Control of the Pancharatnam phase of single q-bits

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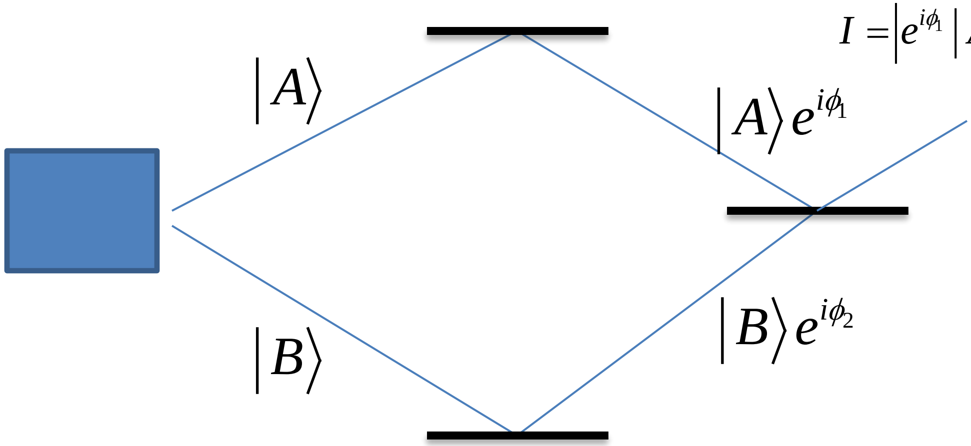
- Phase in quantum mechanics
- Control of Pancharatnam phase with classical light: Two-level system in semi-classical regime
- Control of Pancharatnam phase with quantized field: Trapped-ion
- Control of Pancharatnam phase with static field: Cooper pair box in a SQUID configuration

Phase in quantum mechanics

- Relative phase between two states $|A\rangle$ and $|B\rangle$

$$\Phi_{PP} = \arg\langle A|B\rangle : Pancharatnam\ phase$$

$$I = \left|e^{i\phi_1}|A\rangle + e^{i\phi_2}|B\rangle\right|^2 = 2 + 2\left|\langle A|B\rangle\right|\cos\left(\phi - \arg\langle A|B\rangle\right)$$



$$\phi = \phi_1 - \phi_2$$

$$I\ max\ when\ \phi = \Phi_{PP}$$

Phase in quantum mechanics

- Dynamical phase of state $|\psi\rangle$

$$\Phi_{dyn} = \int \mathrm{Im}\left(\langle\psi|\frac{d}{dt}|\psi\rangle \right) dt = -\frac{i}{\hbar}\int \langle\psi|H|\psi\rangle\, dt$$

If $|\psi\rangle(t) \simeq |\psi_n\rangle(t)$ $\longrightarrow$ $\Phi_{dyn} = -\frac{i}{\hbar}\int E_{adiab.}(t)\,dt$

$$H(t)|\psi_n\rangle(t) = E_n(t)|\psi_n\rangle(t)$$

Phase in quantum mechanics

- Geometric phase of state $|\psi\rangle$

$$\Phi_{Geom} = \arg\left(\langle\psi(0)|\psi(t)\rangle\right) - \int_0^t \mathrm{Im}\left(\langle\psi|\frac{d}{dt}|\psi\rangle\right) dt = \Phi_{PP} - \Phi_{dyn.}$$

- Connection with Berry phase :

$$H = H(\vec{R}(t))$$

$$H\left|\psi_n(\vec{R})\right\rangle = E_n\left|\psi_n(\vec{R})\right\rangle$$

$$\left|\psi(0)\right\rangle = \left|\psi_n(\vec{R}(0))\right\rangle$$

If cyclic and adiabatic evolution

$$\left|\psi(t)\right\rangle = e^{i\Phi_{Dyn}} e^{i\Phi_{Geom}} \left|\psi_n(\vec{R}(t) = \vec{R}(0\ )\right\rangle$$

$$\Phi_{Geom} = \Phi_{Berry} = i\oint_C d\vec{R}\left\langle\psi_n(\vec{R})\middle|\vec{\nabla}_R\,\psi_n(\vec{R})\right\rangle$$

Phase in quantum mechanics

- Important properties of Berry phase:

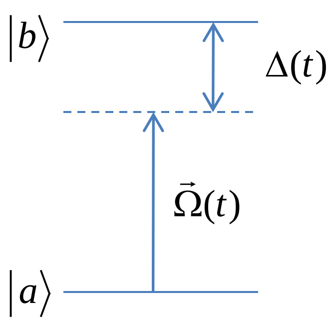
$$\Phi_{Berry} = i \oint_C d\vec{R} \left\langle \psi_n(\vec{R}) \middle| \vec{\nabla}_R \psi_n(\vec{R}) \right\rangle$$

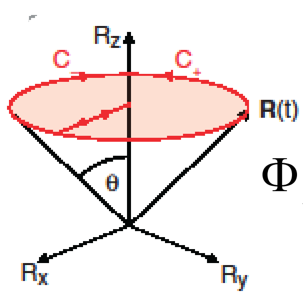
Depends only on the (close) path.

