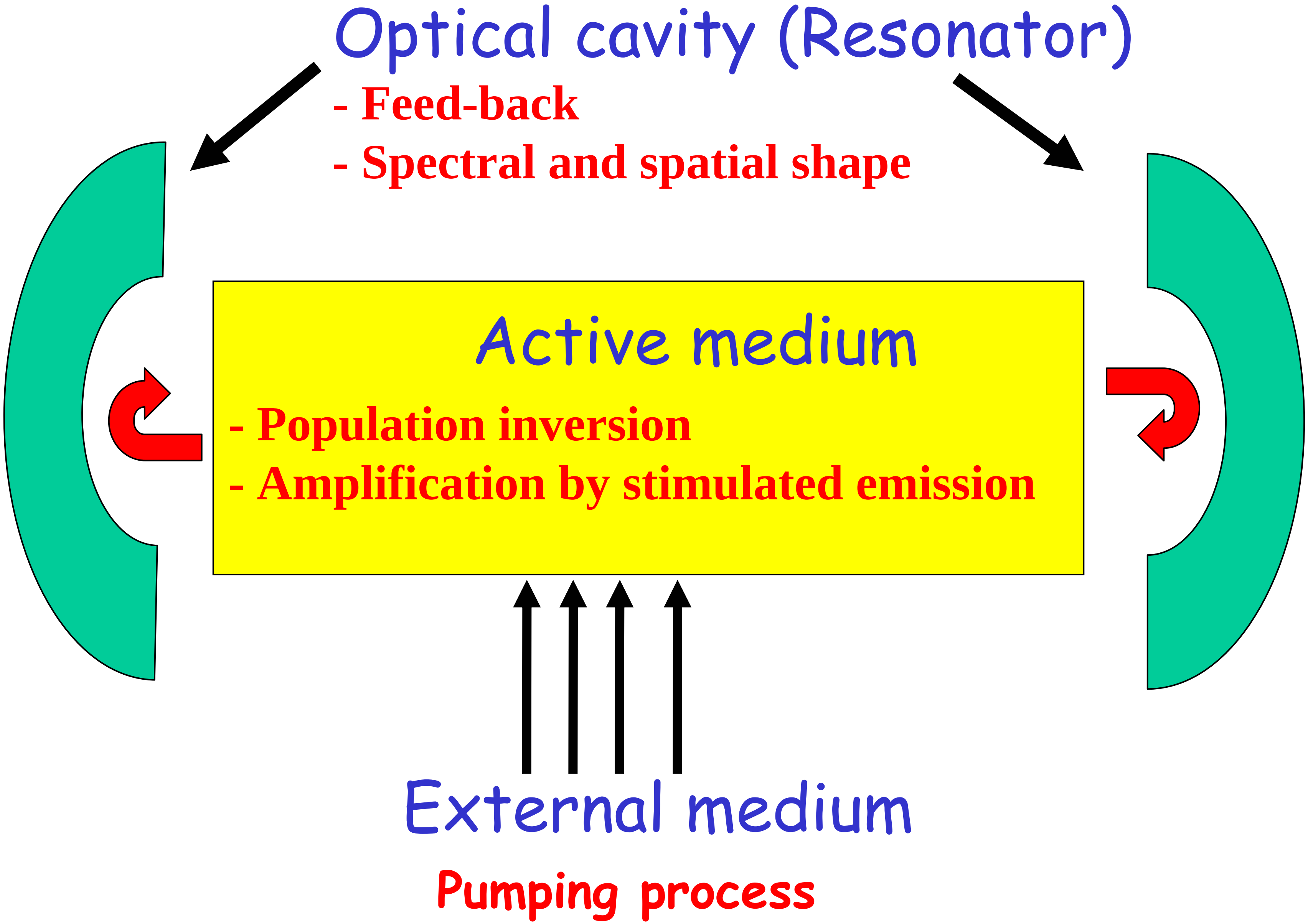


Photons with:

- same energy : Temporal coherence

- same direction of propagation : Spatial coherence

Light Amplification by Stimulated Emission of Radiation

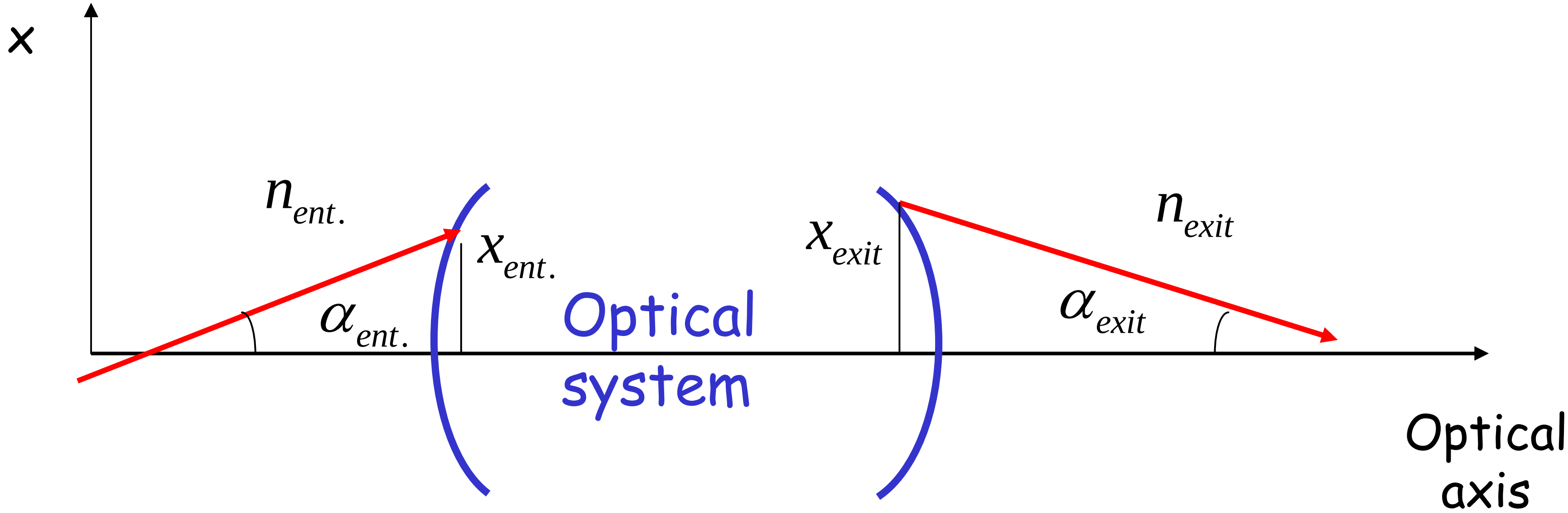


Some properties…

	Laser	Spectral lamp
$\Delta\omega$ (Spectral bandwidth)	CW: 10MHz (standard) $20\,Hz$	10GHz
$\Delta\Omega$ (Solid angle)	$10^{-7}\ st$	$4\pi\ st$
Δt (Pulse duration)	$\mu s\ (10^{-6}s) \rightarrow fs(10^{-15}s)$ $3.5\ fs$	CW
$P\ (Average\ power)$ $P_{peak}(Peak\ power) = \frac{E_{pulse}}{\Delta t}$	$P : mW - few\ W$ $CW\ : 100\ kW$ $P_{peak}\ : MW\,(10^6 W) - 100\ TW\,(10^{14} W)$	$few\ hunbdred\ of\ W$
$I\ (Intensity) : \frac{P}{S}$	$CW\ : kW - MW\ / cm^2$ $Pulsed\ : 10^{22} W\ / cm^2$!!!	

2- Optical cavity

Transfer matrix

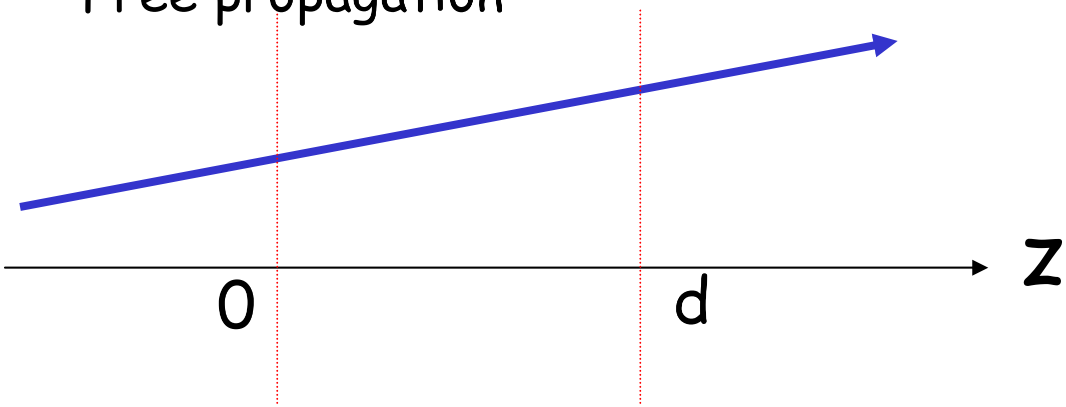


In the paraxial approximation $\begin{pmatrix} x_{exit} \\ n_{exit}\alpha_{exit} \end{pmatrix} = M \begin{pmatrix} x_{ent.} \\ n_{ent.}\alpha_{ent.} \end{pmatrix}$

$$M = \begin{bmatrix} A & B \\ C & D \end{bmatrix} : \ \textit{transfer matrix}, \det M = 1$$

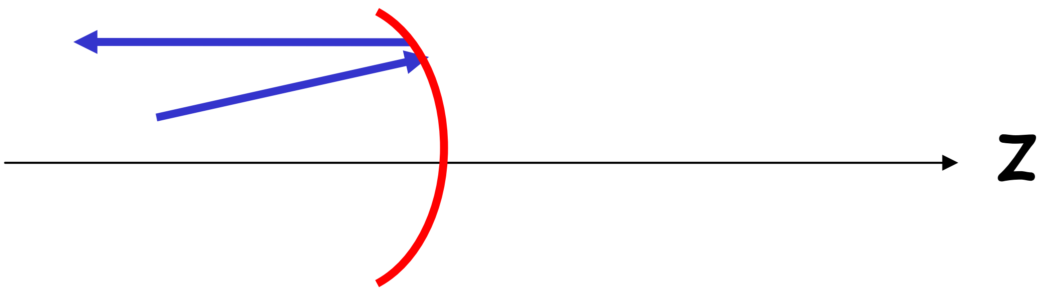
Some examples

Free propagation



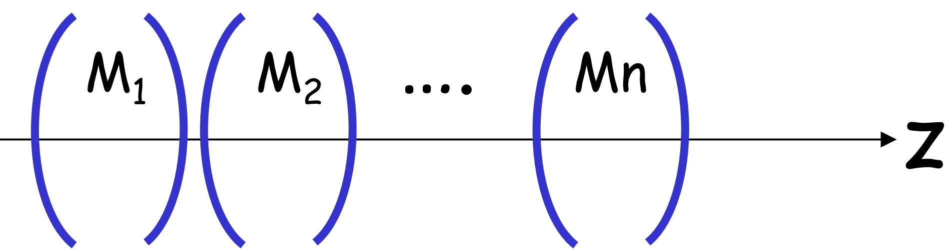
$$\begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix}$$

mirror



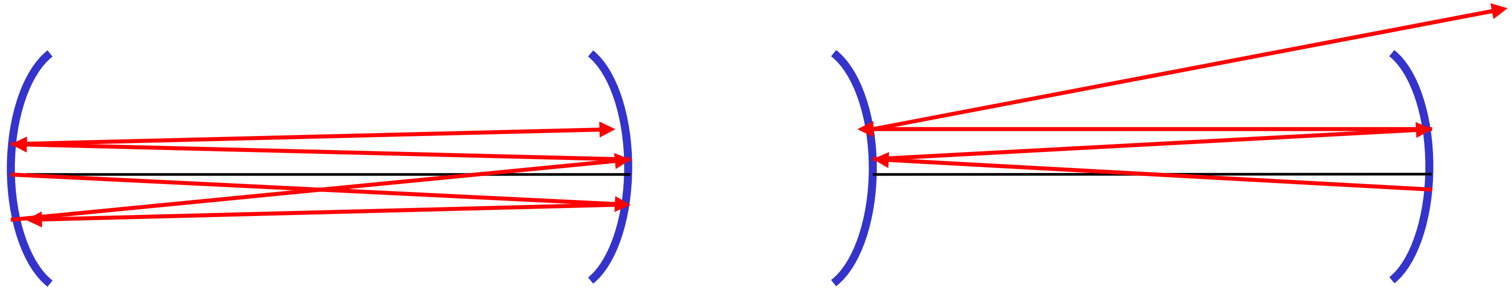
$$\begin{pmatrix} 1 & 0 \\ -\frac{2}{R} & 1 \end{pmatrix}$$

