

Atomic physics in Intense Laser Field

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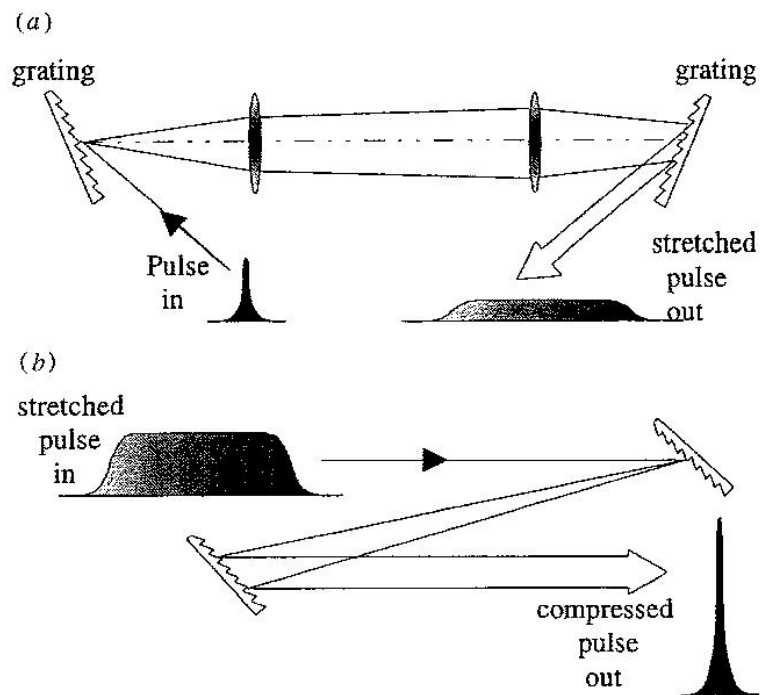
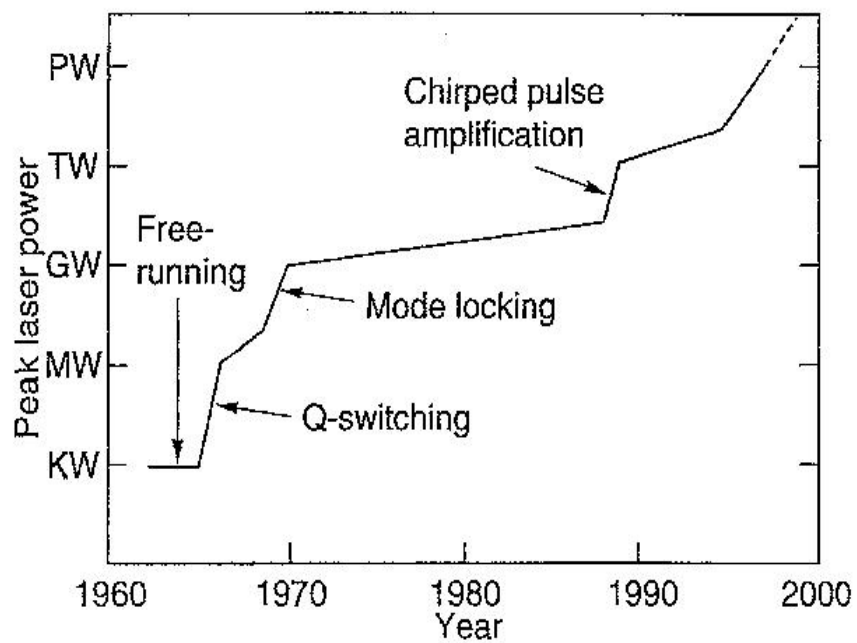
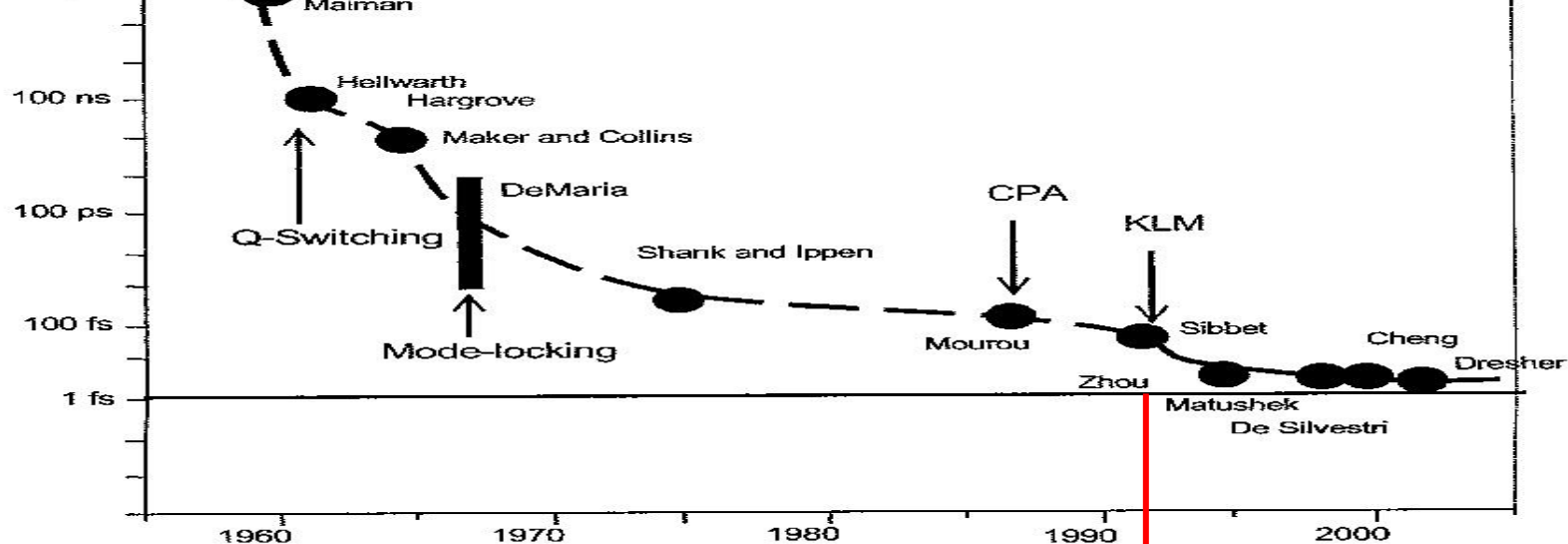


Table 1.1 Multi-Terawatt laser systems and laboratories worldwide.

Name	Laboratory	Country	Type	λ (nm)	Energy (J)	Pulse length (fs)	Power (TW)	Focal spot (μm)	Intensity (Wcm^{-2})
Petawatt ^a	LLNL	USA	Nd:glass	1053	700	500	1300	-	$> 10^{20}$
VULCAN ^b	RAL	UK	Nd:glass	1053	423	410	1030	10	1.06×10^{21}
PW laser ^c	ILE	Japan	Nd:glass	1054	420	470	1000	30	10^{20}
PHELIX ^d	GSI	Germany	Nd:glass	1064	500	500	1000	-	-
LULI 100TW	LULI	France	Ti:Sa	800	30	300	100	-	-
APR 100 TW	APR	Japan	Ti:Sa	800	2	20	100	11	2×10^{19}
HERCULES	FOCUS	USA	Ti:Sa	800	1.2	27	45	(1)	(8×10^{21})
ALFA 2	FOCUS	USA	Ti:Sa	800	4.5	30	150	(1)	(10^{22})
Salle Jaune	LOA	France	Ti:Sa	800	0.8	25	35	-	10^{19}
Lund TW	LLC	Sweden	Ti:Sa	800	1.0	30	30	10	$> 10^{19}$
MBI Ti:Sa	MBI	Germany	Ti:Sa	800	0.7	35	20	-	$> 10^{19}$
Jena TW	IOQ	Germany	Ti:Sa	800	1.0	80	12	3	5×10^{19}
ASTRA	RAL	UK	Ti:Sa	800	0.5	40	12	-	10^{19}
USP	LLNL	USA	Ti:Sa	800	1 (10)	100 (30)	10 (100)	-	5×10^{19}
UHI 10	CEA	France	Ti:Sa	800	0.7	65	10	-	5×10^{19}

^a 1996-1999

^b Petawatt performance achieved on October 5, 2004.