- Interrest for quantum information:

Insensitive to parameters that makes this path unchanged

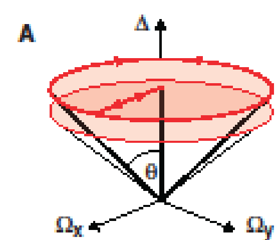
Ex:

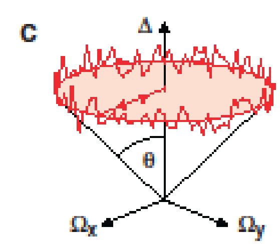




$$\vec{R} = \left(\Omega_x, \Omega_y, \Delta\right)$$

$$\Phi_{Berry} = 2\pi\left(1 - \cos\theta\right)$$





Control of the Pancharatnam phase

- HERE:

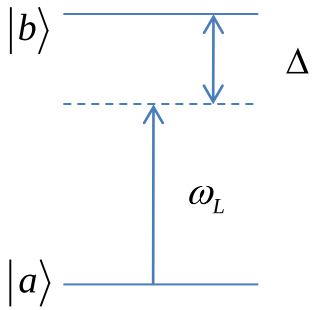
Non cyclic regime.

Behaviour of the PP with excitation parameters and comparison with population dynamics

Investigation restricted to simple q-bits

Control of the Pancharatnam phase

- Simple case : two level system in semi-classical regime



$$\hat{W} = \hbar W_0 e^{-i\varphi} e^{i\omega_L t} |a\rangle\langle b| + hc$$

$$n_b(t) = C(W_0,\Delta,t) + D(W_0,\Delta,t)\cos\phi_0$$

$$\tan\Phi_{PP} = \frac{A(W_0,\Delta,t) + B(W_0,\Delta,t)\cos\phi_0}{A'(W_0,\Delta,t) + B'(W_0,\Delta,t)\cos\phi_0}$$

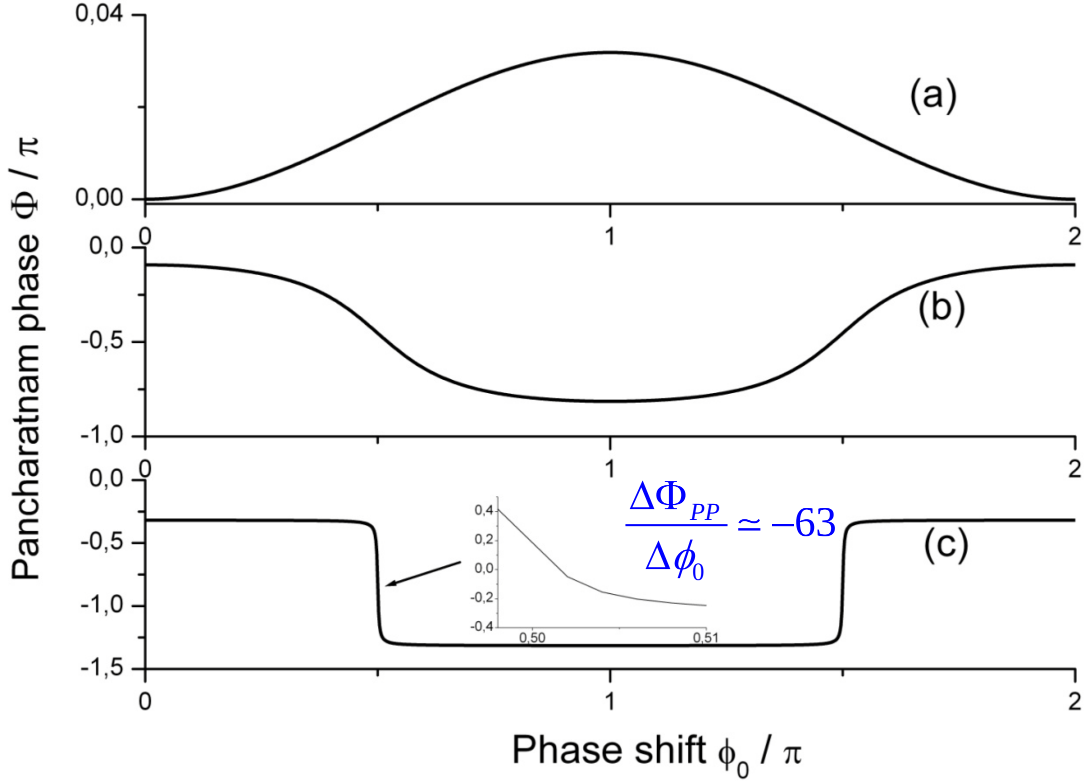
•Interresting parameter : $\phi_0 = \varphi - \arg\left(a(0)b^*(0)\right)$