....



$$M = M_n \ldots\ldots M_2 M_1$$

Geometric stability



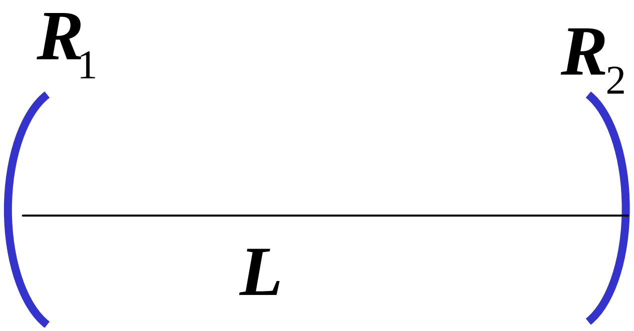
Stable

Unstable

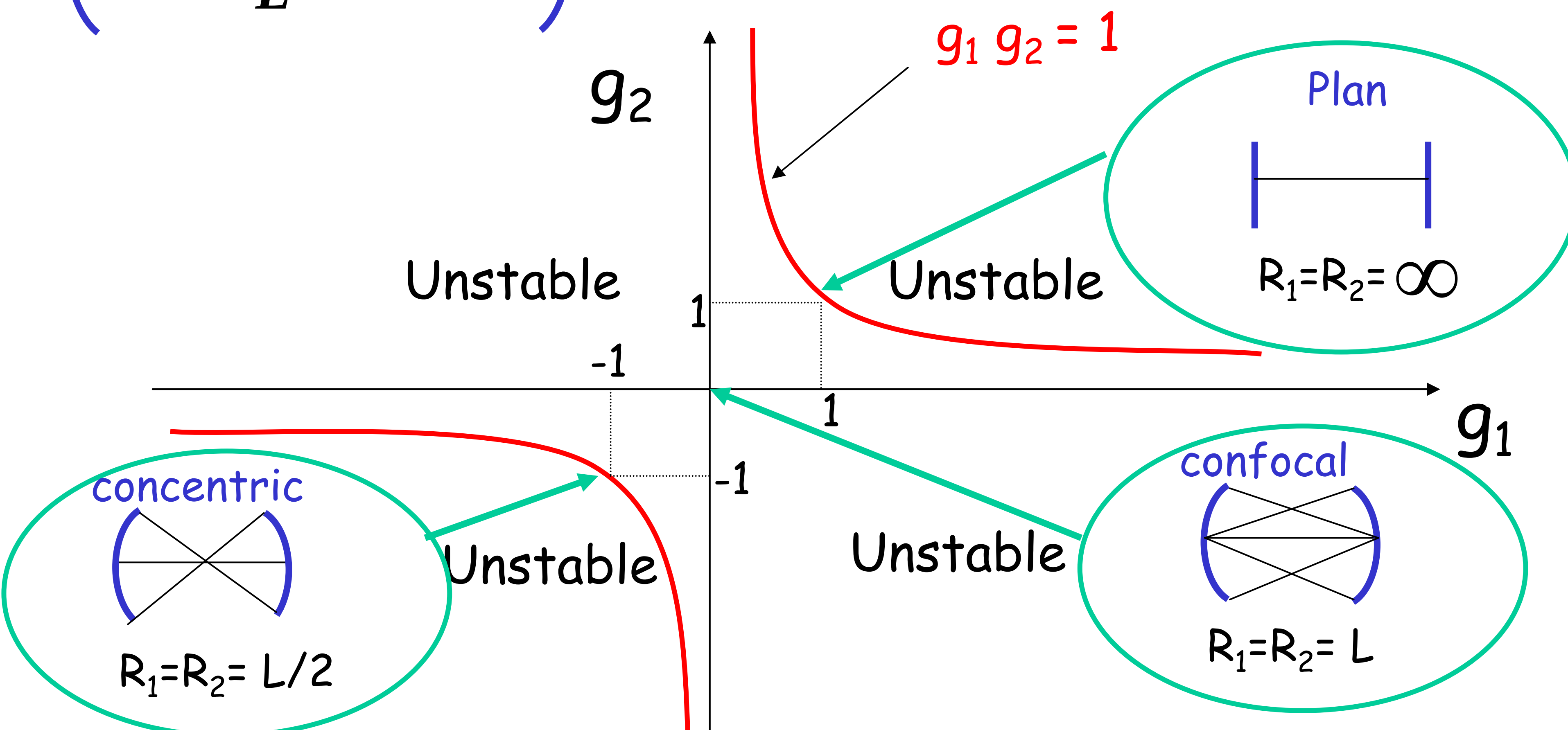
Condition:

$$0 \le \left| Tr \frac{M}{2} \right| \le 1$$

Geometric stability



$$0 \le g_1 g_2 \le 1, \quad g_i = 1 - \frac{L}{R_i}$$



Cavity Modes

$$\mathbf{E}(\vec{r},t) = E\ e^{i\left(kz-\omega t\right)}$$

Elementary solution of Maxwell equation in **vacuum**

Cavity Modes

$$\mathbf{E}(\vec{r},t) = \int E(\omega)\, e^{i(kz-\omega t)} d\omega$$

Arbitrary (plane-wave) solution of Maxwell equation in **vacuum**

Cavity Modes

$$\mathbf{E}(\vec{r},t)=\sum_q E_q\, e^{i\left(k_q z-\omega_q t\right)}$$

Arbitrary (plane-wave) solution of Maxwell equation in **cavity**

ω_q : Define longitudinal modes

Cavity Modes

$$\mathbf{E}(\vec{r},t) = \sum_q E_q(\vec{r})\, e^{i\left(k_q z - \omega_q t\right)}$$

Arbitrary solution of Maxwell equation in **cavity**

ω_q : Define longitudinal modes

$E_q(\vec{r}) \neq cte$: **beyond plane-wave approximation**

Cavity Modes

$$\mathrm{E}(\vec{r},t) = \sum_q \left(\sum_p C_{pq} E_p^{trans}(\vec{r}) \right) e^{i\left(k_q z - \omega_q t\right)}$$

Arbitrary solution of Maxwell equation in **cavity**

ω_q : Define longitudinal modes

$E_q(\vec{r}) \neq cte$: **beyond plane-wave approximation**

$E_p^{trans}(\vec{r})$: Define transverse modes

Cavity Modes

$$\mathrm{E}(\vec{r},t)=\sum_{q}\sum_{p}C_{pq}E_{p}^{trans}(\vec{r})\; e^{i\left(k_q z-\omega_q t\right)}$$

Arbitrary solution of Maxwell equation in **cavity**

ω_q : Define longitudinal modes

$E_q(\vec{r})\neq cte$: **beyond plane-wave approximation**

$E_{p}^{trans}(\vec{r})$: Define transverse modes

Mode: elementary solution of Maxwell equation inside cavity

→ **Complete basis set**

Transverse modes

Monochromatic wave: $\mathrm{E}(\vec{r},t) = E(\vec{r})\, e^{-i\omega t}$

$E(\vec{r})$: Solution of propagation equation inside the cavity within the paraxial approximation

$$\frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} = 2ik\frac{\partial E}{\partial z}$$

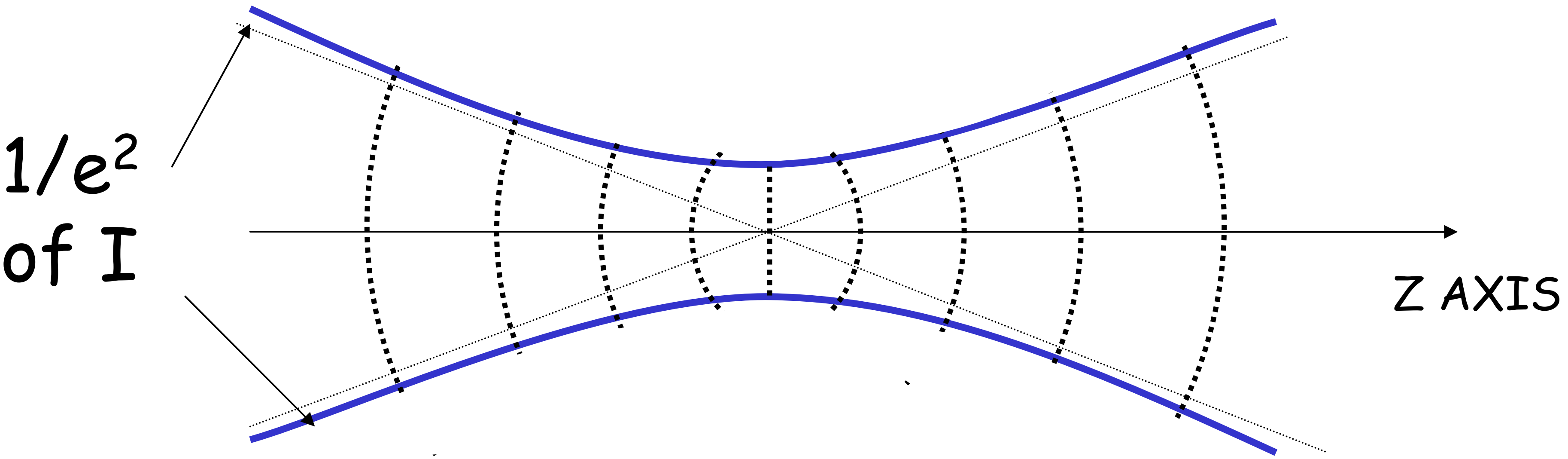
One possible solution: Gaussian beam

$$E(x,y,z) = \frac{w_0}{w(z)}\, e^{-\frac{x^2+y^2}{w^2(z)}}\, e^{-i\phi(x,y,z)}$$

$$\phi(x,y,z) = kz + \frac{k\left(x^2+y^2\right)}{2R(z)} - \arctan\left(\frac{z}{z_R}\right)$$