^c Projected upgrade of PWM — PetaWatt Module.

^d Commissioned for end 2005.

Table 1.1. Light intensities I (in units of W/cm^2) from the very dim to the extremely bright.

may be	10^{+30}	→ generation of real electron–positron pairs from vacuum
	10^{+28}	→ electron acceleration by light comparable to edge of black hole
	10^{+26}	
extr. LO: reality	10^{+24}	→ nonlinear optics of the vacuum ?
	10^{+22}	
	10^{+20}	→ photonuclear fission – light splits nuclei
	10^{+18}	→ relativistic nonlinear optics of vacuum electrons
	10^{+16}	
audit. LO	10^{+14}	→ electrostatic tunneling of electrons from atoms
	10^{+12}	→ Rabi flopping in semiconductors becomes optical
	10^{+10}	
	10^{+8}	
	10^{+6}	→ laser intensity in the first experiment on nonlinear optics in 1961
	10^{+4}	
	10^{+2}	→ a continuous-wave laser of that intensity hurts
	1	→ total intensity of the sun on the earth’s surface ($10^{-1} \text{ W}/\text{cm}^2$)
	10^{-2}	→ thermal radiation from a human
	10^{-4}	
10^{-6}		
10^{-8}		
10^{-10}	→ total intensity of the cosmic 2.8 K background radiation	
10^{-12}		
10^{-14}		
10^{-16}		
10^{-18}		
10^{-20}		
10^{-22}	→ visible intensity in a “dark” room at 300 K ($10^{-23} \text{ W}/\text{cm}^2$)	

OUTLOOK

- I- Free electron in an EM field:
 - Classical treatment
 - Relativistic treatment
 - Quantum treatment

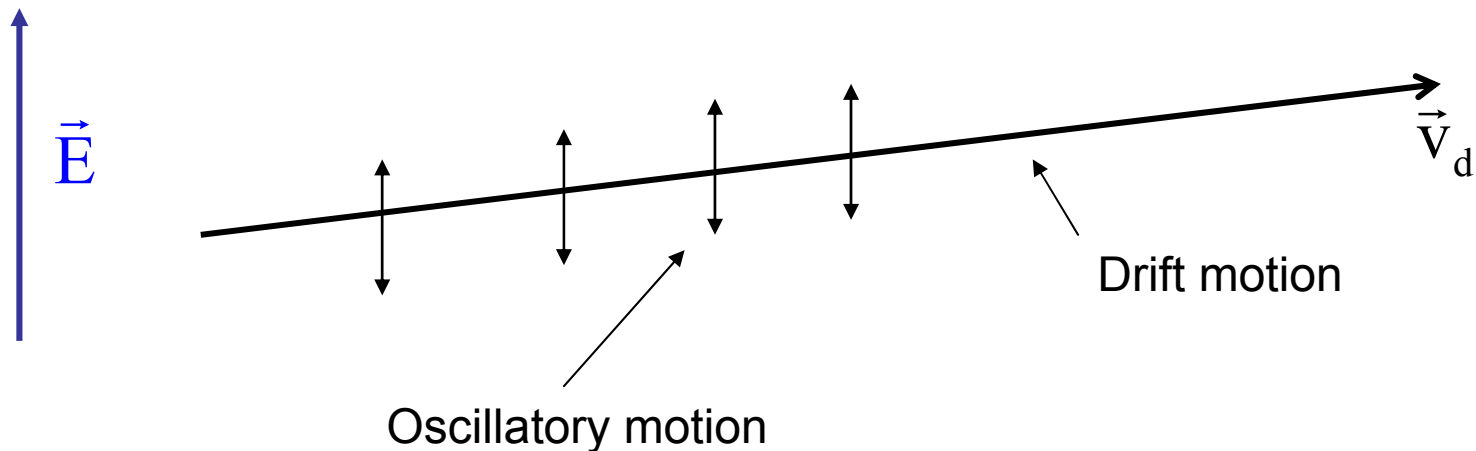
- II- Atom in a strong Laser field:
 - Scenario with I
 - Experimental results

- III- Above Threshold Ionisation

- IV- Tunnel ionisation

I-Free electron in an EM field: Classical treatment

$$\vec{F} \simeq q\vec{E} \quad \left(\frac{F_{\text{magn}}}{F_{\text{elec}}} = \frac{|\vec{v} \times \vec{B}|}{E} = \frac{v}{c} \ll 1 \right); \quad E = E_0 \cos \omega t$$



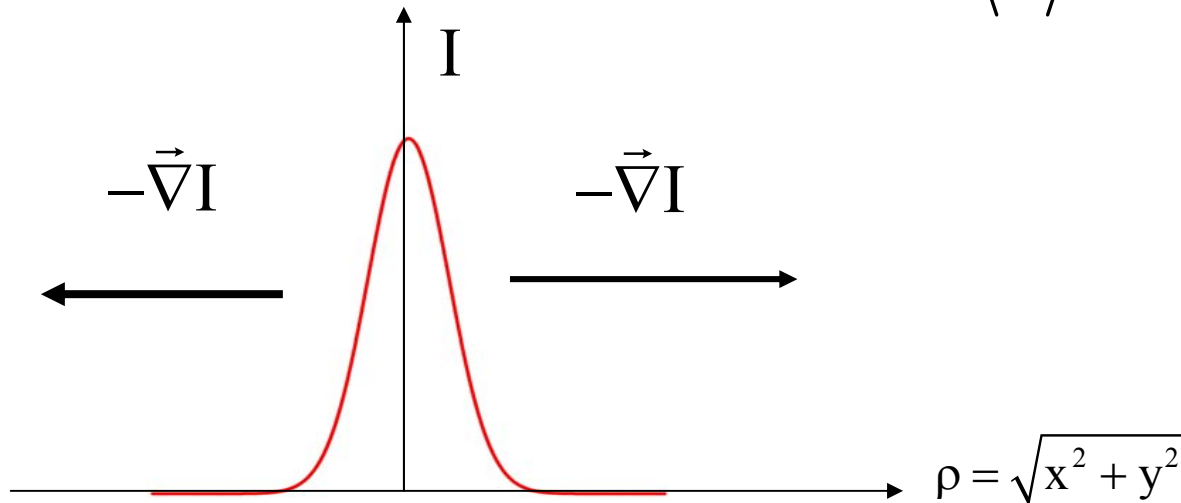
$$= m \frac{d\vec{v}}{dt} \rightarrow \langle E_k \rangle_{T=\frac{2\pi}{\omega}} = m \frac{v_d^2}{2} + \frac{q^2 E_0^2}{4m\omega^2}$$

PONDEROMOTIVE ENERGY
 U_p

$$U_p (\text{eV}) = 9.3410^{-14} \lambda_{\mu\text{m}}^2 I_{\text{W/cm}^2}$$

$$U_p = \begin{cases} 1\text{eV} & \text{for } I \approx 10^{13} \text{ W/cm}^2 \\ 100\text{keV} & \text{for } I \approx 10^{18} \text{ W/cm}^2 \quad (U_p \approx 0.2mc^2) \end{cases}$$