$$D,B,B' \propto \sqrt{n_a(0)n_b(0)}$$

→ Needs initial coherence: Ramsey-like configuration

Two-level system in semi-classical regime

$$n_a(0) = n_b(0) = 1/2; \theta = \frac{\pi}{4}$$



$$\frac{\Delta\Phi_{PP}}{\Delta\phi_0} \simeq -63$$

$W_0\, t = 0.05$

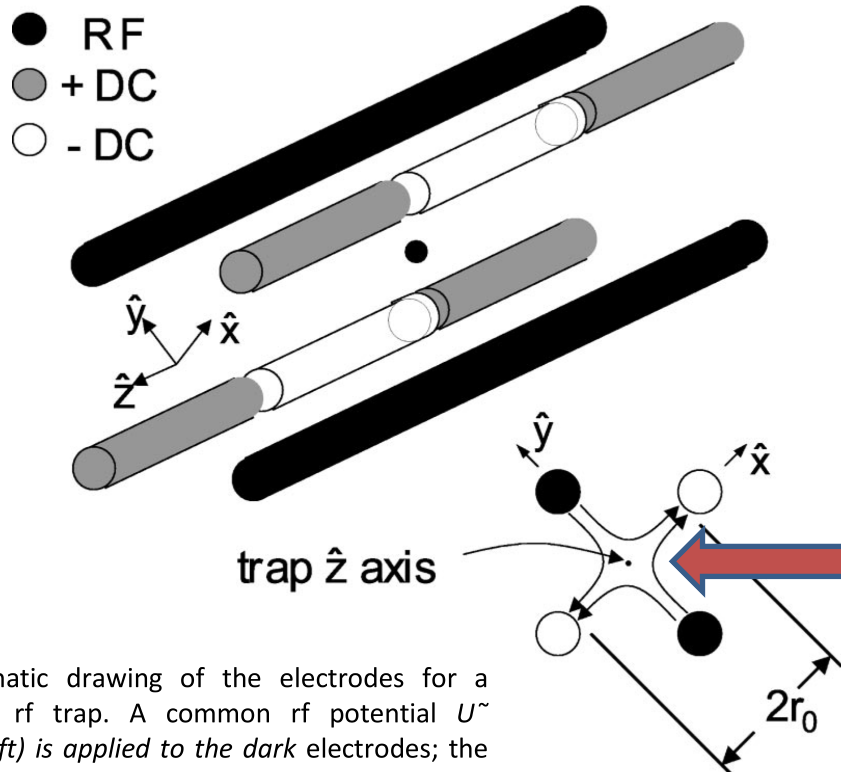
$W_0\, t = 1$

$W_0\, t = 10.$

Highly non-linear dependence

(Attosecond resolution in optical domain)

Trapped-ion



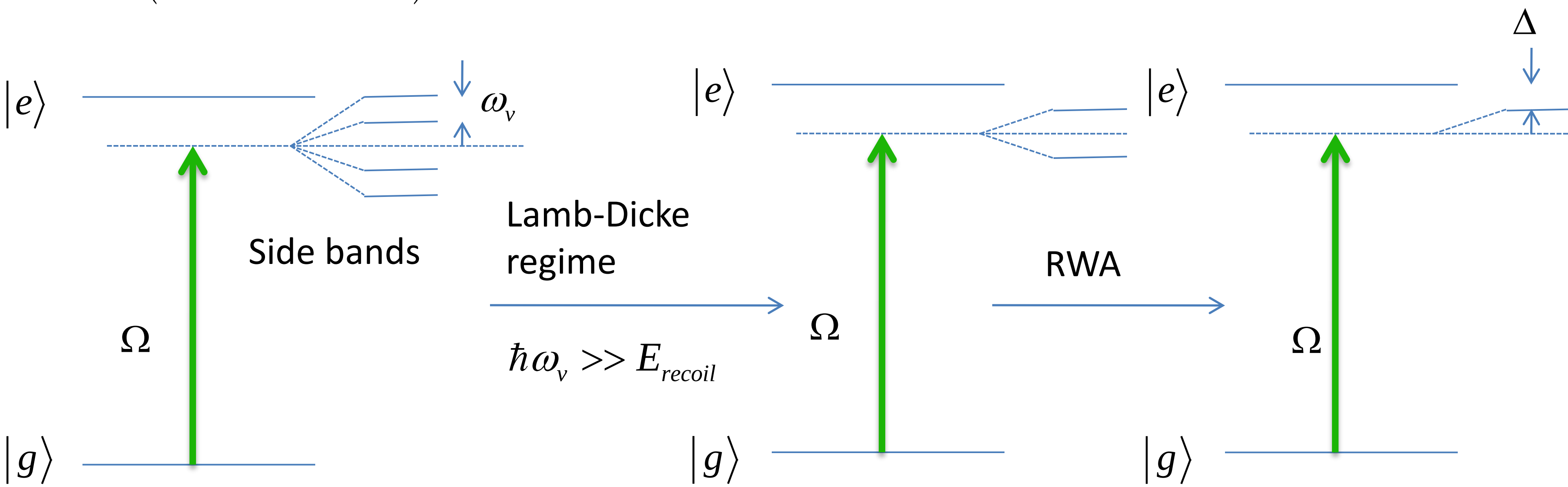
$$\hat{H}_{trans.ion} \simeq \hbar\omega_v\left(\hat{a}^+\hat{a}+1/2\right)$$

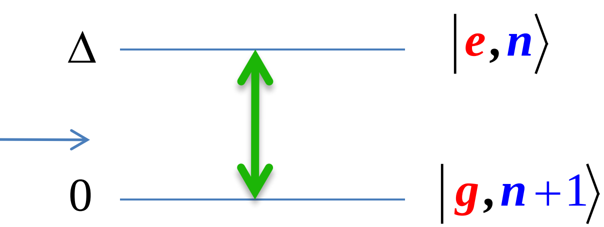
laser: $E = \varepsilon\, e^{i(kz-\Omega t)}\, e^{i\varphi} + cc$

Schematic drawing of the electrodes for a linear rf trap. A common rf potential *U*˜ *cos(vrft) is applied to the dark* electrodes; the other electrodes are held at rf ground through capacitors (not shown) connected to ground. The lower right portion of the figure shows the *x-y electric fields from the* applied rf potential at an instant when the rf potential is positive relative to the ground. A static electric potential well is created (for positive ions) along the *z axis by applying a positive* potential to the outer segments (gray) relative to the center segments (white).