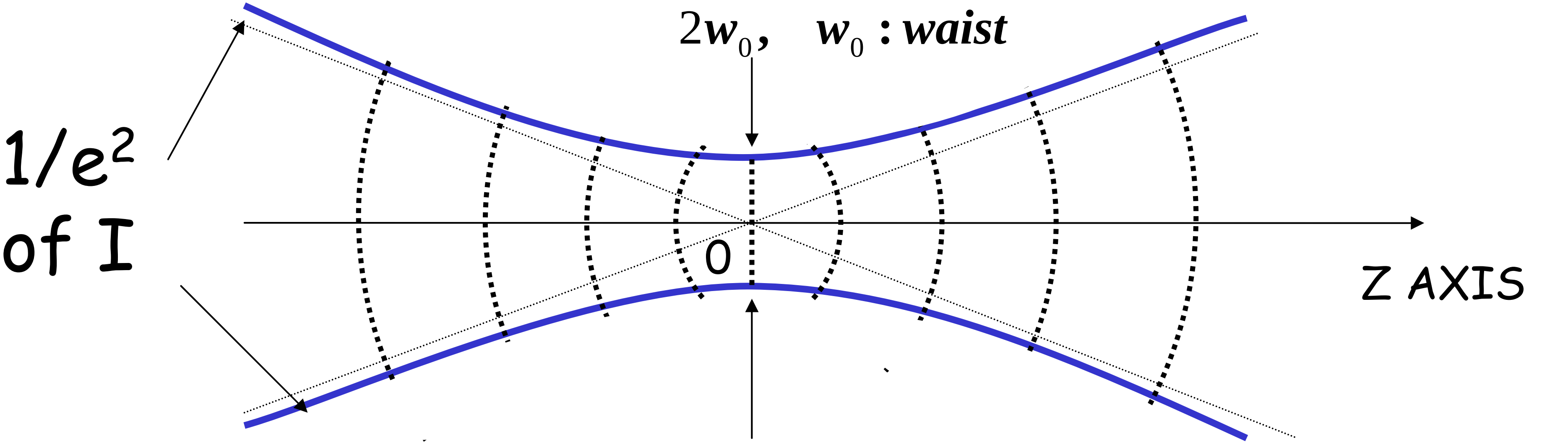
Condition: cavity is stable

FUNDAMENTAL MODE: GAUSSIAN BEAM



$$I = I_0(z)\, e^{-\frac{2\left(x^2+y^2\right)}{w^2(z)}}$$

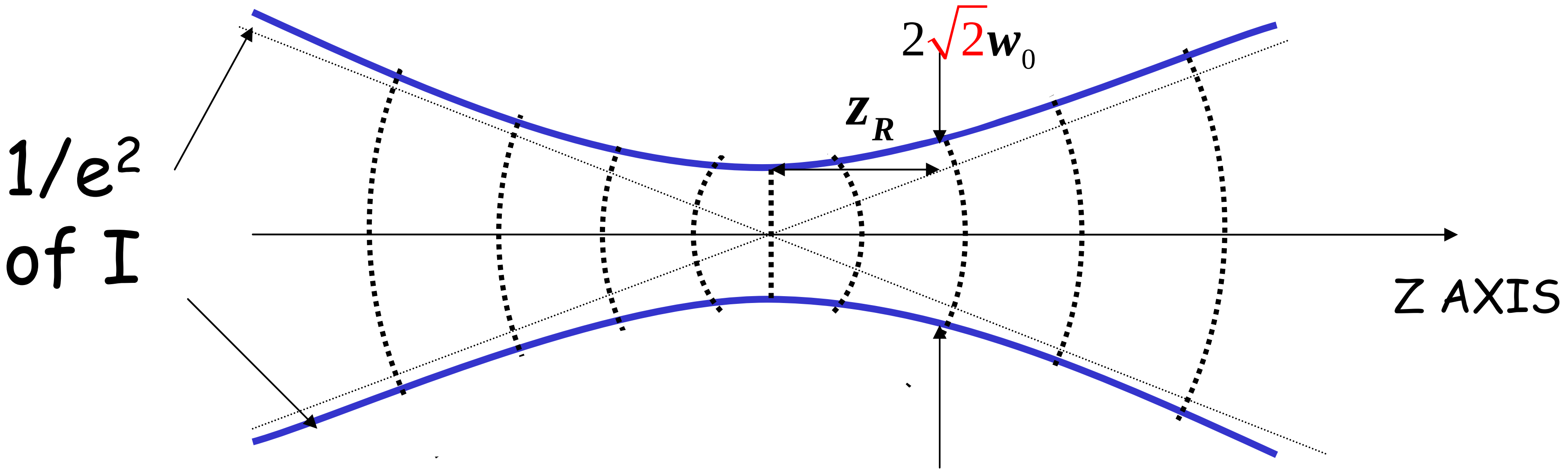
FUNDAMENTAL MODE: GAUSSIAN BEAM



Z=0 plane wave

$$I = I_0(z)\, e^{-\frac{2\left(x^2+y^2\right)}{w^2(z)}}$$

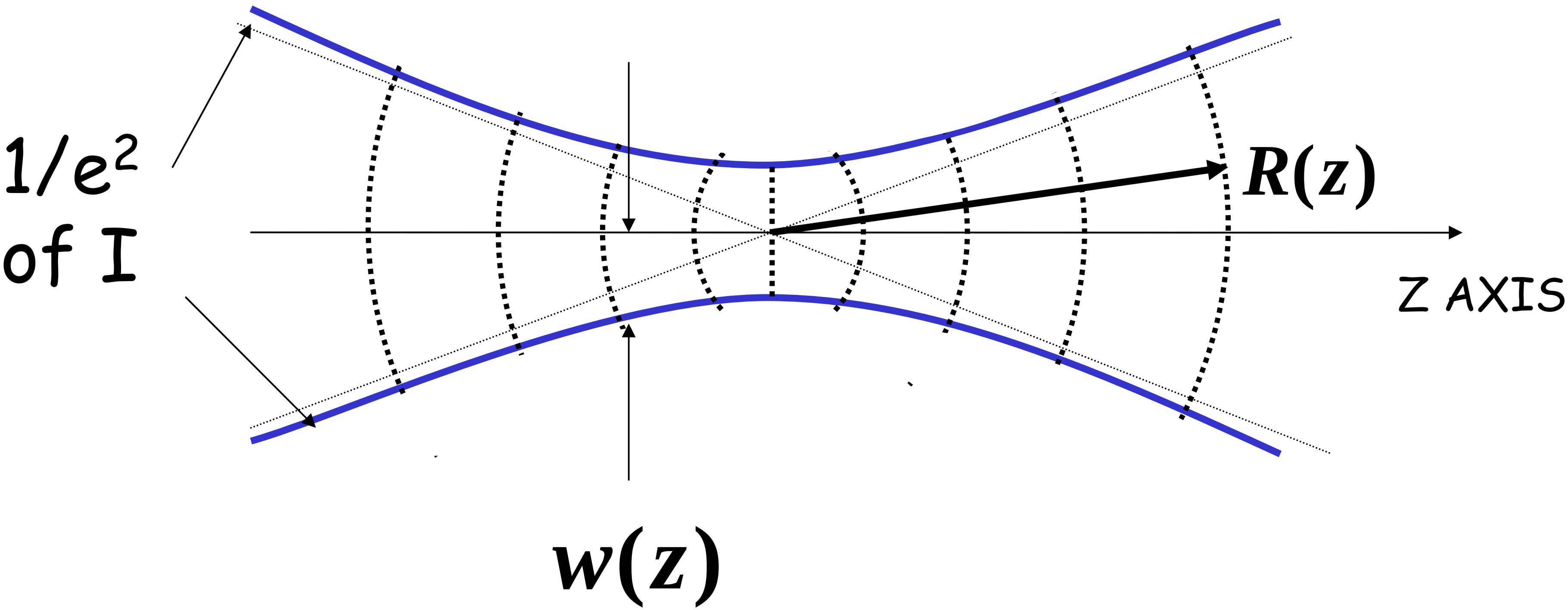
FUNDAMENTAL MODE: GAUSSIAN BEAM



$$I = I_0(z)\, e^{-\frac{2\left(x^2+y^2\right)}{w^2(z)}}$$

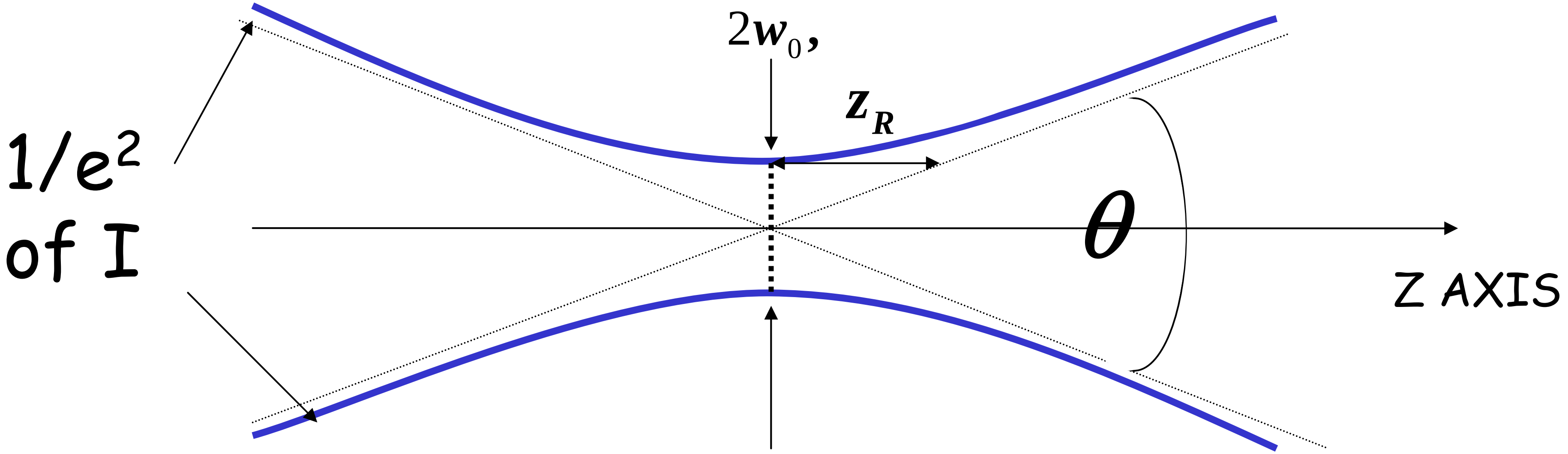
$$z_R = \frac{\pi w_0^2}{\lambda}$$: ***Rayleigh length***

FUNDAMENTAL MODE: GAUSSIAN BEAM



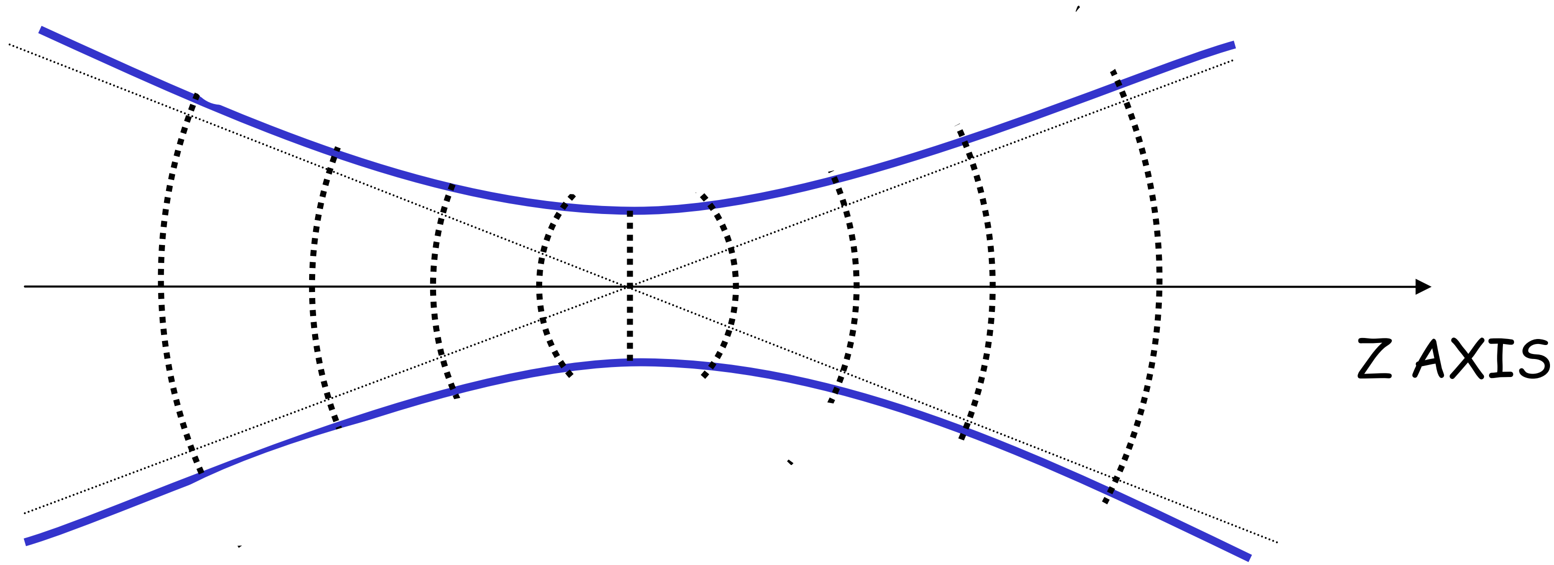
$z_R = \frac{\pi w_0^2}{\lambda}$: ***Rayleigh length*** $R(z) = z\left(1+\frac{z_R^2}{z^2}\right), \quad w(z) = w_0\sqrt{1+\frac{z_R^2}{z^2}}$