Force on the electron in a laser spot: $I = I(\vec{r}) \rightarrow \langle \vec{F} \rangle = -\vec{\nabla} U_p$

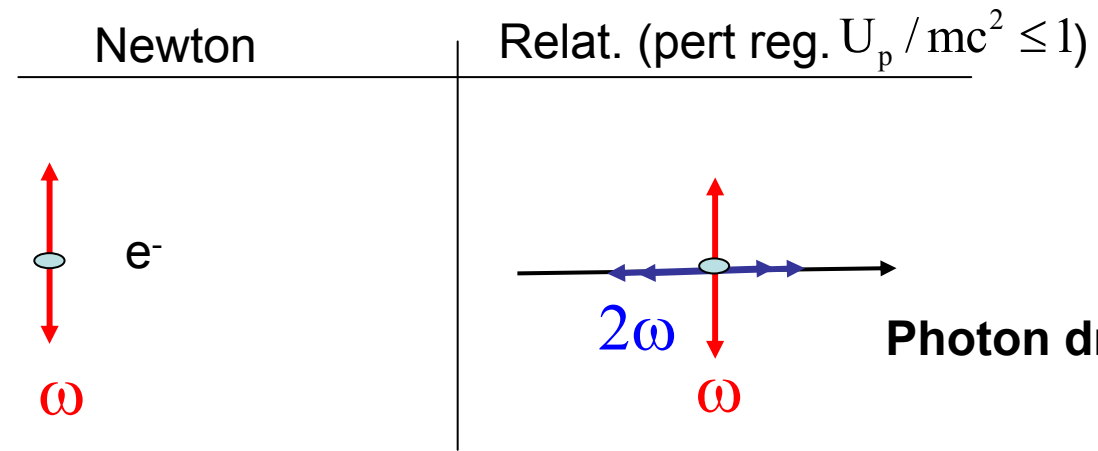
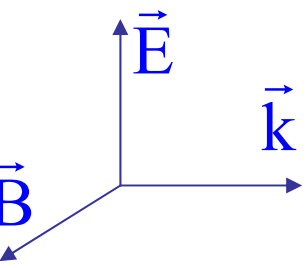


The electron is ejected

I-Free electron in an EM field: Relativistic treatment

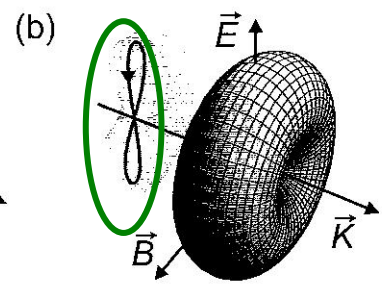
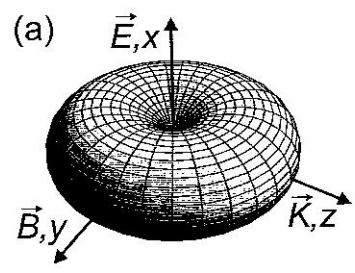
$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}); \quad E_x = E_0 \cos \omega t; \quad B_y = B_0 \cos \omega t$$

electron at rest



Photon dragging

Radiation pattern



ω

2ω

General case: analytical solution

$$\varepsilon = \frac{4U_P}{m_e c^2} \quad \tilde{t} = \omega t; \tilde{x} = kx; \tilde{z} = kz$$

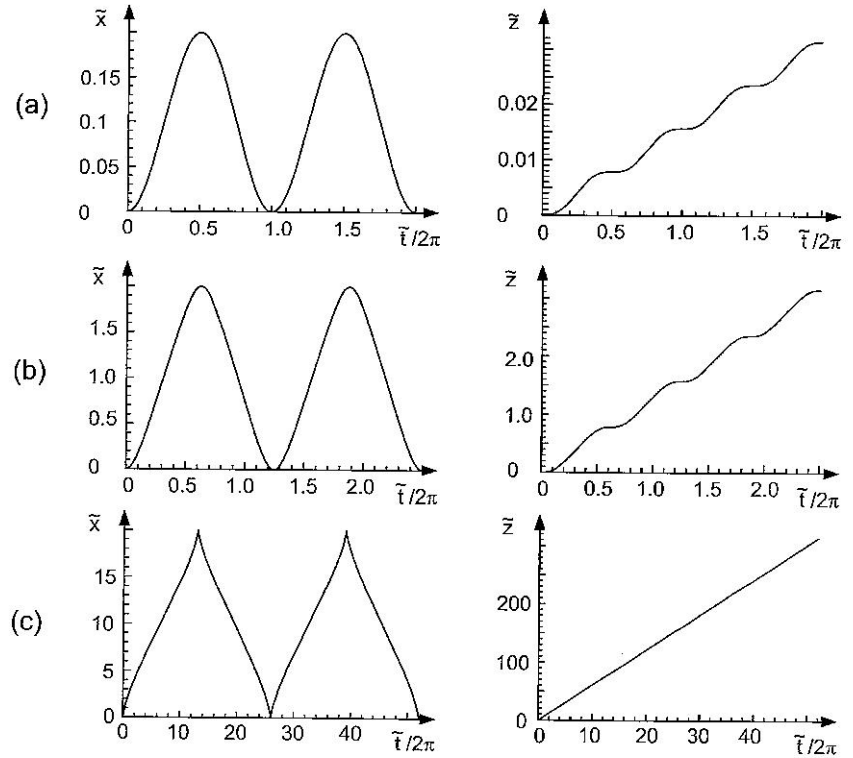
$$\tilde{z}(0) = -\xi_0$$

$$\zeta(\tilde{t}) = (\zeta - \zeta_0) \left[1 + \frac{\varepsilon^2}{2} \left(\frac{1}{2} + \sin^2 \zeta_0 \right) \right]$$

$$+ \frac{\varepsilon^2}{2} \left[-\frac{\sin(2\zeta)}{4} + 2 \cos \zeta \sin \zeta_0 - \frac{3 \sin(2\zeta_0)}{4} \right]$$

$$\zeta(\tilde{t}) = \mathcal{E} \left((\cos \zeta_0 - \cos \zeta) - (\zeta - \zeta_0) \sin \zeta_0 \right),$$

$$\tilde{t}(\zeta) = \tilde{t} - \zeta,$$



Periodic motion

HARMONIC DECOMPOSITION

fundamental oscillation frequency:

$$\frac{\omega_{\text{osc}}}{\omega} = \frac{1}{1 + \frac{\varepsilon^2}{2} \left(\frac{1}{2} + \sin^2 \xi_0 \right)}$$

fundamental emission frequency:



$$\frac{\omega_{\text{em}}}{\omega} = \frac{\xi_0 = 0}{1 + \frac{\varepsilon^2}{2} (1 - \cos \theta)}$$

I-Free electron in an EM field: Quantum treatment

Hamiltonian:

$$H = \frac{(\vec{P} - q\vec{A})^2}{2m}; \quad \vec{E} = -\frac{\partial \vec{A}}{\partial t} = \vec{E}_0 \cos \omega t$$

Schrödinger equation:

$$i\hbar \frac{\partial \Psi(\vec{r}, t)}{\partial t} = H\Psi(\vec{r}, t)$$

Analytical solution!