Trapped-ion

$\hat{H}_{int} = \hbar g\left(\sigma^{+} e^{-i(\Omega t - k\hat{z})} + HC\right); \quad \sigma^{+} = |e\rangle\langle g|$ Doppler effect : coupling between electronic and vibrational motion

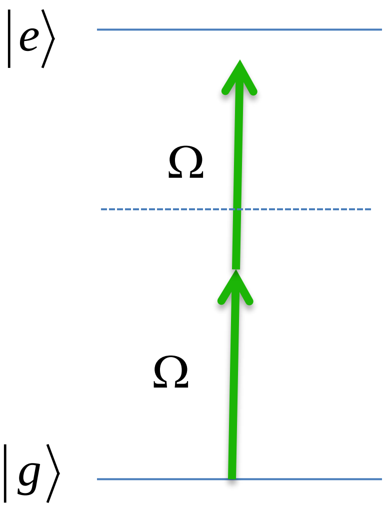




$\hat{H}_{int} = \hbar g'(\Omega)\left(\sigma^{+}\hat{a}\, e^{i\Delta t} + \hat{\sigma}\,\hat{a}^{+} e^{-i\Delta t}\right)$: *Jaynes – Cummings model for atom – phonon interaction assisted by laser*

Trapped-ion

More complex system



Adiabatic elimin.
of interme state +
Lambe-Dicke approx. +
RWA

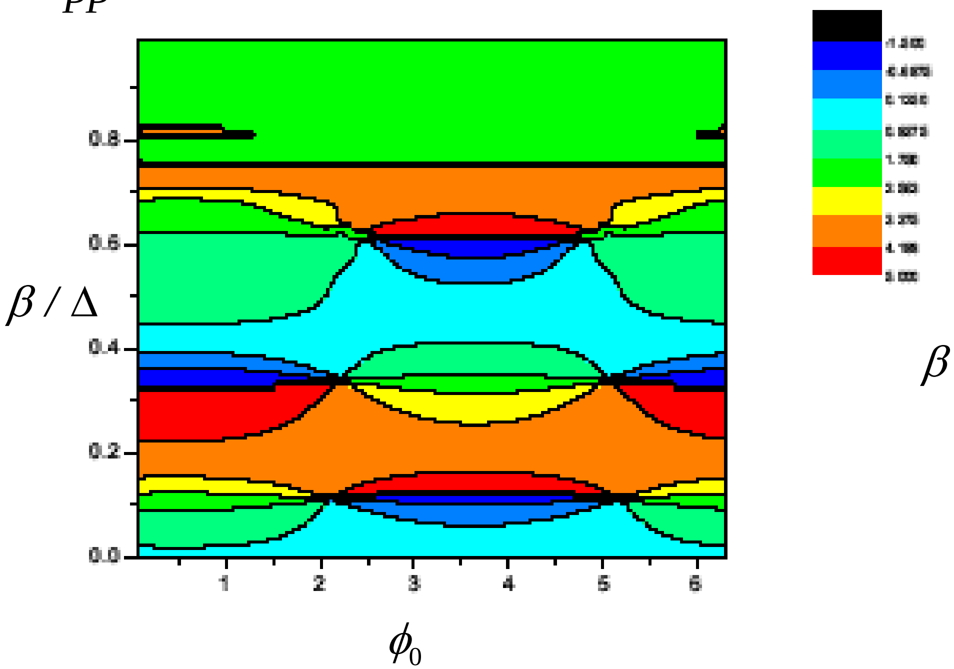
$$\hat{H}_{eff} = \begin{pmatrix} \Delta + \beta n & \lambda_{eg}\, e^{-2i\varphi}\sqrt{(n+1)(n+2)} \\ \lambda_{eg}\, e^{2i\varphi}\sqrt{(n+1)(n+2)} & -\beta(n+2) \end{pmatrix}$$