FUNDAMENTAL MODE: GAUSSIAN BEAM



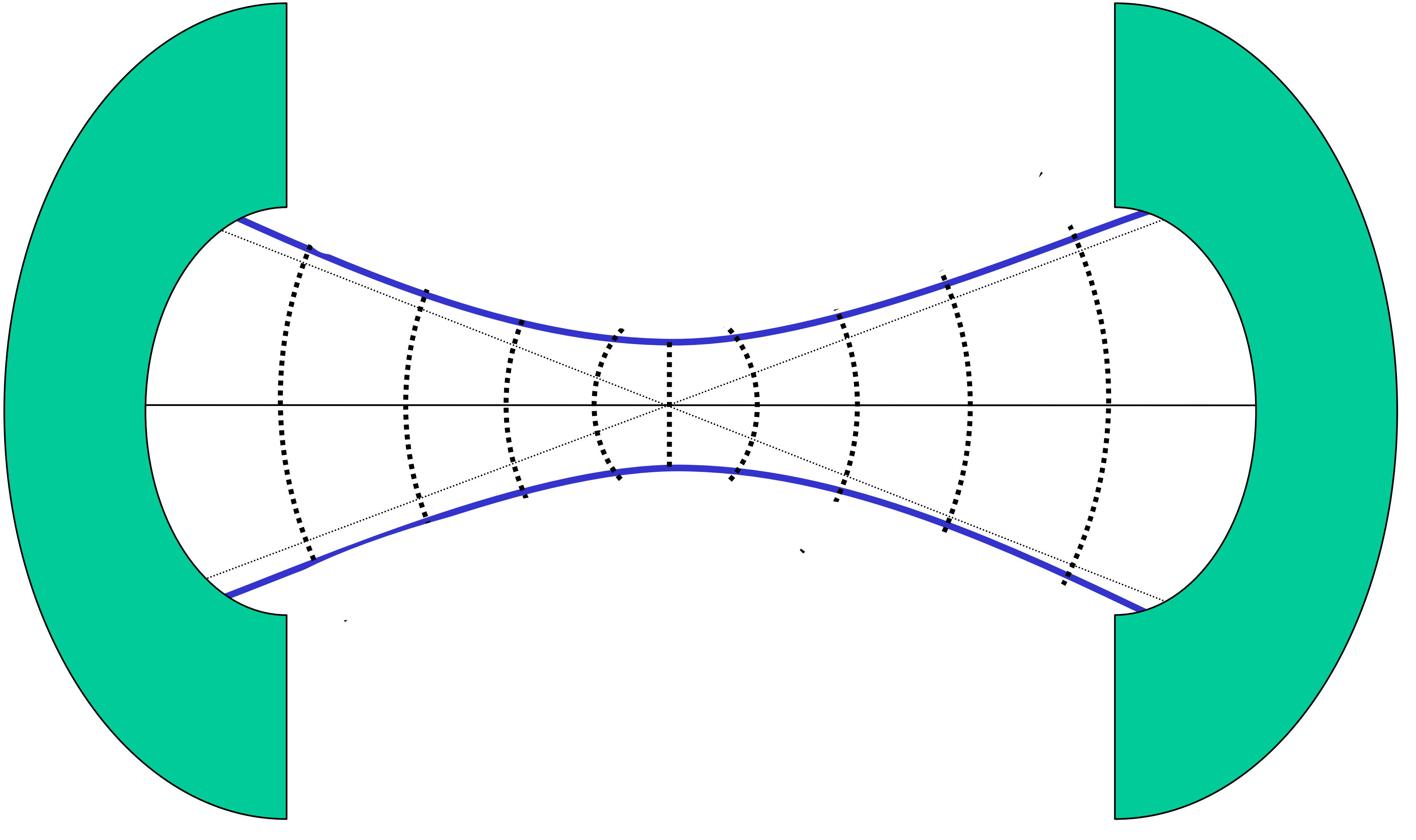
$$\theta = \frac{2w_0}{z_R} = \frac{2\lambda}{\pi w_0} : divergence$$

$$ex: \quad \lambda = 0.5\mu m, w_0 = 33\mu m \rightarrow \theta \simeq 1\, mrad$$



Everything depends on the waist

How to get w_0 and $z=0$ in a cavity ??



In a cavity: The wavefront adapts to the shape of the mirors (⟷ stability)

High order modes

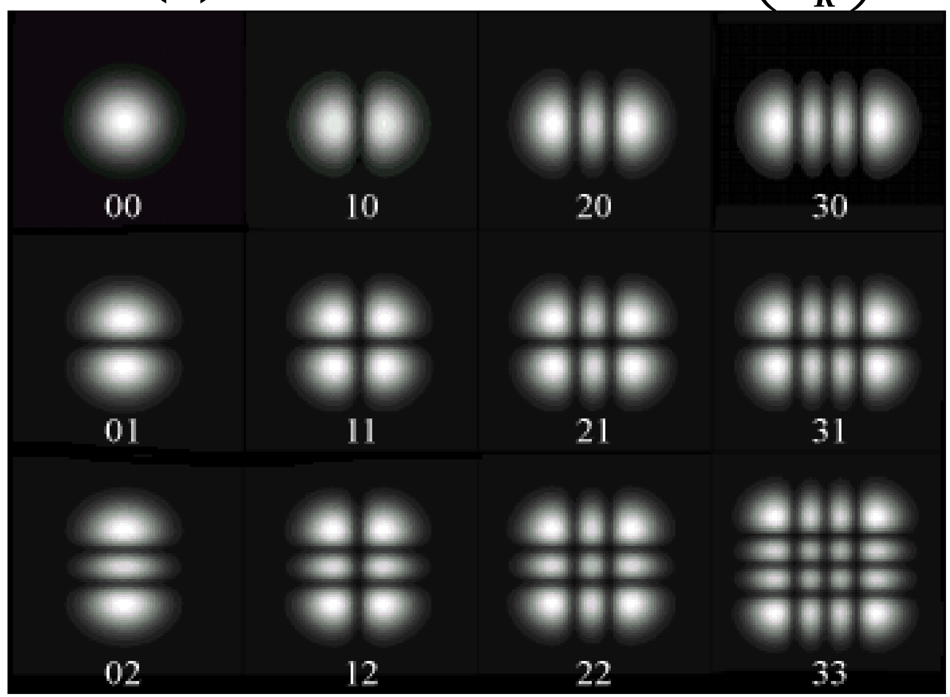
Solutions : **Hermite-gauss** beams (a complete basis set)

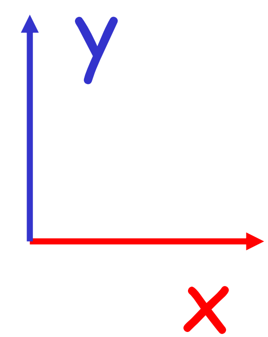
$$E_{m,n}(x,y,z)=\frac{w_0}{w(z)}H_m\left(\frac{\sqrt{2}x}{w(z)}\right)H_n\left(\frac{\sqrt{2}y}{w(z)}\right)e^{-\frac{x^2+y^2}{w^2(z)}}\,e^{-i\phi_{m,n}(x,y,z)}$$

$$\phi_{m,n}(x,y,z)=kz+\frac{k\left(x^2+y^2\right)}{2R(z)}-(m+n+1)\arctan\left(\frac{z}{z_R}\right)$$

$TEM_{m,n}$

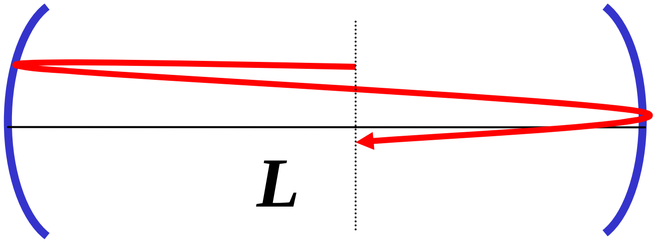
Nodes in x, y





Longitudinal modes

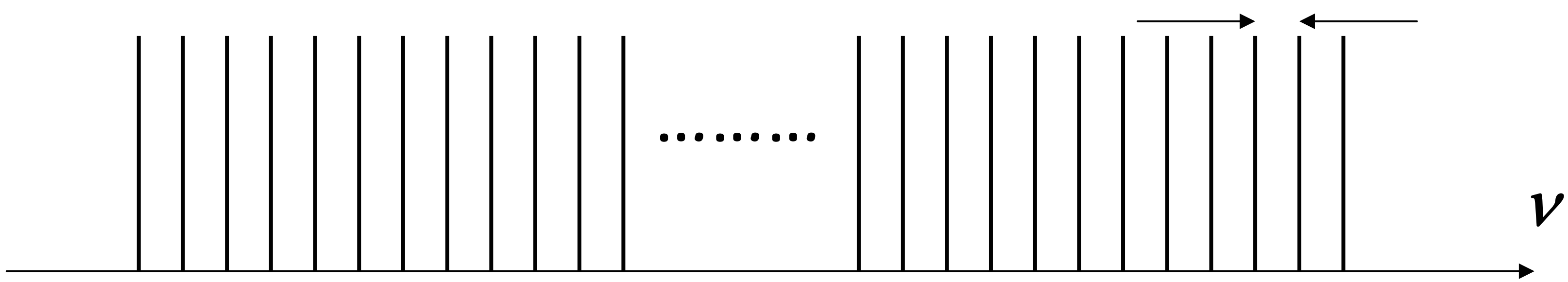
Condition of resonance :