$$\Psi(\vec{r}, t) = \Psi_0 e^{i\left(\vec{k}\vec{r} - \left(\frac{E_d + U_p}{\hbar}\right)t\right)} e^{i\frac{U_p}{\hbar}\left(\frac{\sin 2\omega t}{2\omega}\right)} e^{-i\frac{q\vec{k}\vec{E}_0}{m\omega}(\cos \omega t - 1)} \quad E_{\text{drift}} = \frac{\hbar^2 \mathbf{k}^2}{2m}$$

VOLKOV STATE

- STATIONNARY STATE BUT TIME DEPENDENT ENERGY

VOLKOV STATE

$$\Psi(\vec{r}, t) = \Psi_0 e^{i\left(\vec{k}\vec{r} - \left(\frac{E_d + U_p}{\hbar}\right)t\right)} e^{i\frac{U_p}{\hbar}\left(\frac{\sin 2\omega t}{2\omega}\right)} e^{-i\frac{q\vec{k}\vec{E}_0}{m\omega}(\cos \omega t - 1)}$$

Classical part

$$e^{i\alpha \cos \theta} = \sum_{m=-\infty}^{m=\infty} i^m J_m(\alpha) e^{im\theta}$$

$$\Psi(\vec{r}, t) = \Psi_0 e^{i\left(\vec{k}\vec{r} - \left(\frac{E_d + U_p}{\hbar}\right)t\right)} \sum_n J_n\left(\frac{U_p}{2\hbar\omega}\right) e^{i2n\omega t} \sum_m i^m J_m\left(-2\sqrt{2}\frac{\sqrt{U_p E_d}}{\hbar\omega}\right) e^{im\theta}$$

Even harmonics

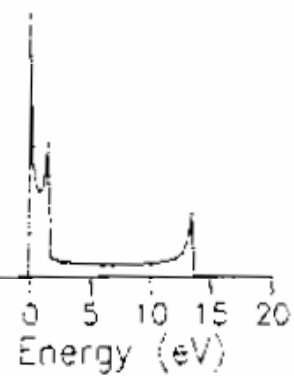
Odd + Even harmonics:
Needs $E_d \neq 0$

New effects : $U_p, E_d \geq \hbar\omega$

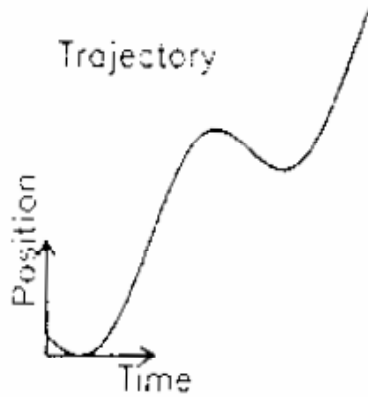
CLASSICAL ELECTRON IN A LASER FIELD

$$U_p = 3\text{eV} \quad m\langle v \rangle^2/2 = 1.5\text{eV}$$

Energy Distribution



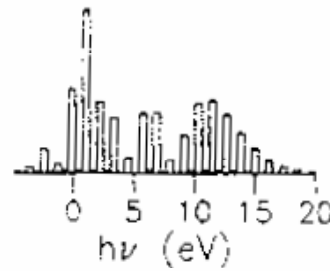
Trajectory



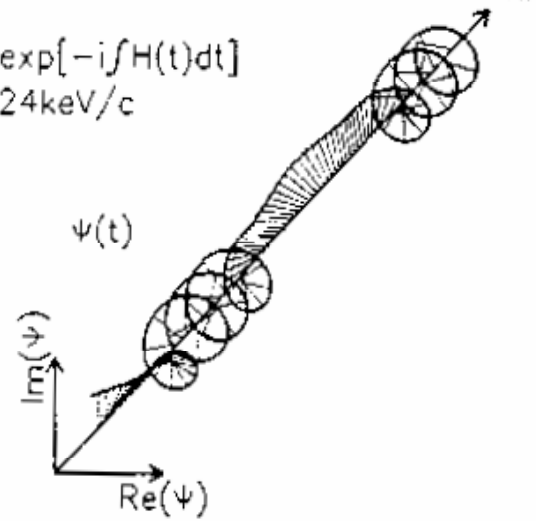
VOLKOV STATE $\Psi(t) = \exp[-i\int H(t)dt]$

$$U_p = 3\text{eV} \quad p = 1.24\text{keV}/c$$

Power Spectrum



$\Psi(t)$



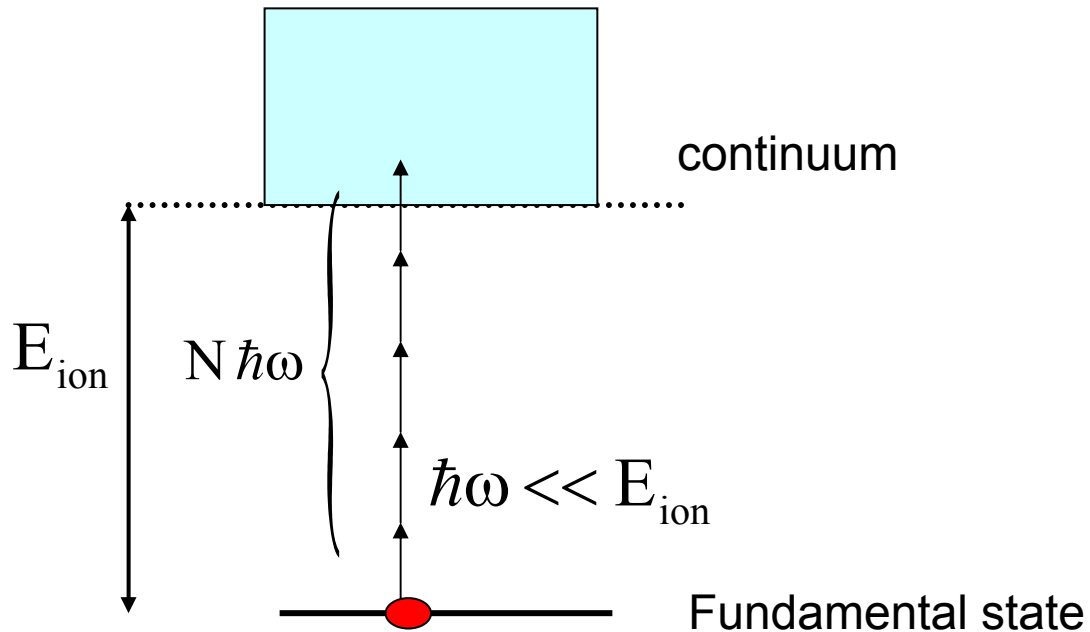
$$\vec{V} = \vec{V}_0 - \frac{q\vec{E}_0}{m\omega} \sin \omega t;$$