Parameters: Laser intensity; Laser phase; Laser frequency; Phonon number

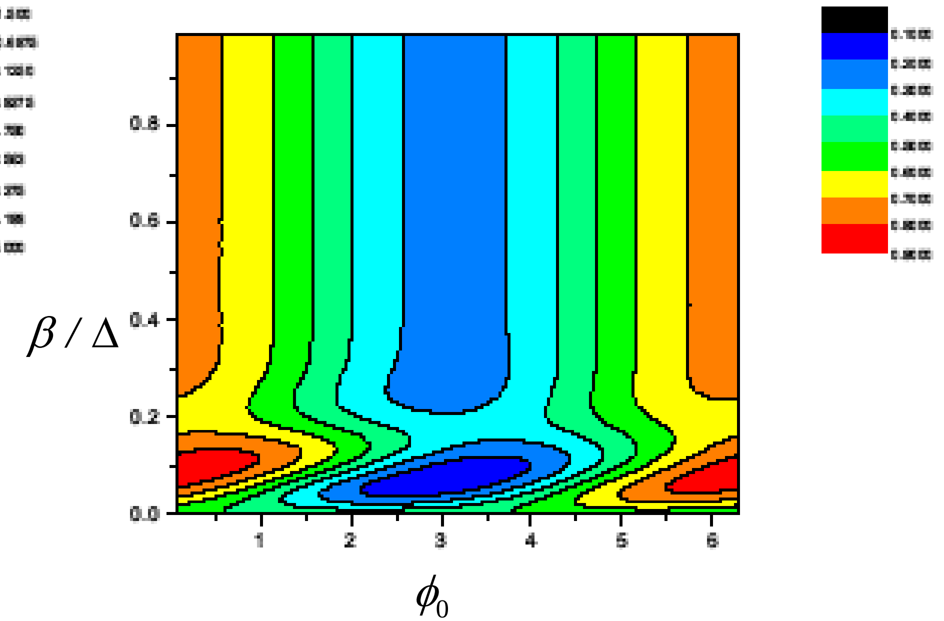
Trapped-ion

Coherent state $\bar{n} = 10,\ \Delta t = 1,\ n_g(0) = n_e(0) = 1/2$

Φ_{PP}

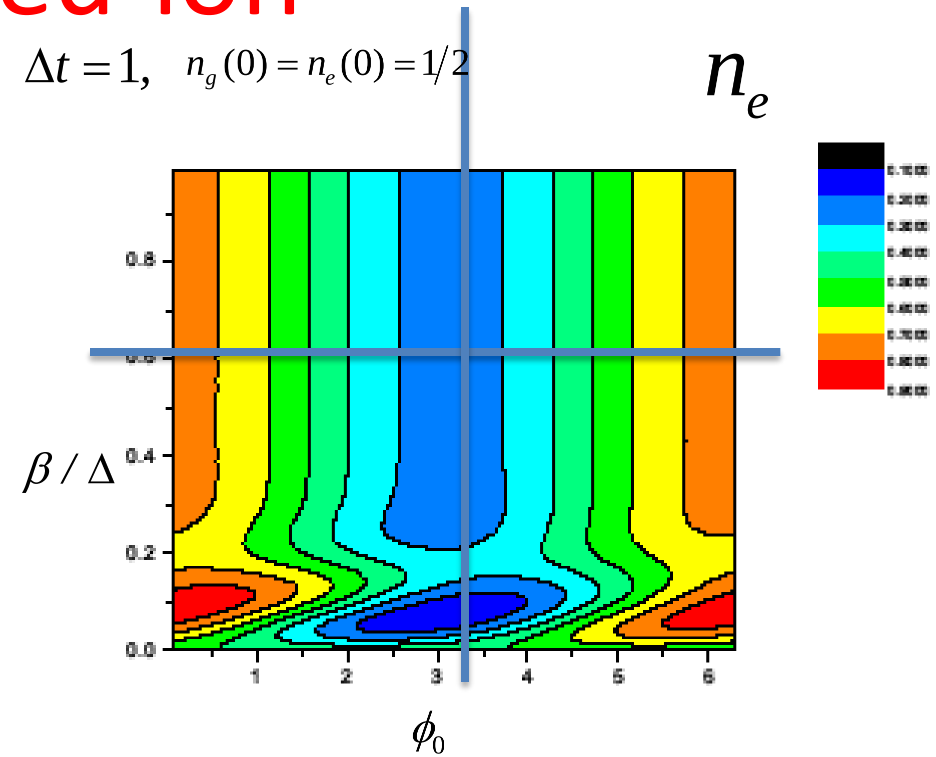


n_e

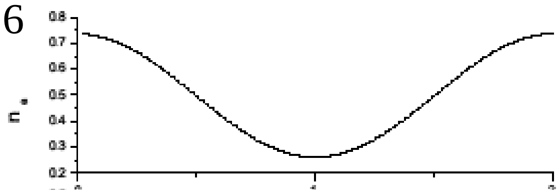


Trapped-ion

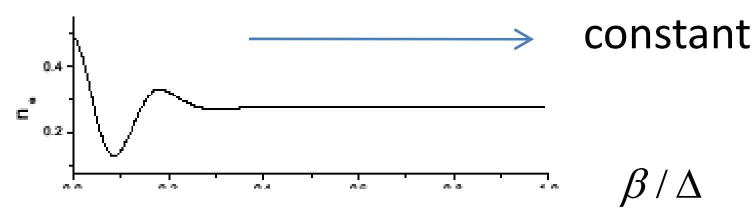
Coherent state $\bar{n} = 10, \ \Delta t = 1, \ n_g(0) = n_e(0) = 1/2$