$$\phi_{round\ trip} = \phi_0 + 2q\pi$$

$$E_p \simeq e^{ikz} \rightarrow 2kL = 2\pi q \quad (q : \text{integer})$$

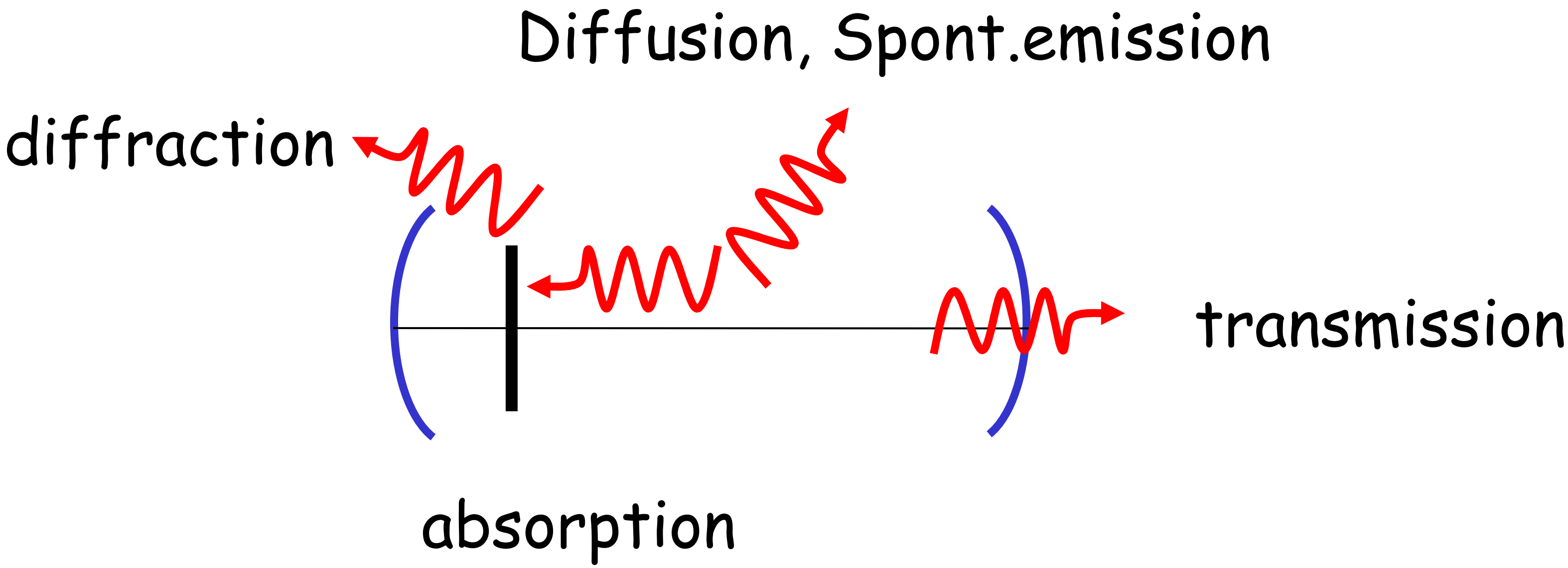
$$\nu_q = c/\lambda = q\frac{c}{2L}$$



In fact, $E_p : TEM_{n,m} \rightarrow$

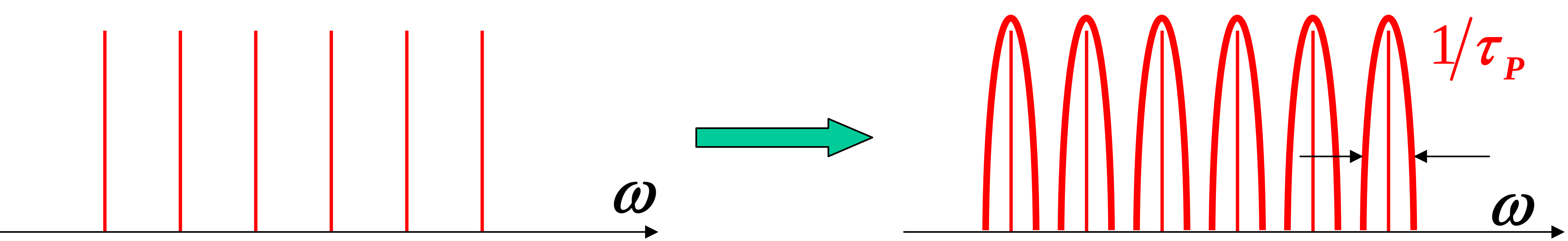
$$\nu_{mnq} = \frac{c}{2L}\left[q + \frac{1}{\pi}(m+n+1)\ ar\cos\left(\pm\sqrt{g_1 . g_2}\right)\right]$$

Cavity losses



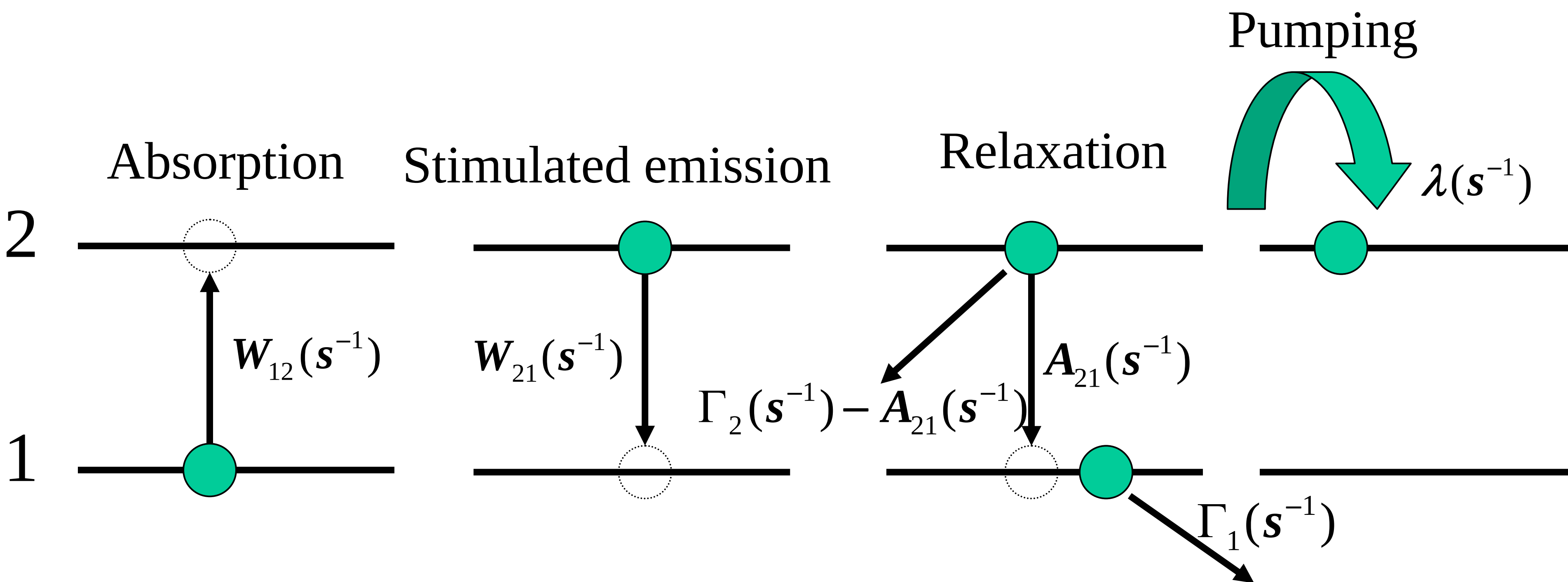
$$I(t) = I_0\, e^{-t/\tau_P}, \tau_P :$$ ***photon lifetime***

Consequence: **broadenning of the longitudinal modes**



3- Energetic model of the Interaction

1- Evolution of populations



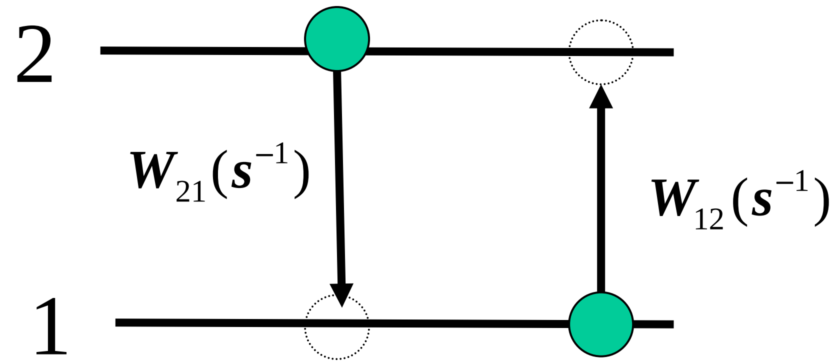
$$\frac{dN_2}{dt} = W_{12}N_1 \quad - W_{21}N_2 \quad - \Gamma_2 N_2 \quad + \quad \lambda$$

$$\frac{dN_1}{dt} = -W_{12}N_1 \quad + W_{21}N_2 \quad - \Gamma_1 N_1 + A_{21}N_2$$

Rate equations

$W_{12} = W_{21} = \sigma I / \hbar\omega, \quad \sigma : cross\ section,\ I : Intensity$

2- Evolution of Intensity



Energy variation

$(J\ m^{-3}s^{-1})$

$$\frac{dI}{dz} = \left(W_{21}N_2\right)\hbar\omega - \left(W_{12}N_1\right)\hbar\omega$$

Stimulated emission　　Absorption

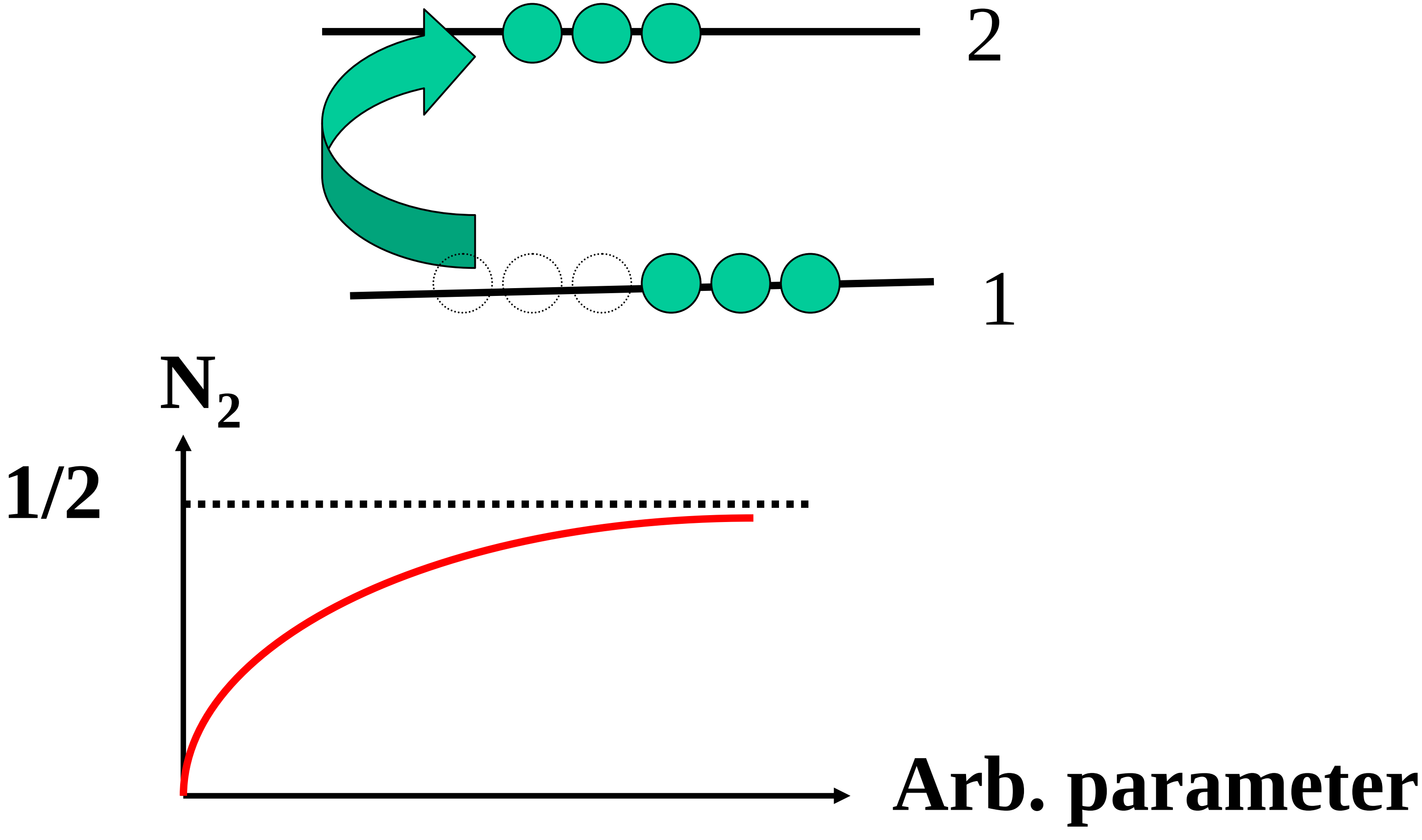
$$\frac{dI}{dz} = \sigma I\left(N_2 - N_1\right) > 0 \quad \textbf{\textit{if}} \quad N_2 > N_1 :$$