$$E_k = \left(\sqrt{E_d} - \sqrt{2U_p} \sin \omega t \right)^2$$

$$E = E_d + U_p + n\hbar\omega$$

II- Atom in a strong laser field

Model: Bound - Unbound transition for a single electron



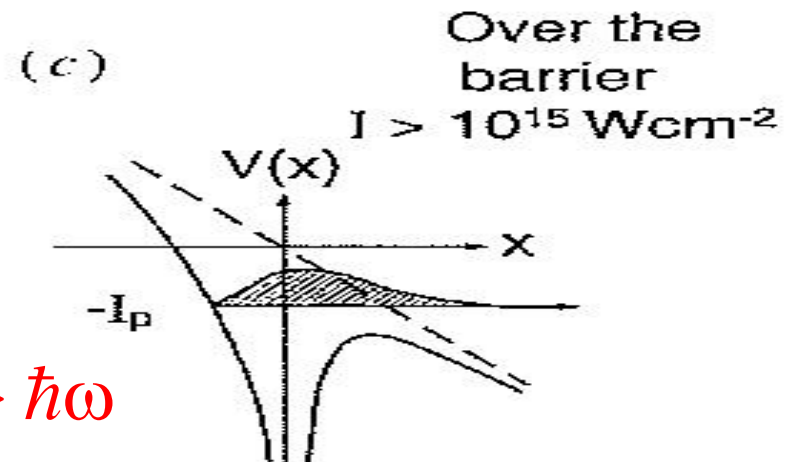
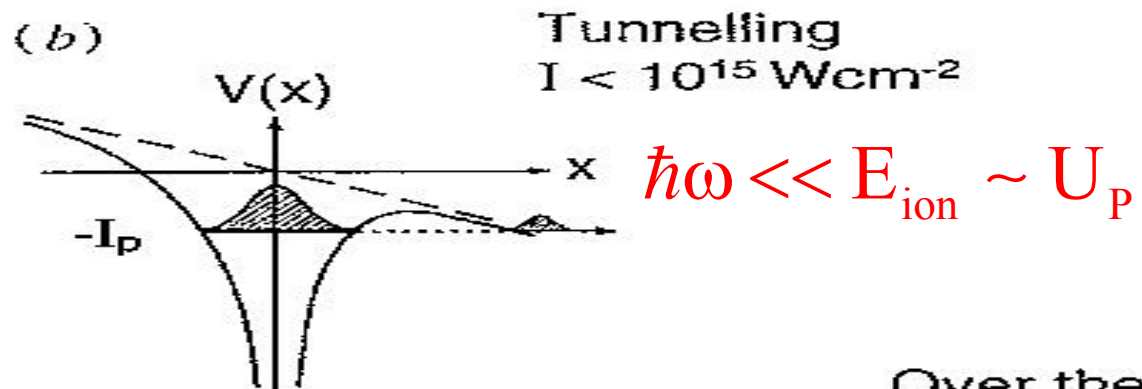
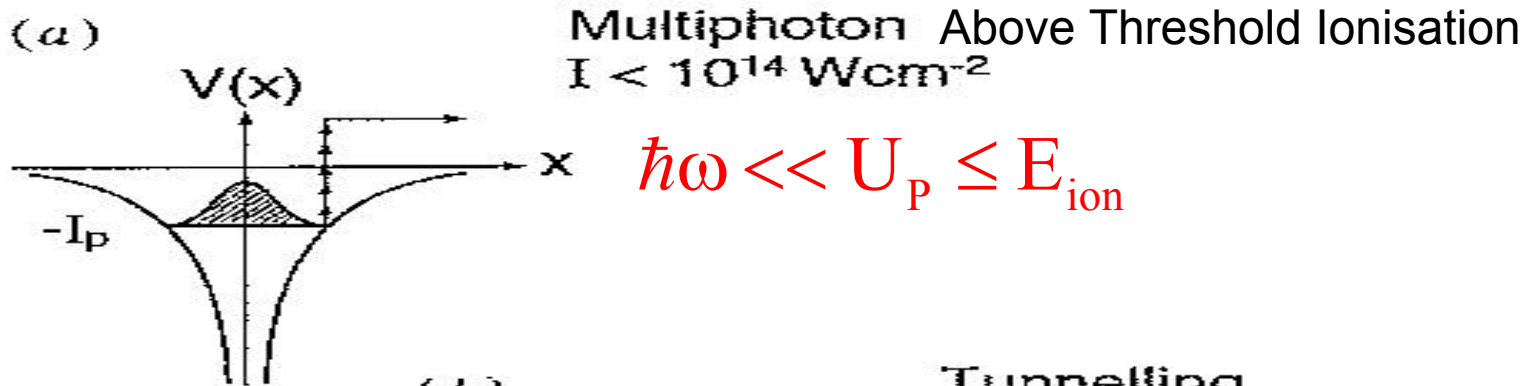
perturbative multiphoton ionisation :

$$U_p \ll \hbar\omega \ll E_{\text{ion}}; \quad I \leq 10^{12} \text{ W / cm}^2; \quad P_{\text{ion}} \propto I^N$$

$I \nearrow$

Dynamics depends strongly on the laser intensity

Scenario with I



VALID IF $I \leq I_{\text{crit}}$ (depletion of the ground state)