$\beta / \Delta = 0.6$

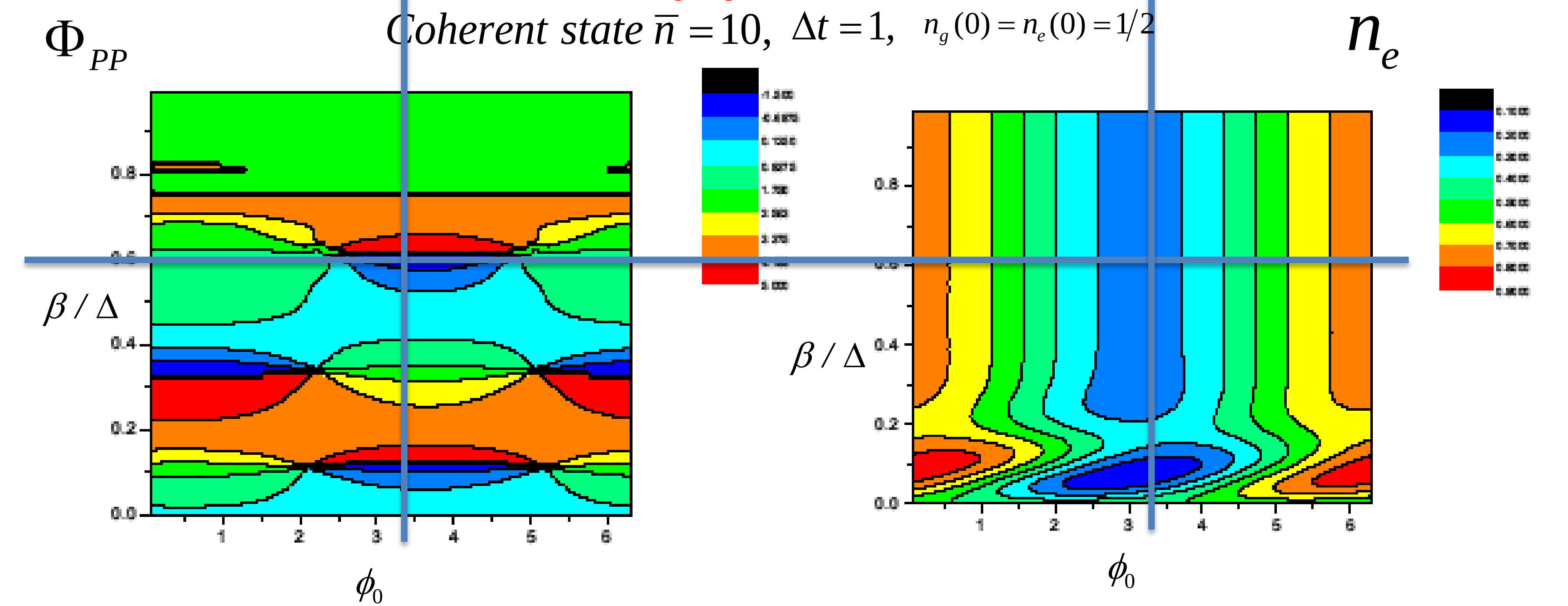


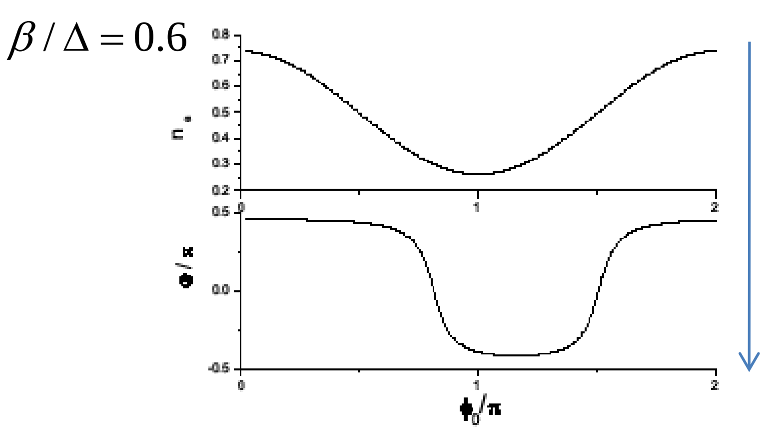
$\phi_0 = 1.1\pi$



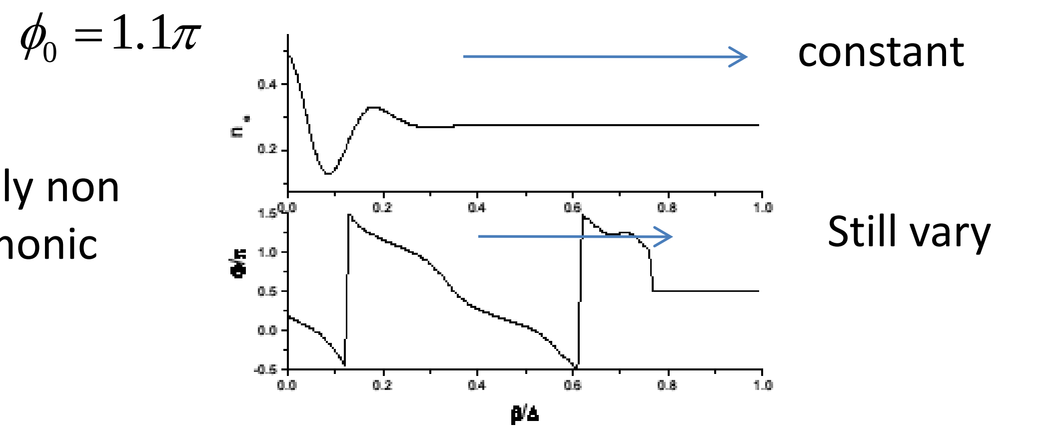
Trapped-ion

Coherent state $\bar{n} = 10,\ \Delta t = 1,\ n_g(0) = n_e(0) = 1/2$

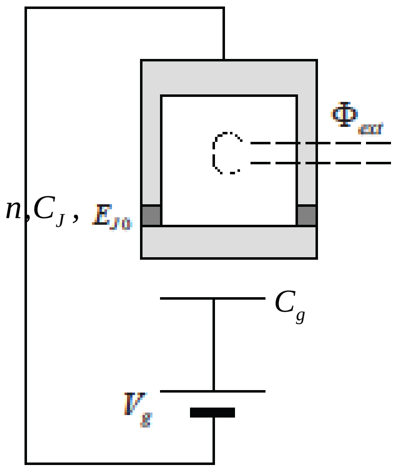




Highly non harmonic

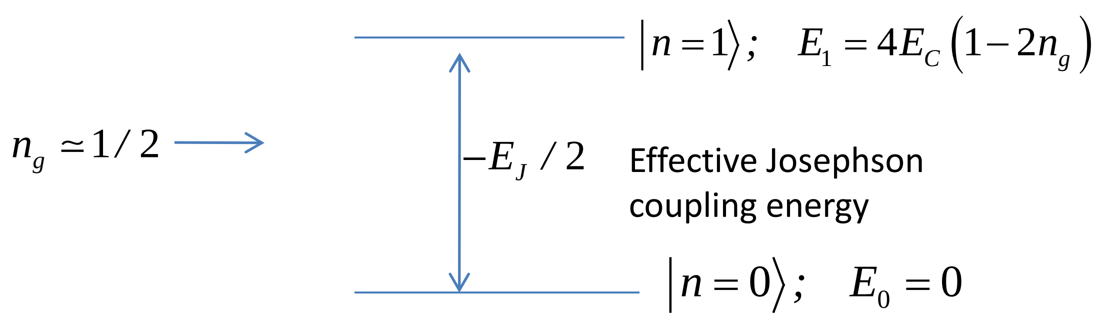