Population inversion

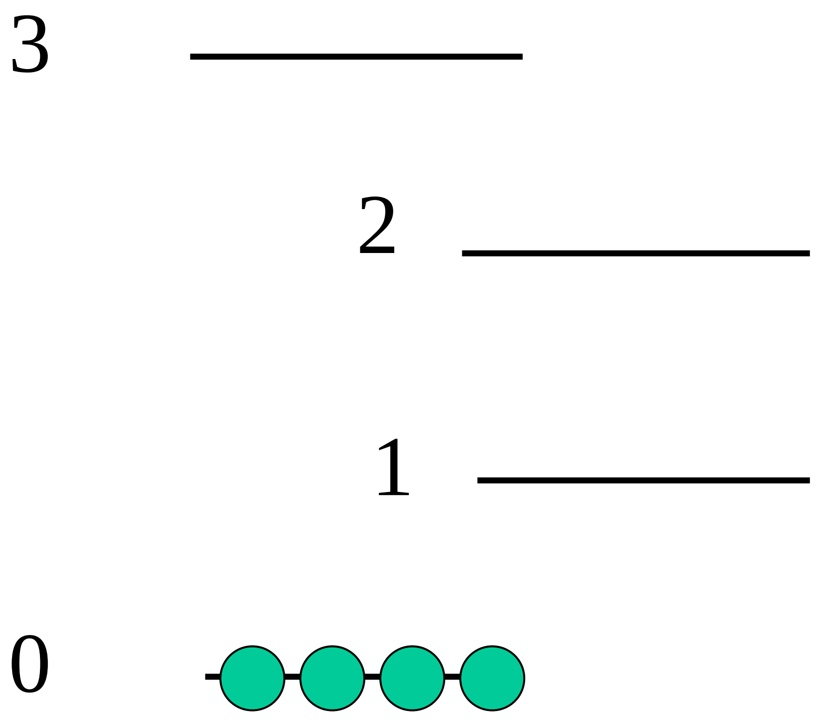
(amplification)

Inversion of population ?

In a CLOSED two-level system, an incoherent excitation can **never** accomplish population inversion