Experimental manifestation

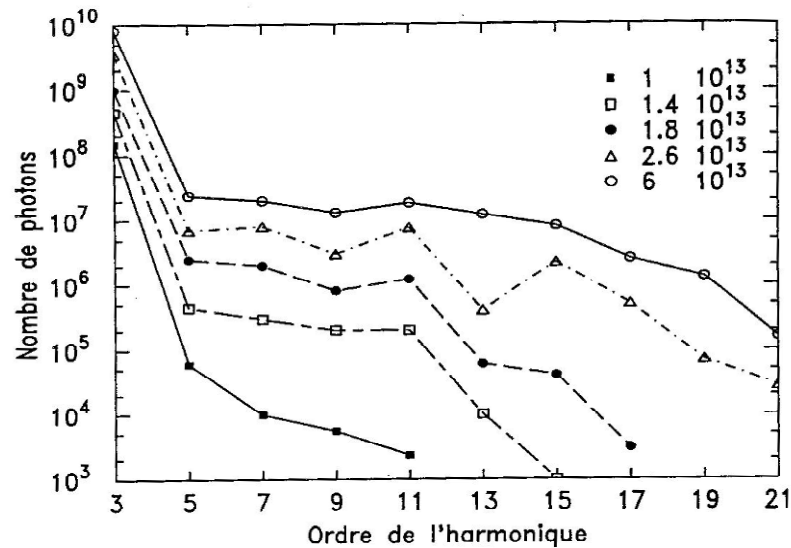
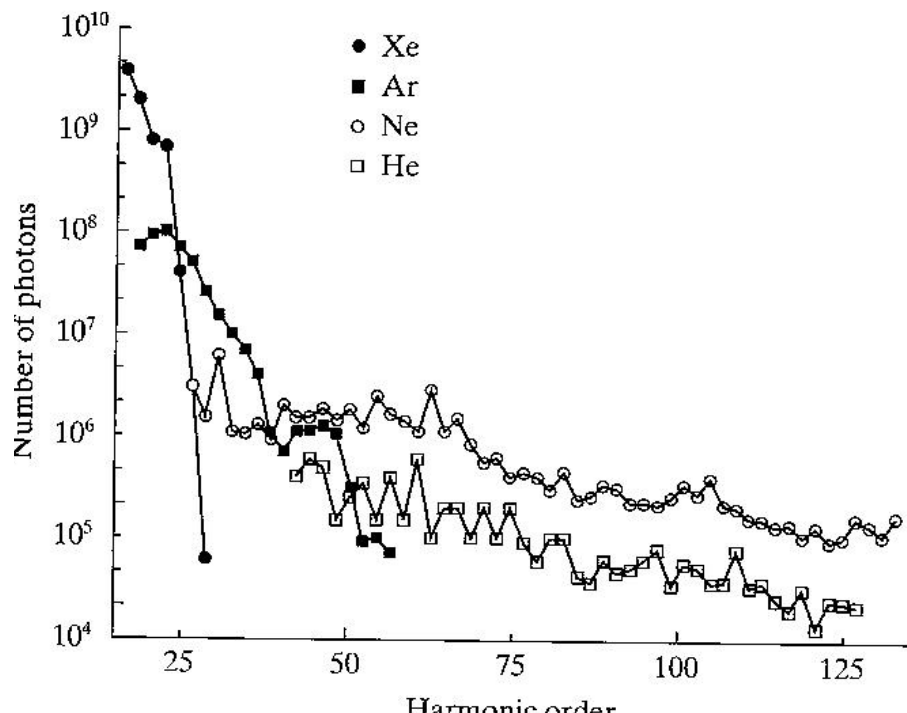
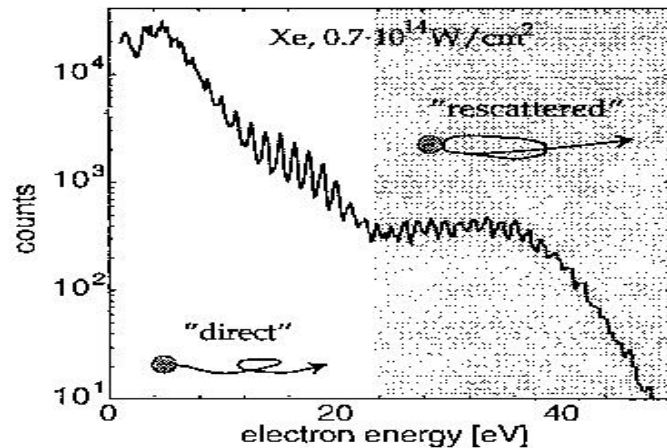
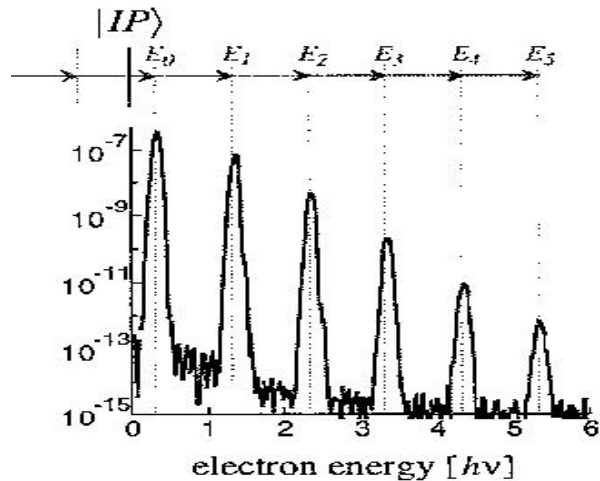
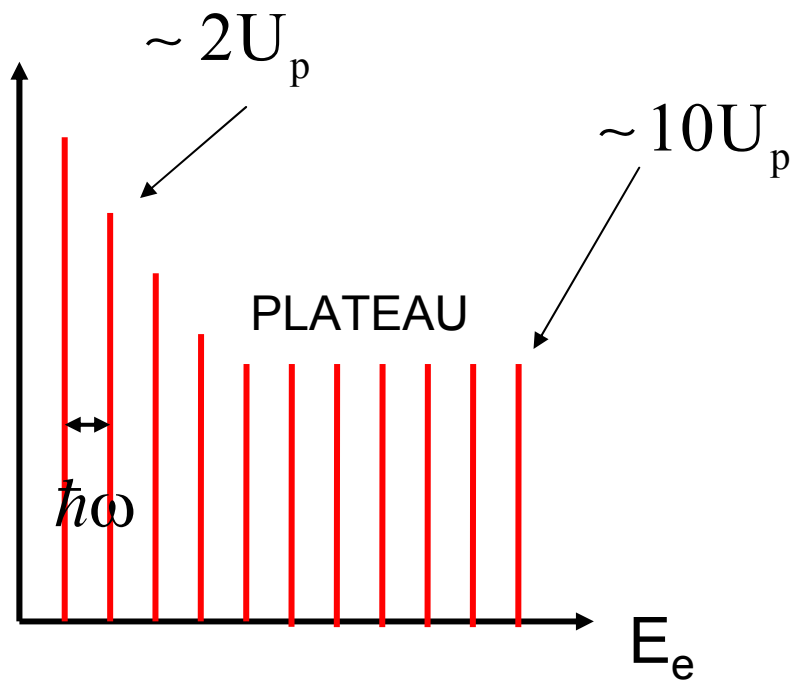


Figure I-1. Spectres d'harmoniques obtenus à 1064 nm dans le xénon, à différents éclaircissements.

Electron spectrum

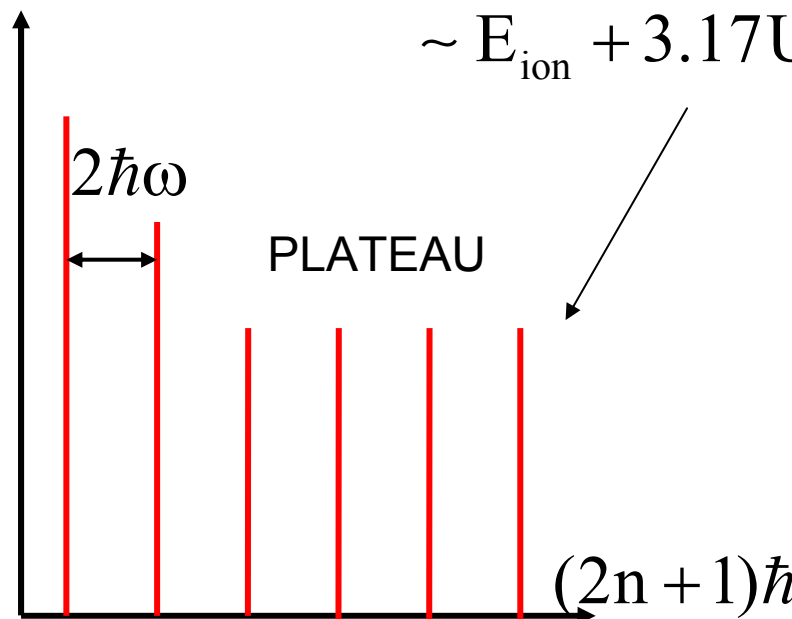


ATI → Tunneling

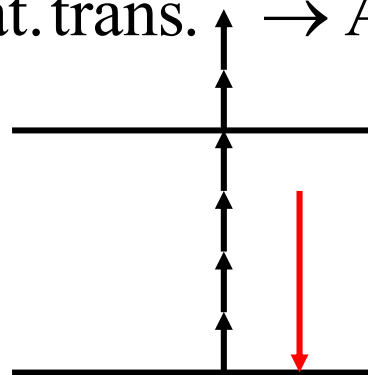
- Quantification: transition into a Volkov state

- Cut-off energy?

Radiation spectrum



Intra at. trans. → ATI + Tunneling



- Odd harmonics: inversion symmetry

- Cut-off energy?