Single Cooper-pair box



$$\hat{H} = 4E_c \sum_n (n-n_g)^2 |n\rangle\langle n| - \frac{E_J}{2} \sum_n \left(|n+1\rangle\langle n| + |n\rangle\langle n+1|\right)$$

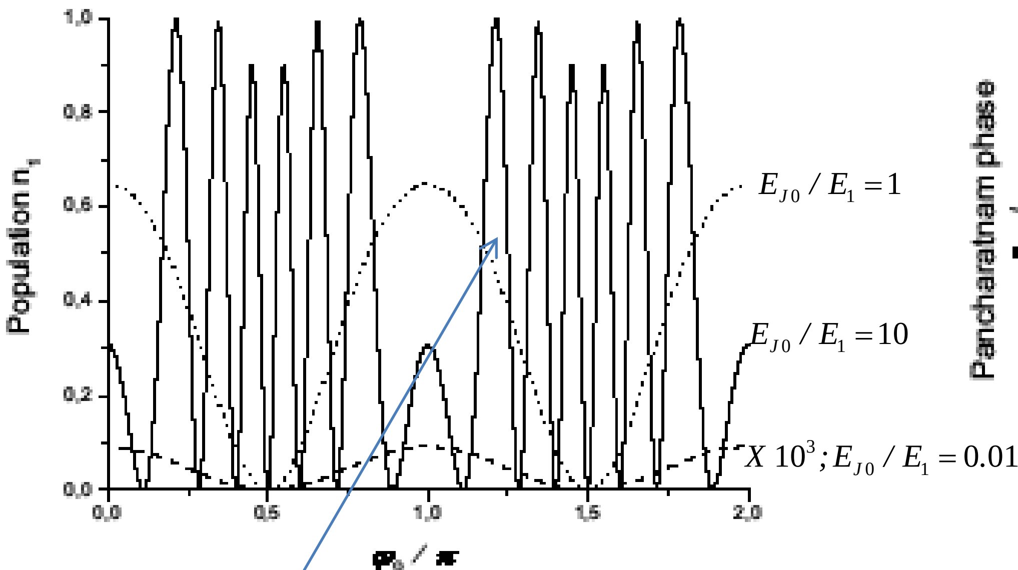
$E_c = e^2/2(C_g + C_J)$ $\quad n_g = C_g V_g / 2e$ $\quad E_J = E_{J0} \cos\left(\frac{\pi \Phi_{ext}}{\Phi_q}\right)$ $\quad \Phi_q = h/2e$