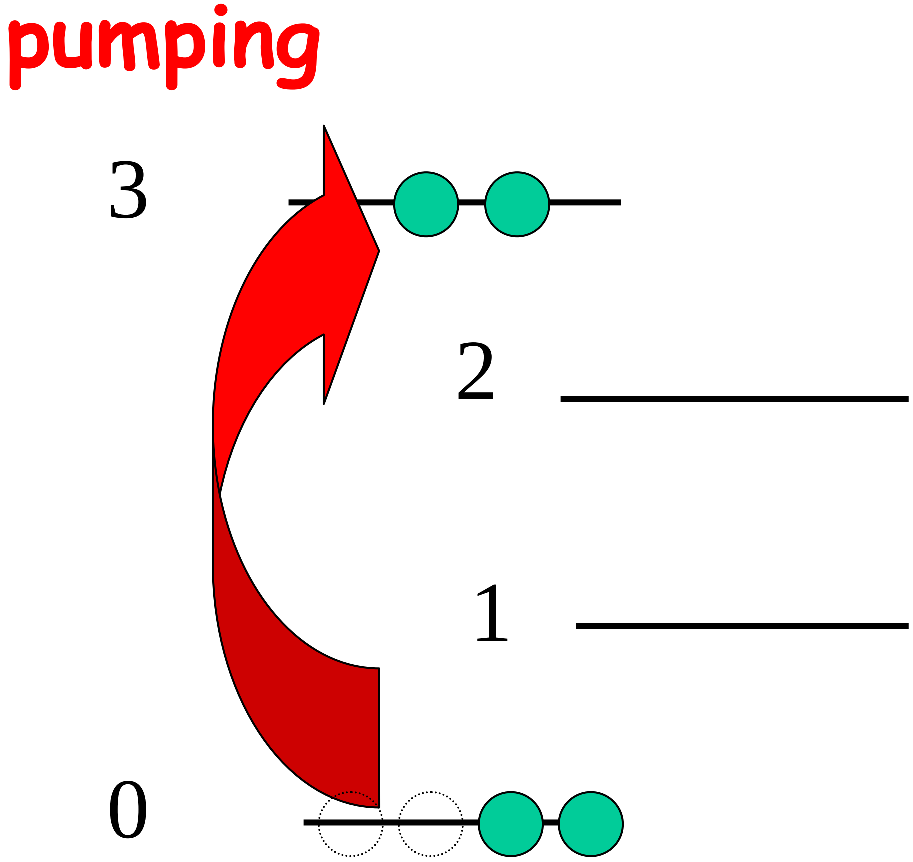


Solution : More levels

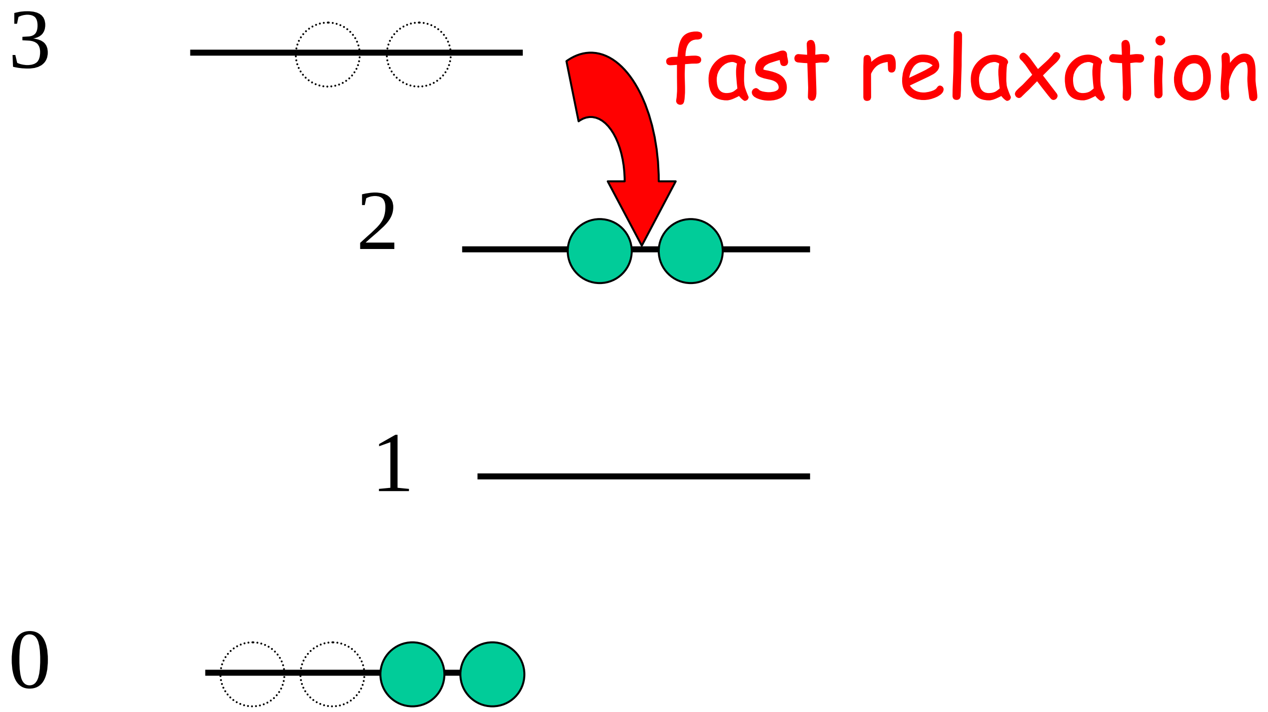


FOUR LEVEL SYSTEM

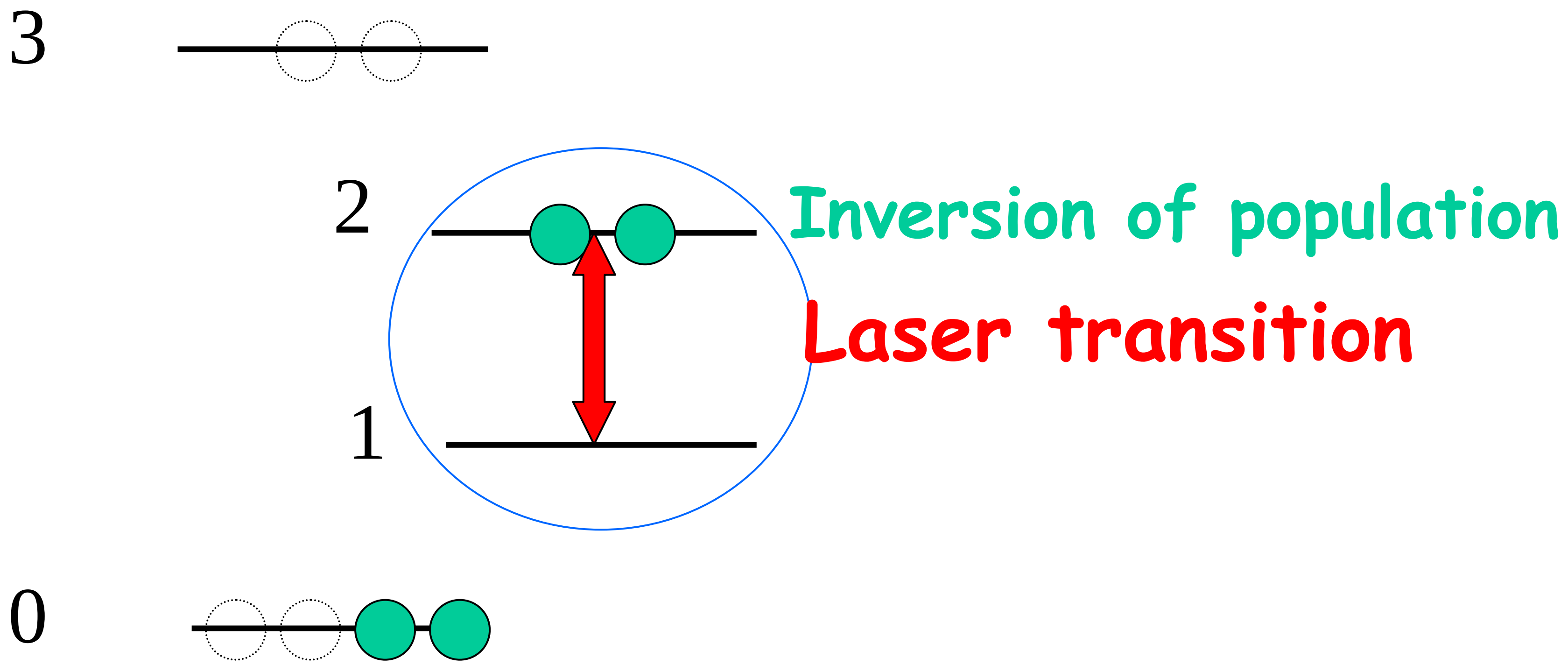
Solution : More levels



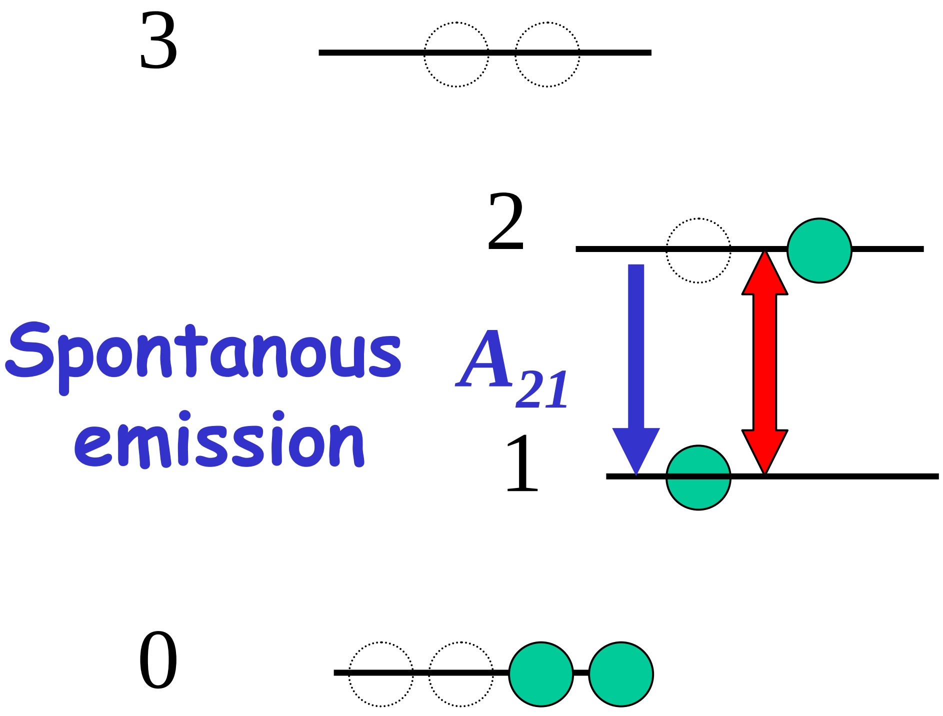
Solution : More levels



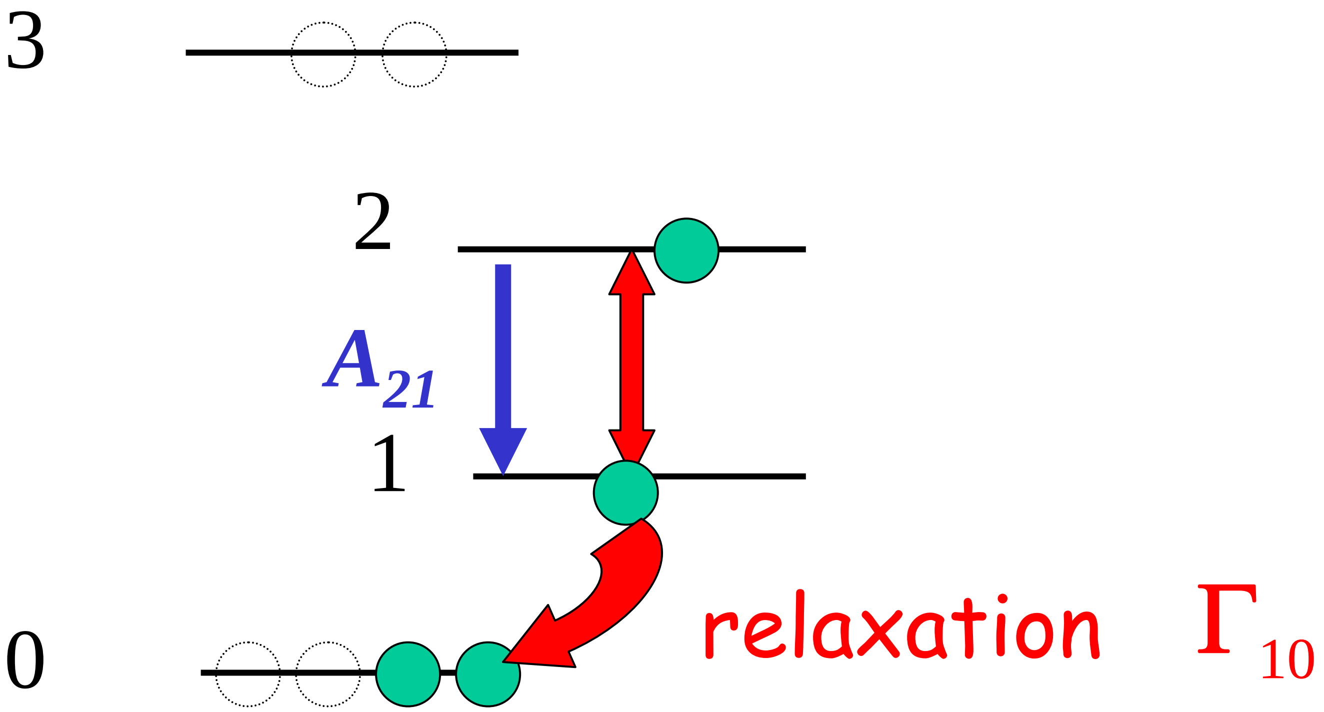
Solution : More levels



Solution : More levels



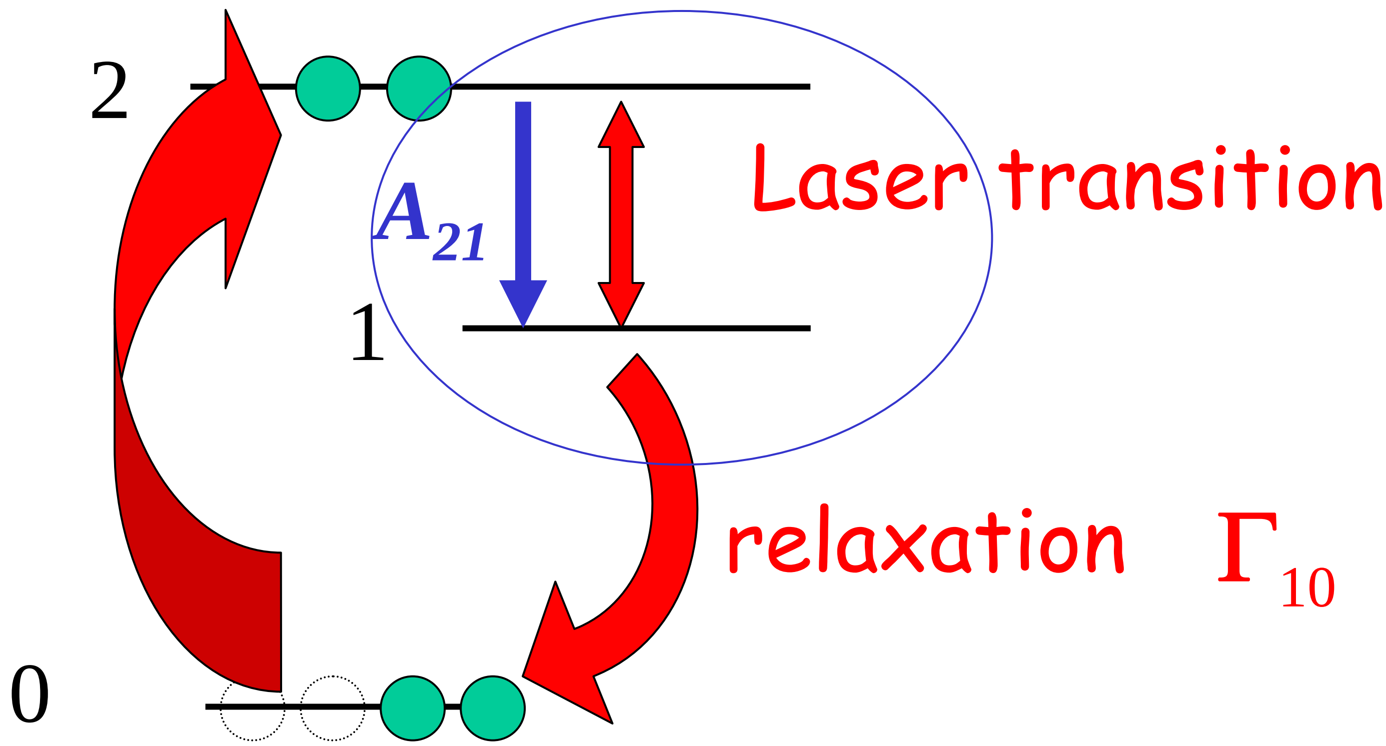
Solution : More levels



CONDITION: $\Gamma_{10} > A_{21}$

Solution : More levels

Three level system



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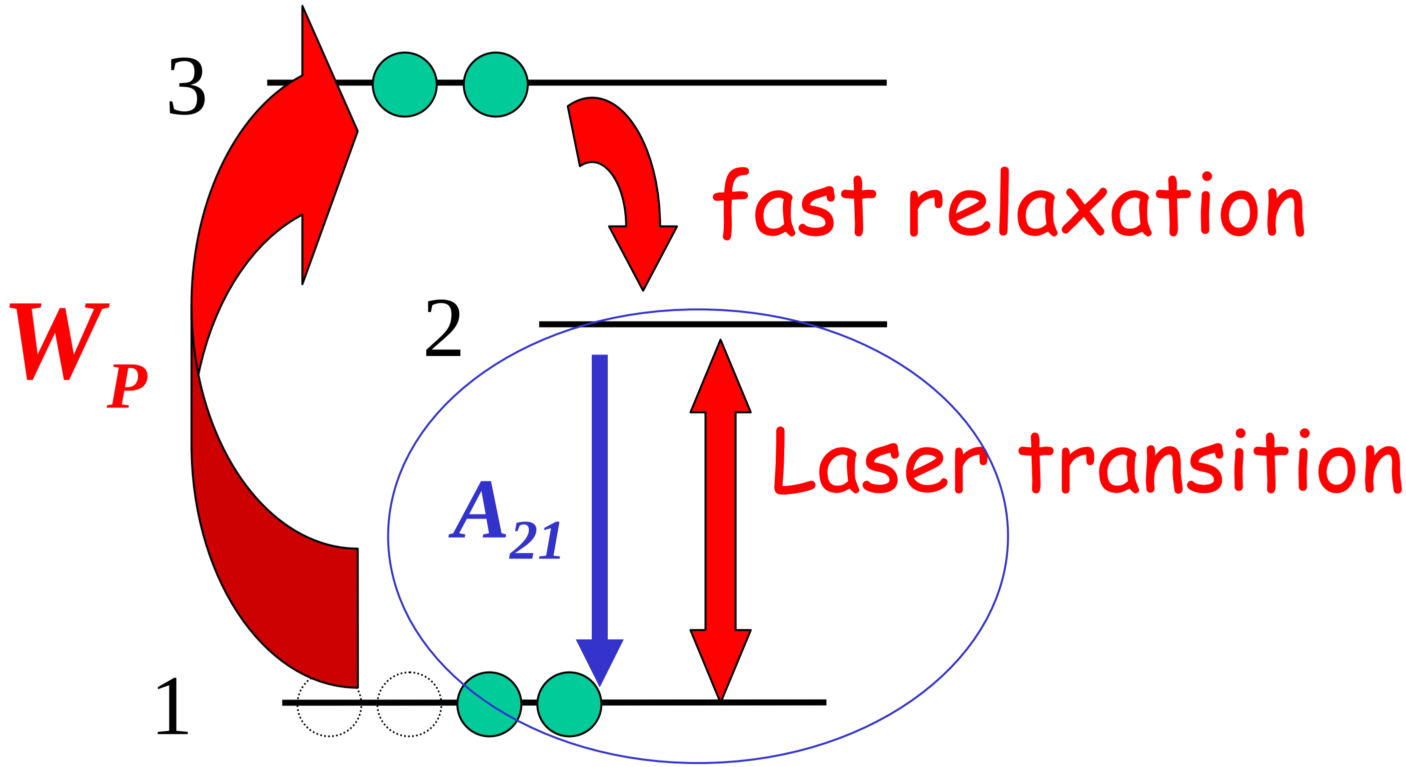
Solution : More levels

Three level system



CONDITION: $W_P > A_{21}$

4- Laser Oscillation (CW laser)

Amplifier in an optical cavity

$$
\begin{cases}
\dfrac{dI}{dz} = \sigma\left(N_2 - N_1\right) I + \alpha I \\[2ex]
\dfrac{dN_2}{dt} = \sigma I (N_1 - N_2) - \Gamma_2 N_2 + \lambda = 0 \\[2ex]
\dfrac{dN_1}{dt} = -\sigma I (N_1 - N_2) - \Gamma_1 N_1 + A_{21} N_2 = 0
\end{cases}
$$

Losses<0 (spread)

Laser equations

$N_2 - N_1 > 0$: Not sufficient!