Above Threshold Ionisation: Important features

1- Cut off energy

$$2U_p$$

Classical explanation.

$$E = E_0 \cos \omega t; v(t_0) = 0 \text{ (electron created at rest)}$$

$$E_{\text{kin}} = 2U_p \cos^2 \omega t_0 : 0 \rightarrow 2U_p$$

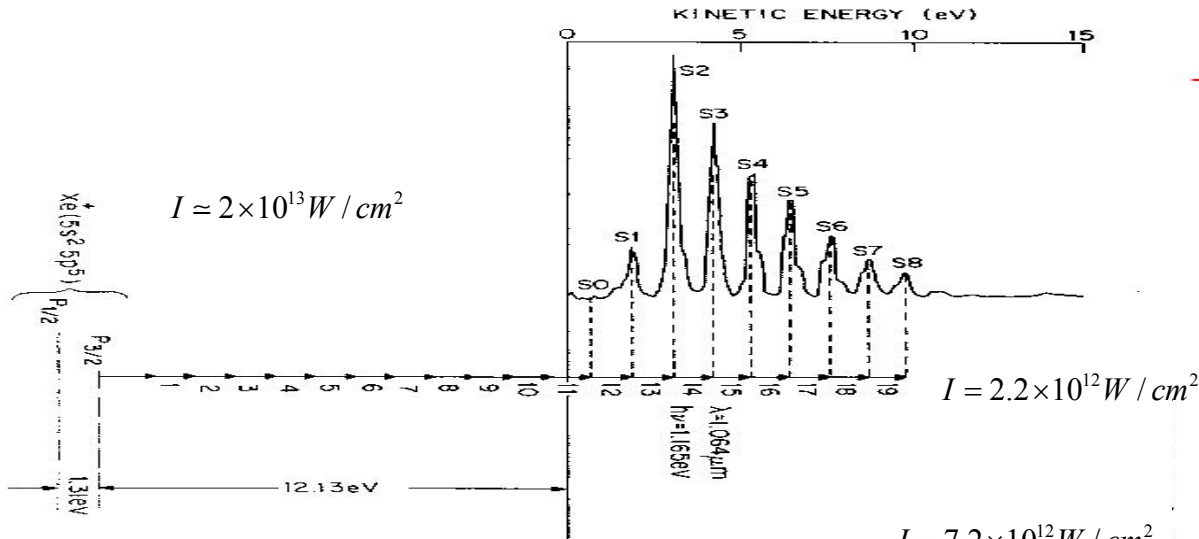
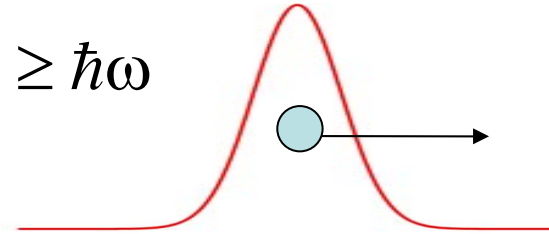
Drift motion even if $v(t_0) = 0$
(phase shift effect)

2- Peak suppression

$$E_{\text{elec.}} = (n + s)\hbar\omega - E_{\text{ion}} \begin{cases} \geq U_p & \text{if conversion potential} \leftrightarrow \text{kinetic} \\ \geq 0 & \text{else} \end{cases} \quad (1)$$

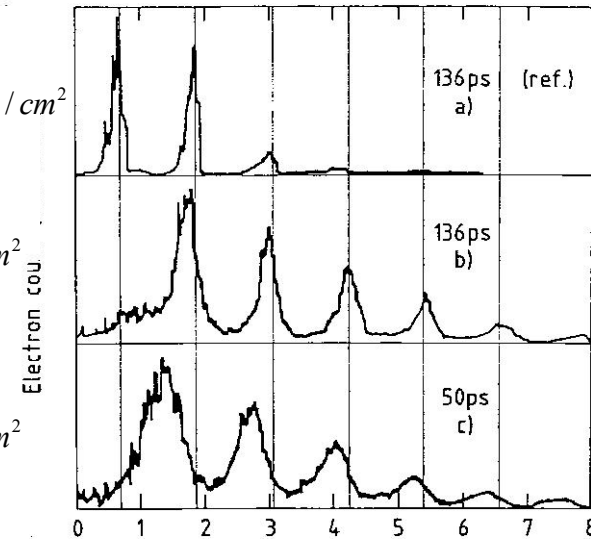
$$(2)$$

(1) long pulse → Peak suppression when $U_p \geq \hbar\omega$



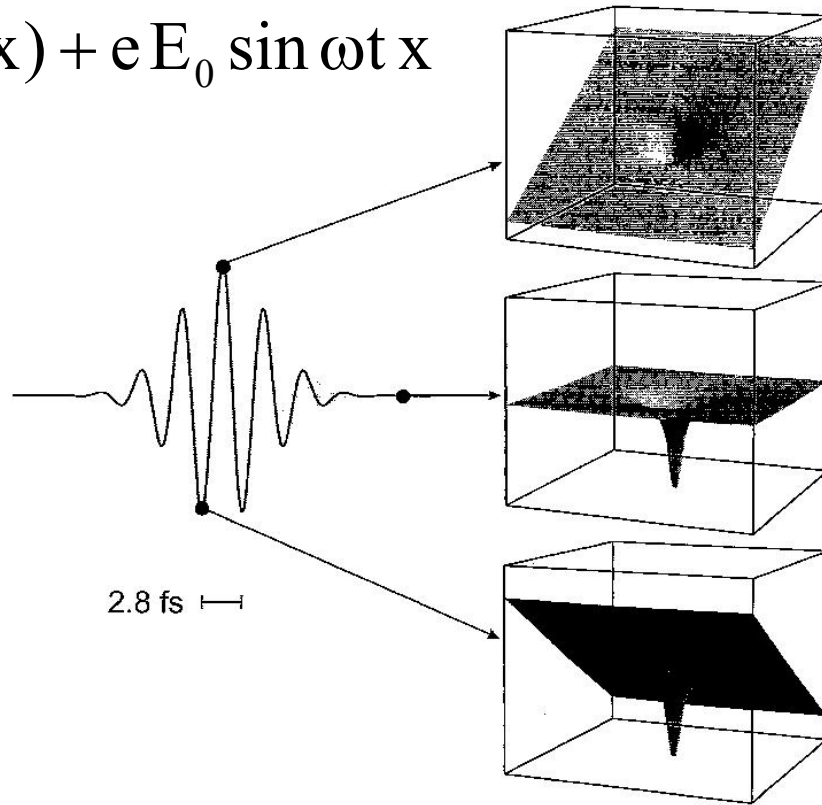
(2) Short pulse

$$I = 7.2 \times 10^{12} W / cm^2$$



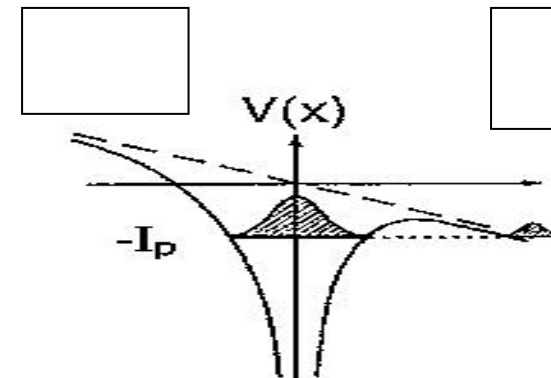
Tunnel Ionisation

$$U(x) = V(x) + eE_0 \sin \omega t x$$



Time dependent Barrier

Maximum of Ionisation when $\omega t = \frac{\pi}{2} [\pi]$



Tunnel Ionisation: Keldysh parameter

$$\gamma = \frac{\text{time needed to escape}}{\text{optical period}}$$

$$\gamma = \sqrt{\frac{E_{\text{ion}}}{2U_p}}$$

$\gamma < 1 \rightarrow$ Tunneling

$\gamma > 1 \rightarrow$ Multiphoton Ionisation

Ionisation rate in the quasi-static regime (**A**mnosov, **D**elone, **K**rainov formula)