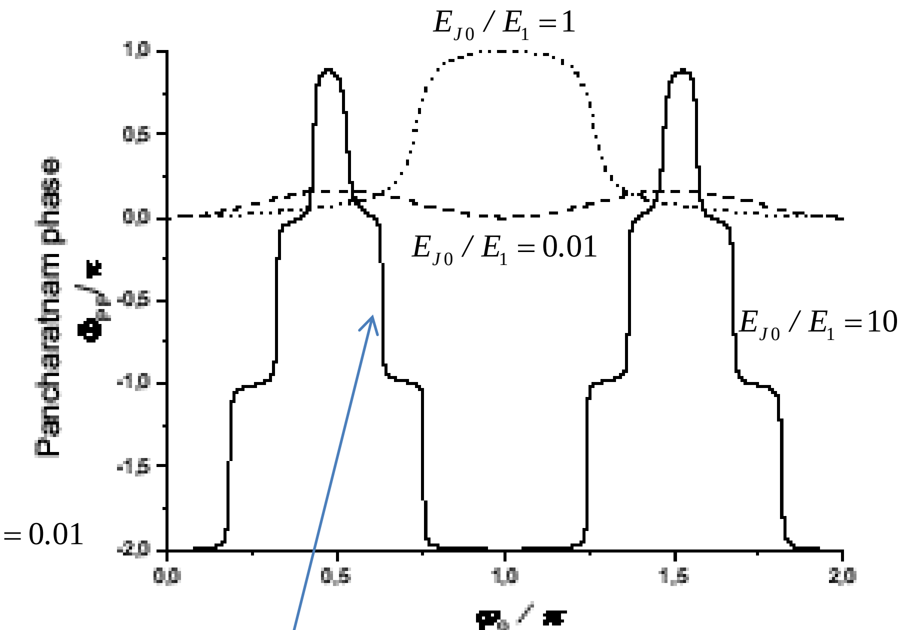


Cooper-pair box with a SQUID loop.
E_{J0} is the Josephson energy,
Vg is the voltage gate and Φ_{ext}
is the magnetic flux

Effective two-level system
Control parameter: $\varphi_0 = \pi\, \Phi_{ext}/\Phi_q$

Single Cooper-pair box





$$\Delta \Phi_{PP} / \Delta \varphi_0 \propto E_{J0}\, t \left(E_{J0} / E_1 \right)$$

$$\Delta n_1 / \Delta \varphi_0 \propto E_{J0}\, t$$

$$>> 1$$

Higher sensitivity to the magnetic field

Conclusion

- Comparison of PP with population: greater sensitivity to control parameters

- Applications:

- Phase, flux: pertinent parameters for q-bits manipulation
- Detection of small magnetic flux (Cooper-box).
- Stabilisation of interferometers.

Two-level system in semi-classical regime

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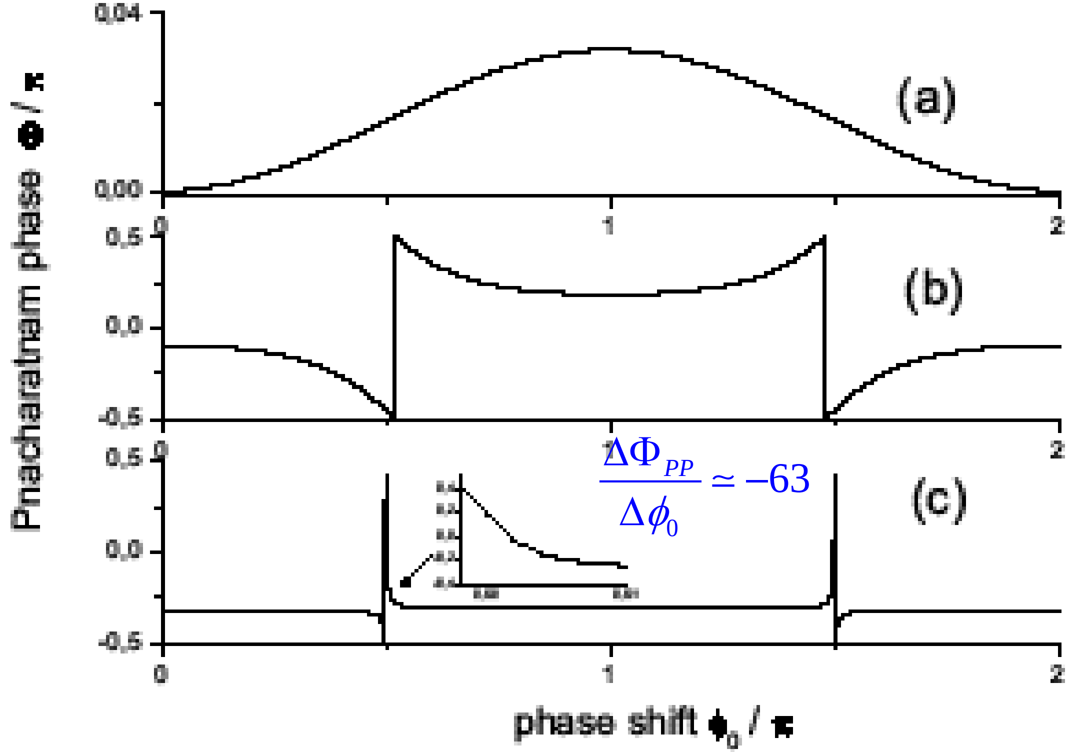

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$W_0\ t = 1$

$W_0\ t = 10.$

Highly non-linear dependence

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