→ **Gain must overcome the losses**

Laser works (Stationnary regime):

$$\frac{dN_1}{dt} = \frac{dN_2}{dt} = 0; \quad \sigma\left(N_2 - N_1\right) = -\alpha;$$

Gain = losses

Photons created on a round trip = photons lost on a round trip

Laser equations

→

$$\begin{cases} \Delta N = \dfrac{\Delta N_0}{1+\dfrac{I}{I_s}} \\ I = -I_s\left(1+\dfrac{\sigma \Delta N_0}{\alpha}\right) \end{cases}$$

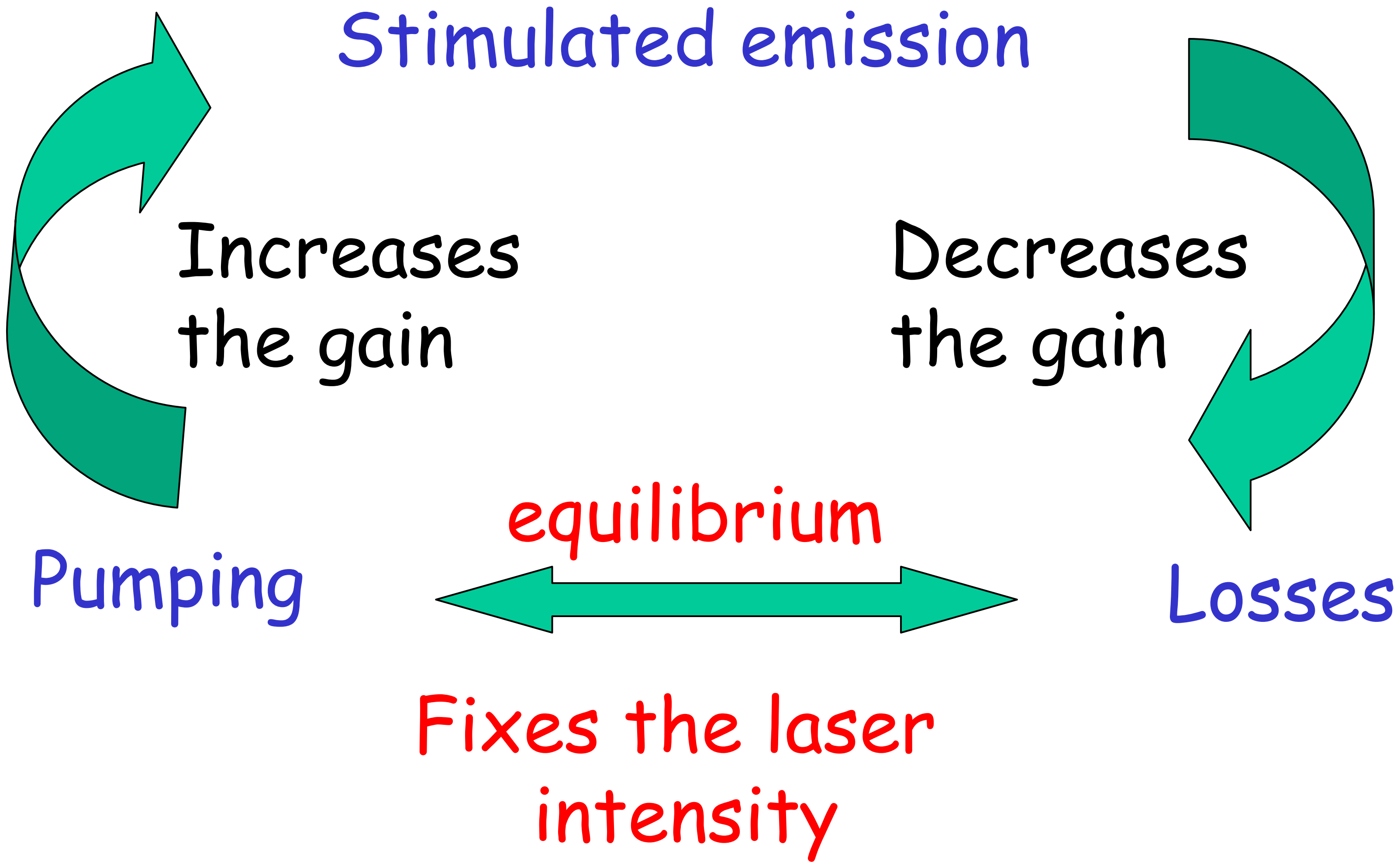
$$\Delta N_0 = \frac{\lambda}{\Gamma_2}\left(1-\frac{A_{21}}{\Gamma_1}\right):$$ **maximal population inversion**

$$I_s = \frac{\sigma}{\hbar\omega}\left(\frac{1}{\Gamma_1}+\frac{1}{\Gamma_2}-\frac{A_{21}}{\Gamma_1\Gamma_2}\right):$$ **saturation intensity**

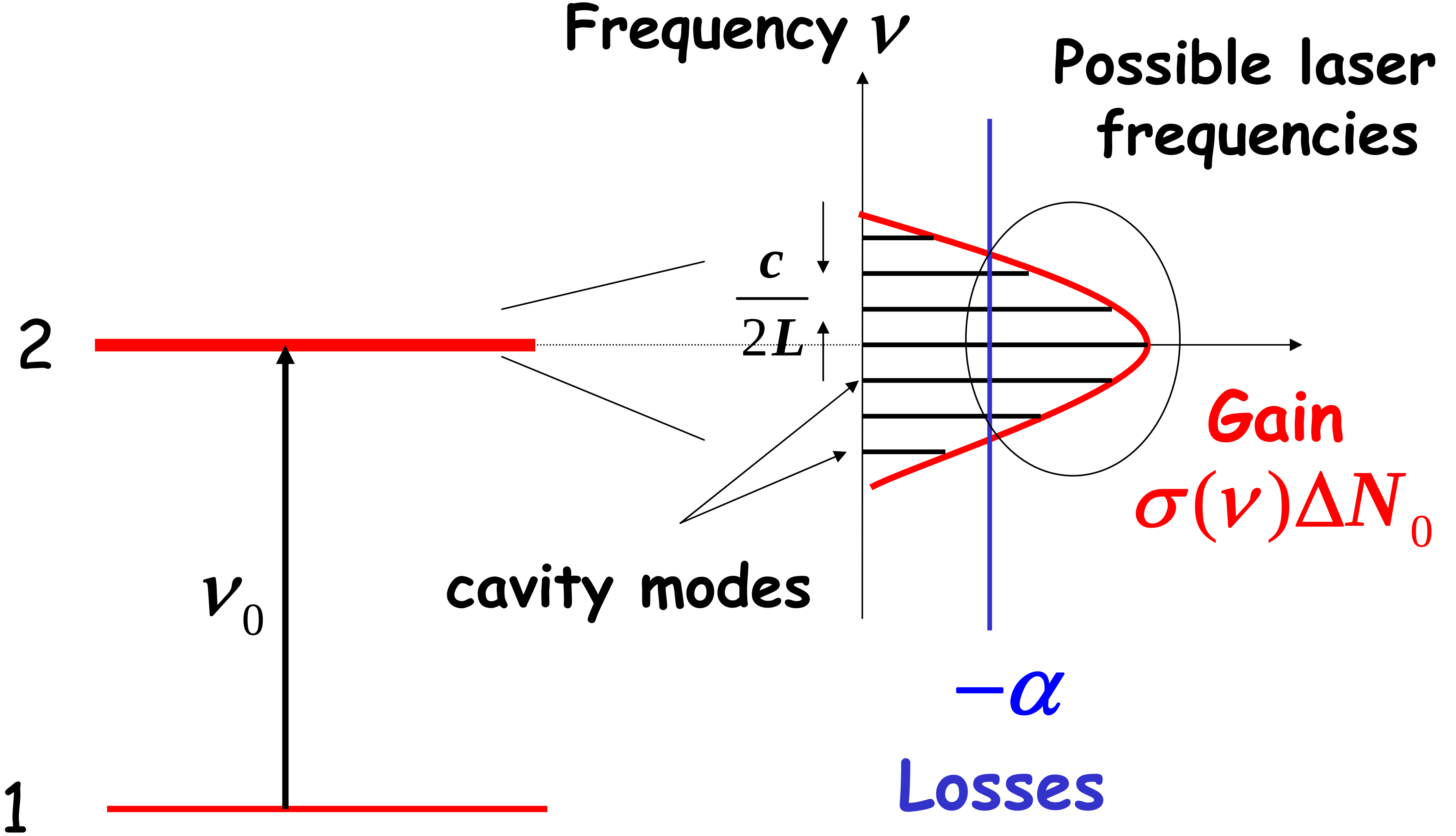
Two regimes:

Threshold regime: $I \ll I_s, \Delta N \simeq \Delta N_0 = -\dfrac{\alpha}{\sigma}$: *laser oscillation condition*

Gain saturation regime: $I \geq I_S, \Delta N = \dfrac{\Delta N_0}{1+\dfrac{I}{I_S}} = -\dfrac{\alpha}{\sigma}$: *decreases when intensity increases!!*

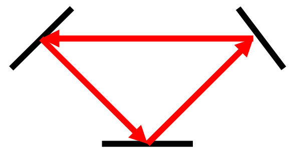

Stimulated emission
Increases
the gain
Decreases
the gain
Pumping
equilibrium
Losses
Fixes the laser
intensity

5- Laser frequency



Frequency ν
Possible laser frequencies
$\frac{c}{2L}$
2
1
ν_0
cavity modes
Gain
$\sigma(\nu)\Delta N_0$
$-\alpha$
Losses

travelling wave ↓ Spatial stationnary wave ↓

	Ring laser 	Linear laser 
Homogenous linewidth	Monomode	Multimode
Inhomogenous linewidth	Multimode	Multimode

6- Pulsed regime (summary)

Relaxation regime:

μs pulses (10^{-6} s)

Q-switch regime :

Nanosecond pulses (10^{-9}s)

Mode-locking Regime:

Picosecond (10^{-12} s) and femtosecond pulses (10^{-15} s)

Bibliographie

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- - RABIN&TANG: "Quantum Electronics" (Academic Press 1975)
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