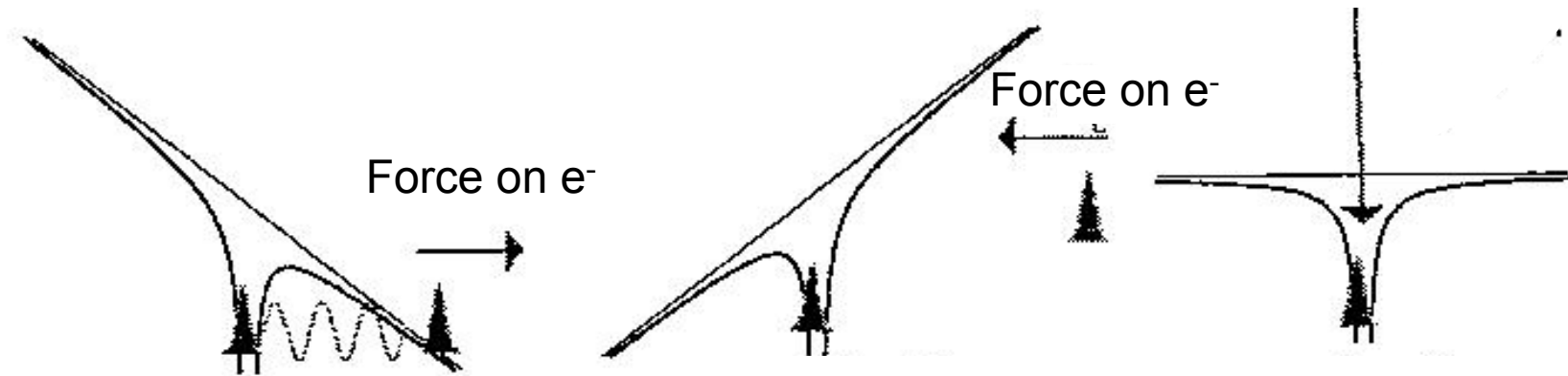
Initially electron $|n^*, l, m\rangle$; $l^* = n^* - 1$; $m = Z(2E_{\text{ion}})^{1/2} \quad m_e = \hbar = c =$

$$\Gamma_{\text{ion}} = \sqrt{\frac{3E_0}{\pi(2E_{\text{ion}})^{3/2}}} |C_{n^*l^*}|^2 f(l, m) E_{\text{ion}} \left(\frac{2(2E_{\text{ion}})^{3/2}}{E_0} \right)^{\left(\frac{2Z}{\sqrt{2E_{\text{ion}}}} - |m| - 1 \right)} e^{-\frac{2}{3E_0}(2E_{\text{ion}})}$$

$$f(l, m) = \frac{(2l+1)(1+|m|)}{2^{|m|} |m|! (1-|m|)!} \quad |C_{n^*l^*}|^2 = \frac{2^{ln^*}}{n^* \Gamma(n^* + l^* + 1) \Gamma(m^* - l^*)}$$

Tunnel Ionisation: cut-off energies

Three-step model $E = E_0 \sin \omega t$



① Tunneling ionization

② Oscillation in the laser field

③ Re-collision

$$t = t_0$$

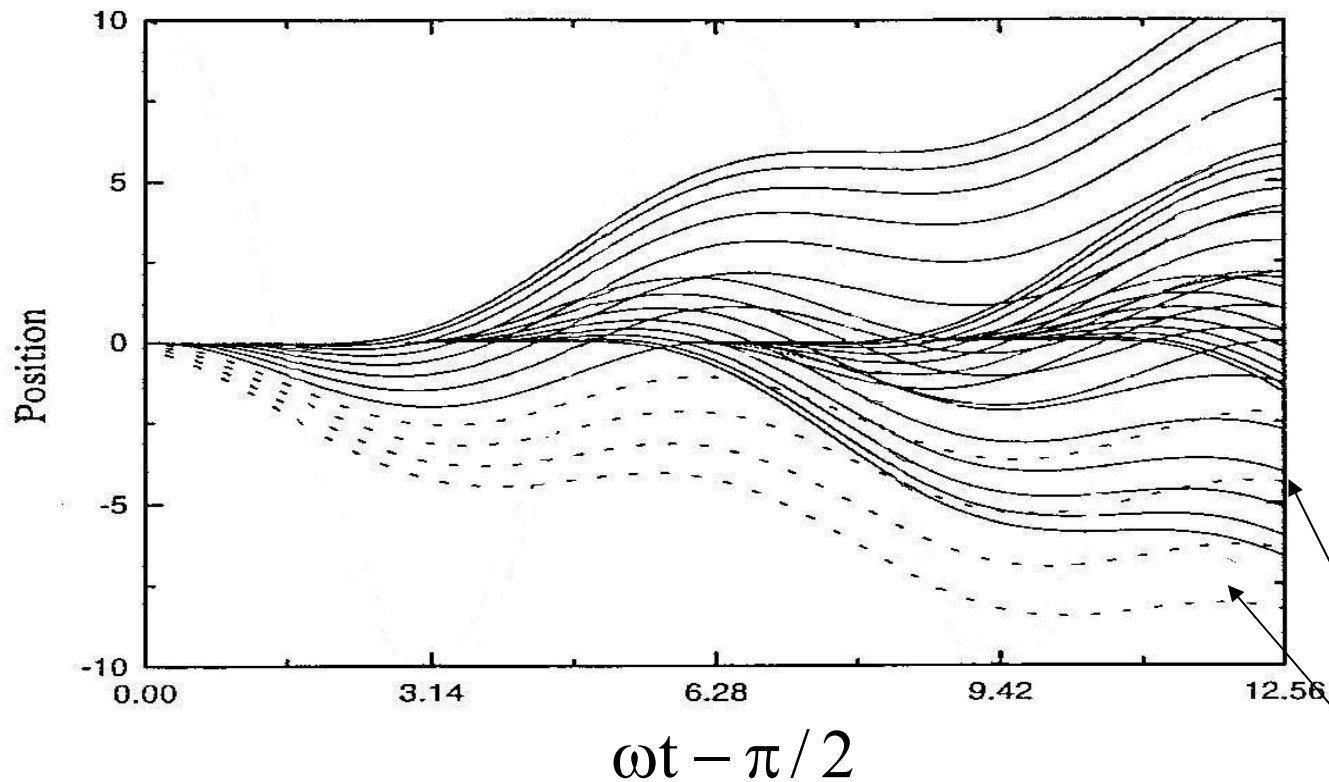
$$\omega t_0 \approx \frac{\pi}{2} [\pi]$$

$$x = \frac{-eE_0}{m\omega^2} (\sin \omega t - \sin \omega t_0)$$

$$+ \frac{eE_0}{m\omega^2} (\omega t - \omega t_0)$$

$$t = t_1(t_0)$$

TRAJECTORIES



..... : No return



$$0 \leq \omega t_0 \leq \frac{\pi}{2} \quad [\pi]$$

— : Return



$$\frac{\pi}{2} \leq \omega t_0 \leq \pi \quad [\pi]$$

Recombination \longrightarrow HHG plateau

$$t = t_1 \quad \mathbf{x}(t_1) = 0$$

$$\langle E_{\text{kin}} \rangle = \frac{e^2 E_0^2}{2m\omega^2} (\cos \omega t_1 - \cos \omega t_0)^2$$

$$\text{max when } \omega t_0 \simeq 108^\circ \rightarrow \omega t_1 \simeq 342^\circ \rightarrow \langle E_{\text{kin}} \rangle = 3.17U_p$$

$$\rightarrow N_{\text{max}} \hbar\omega = E_{\text{ion}} + 3.17U_p$$

e-collision:

Diffusion \longrightarrow ATI Plateau

$$t = t_1^+ \quad \dot{\mathbf{x}}(t_1^+) = -\dot{\mathbf{x}}(t_1) \text{ (best situation)}$$

$$E_{\text{drift}} = \frac{eE_0^2}{2m\omega^2} (2 \cos \omega t_1 - \cos \omega t_0)^2$$

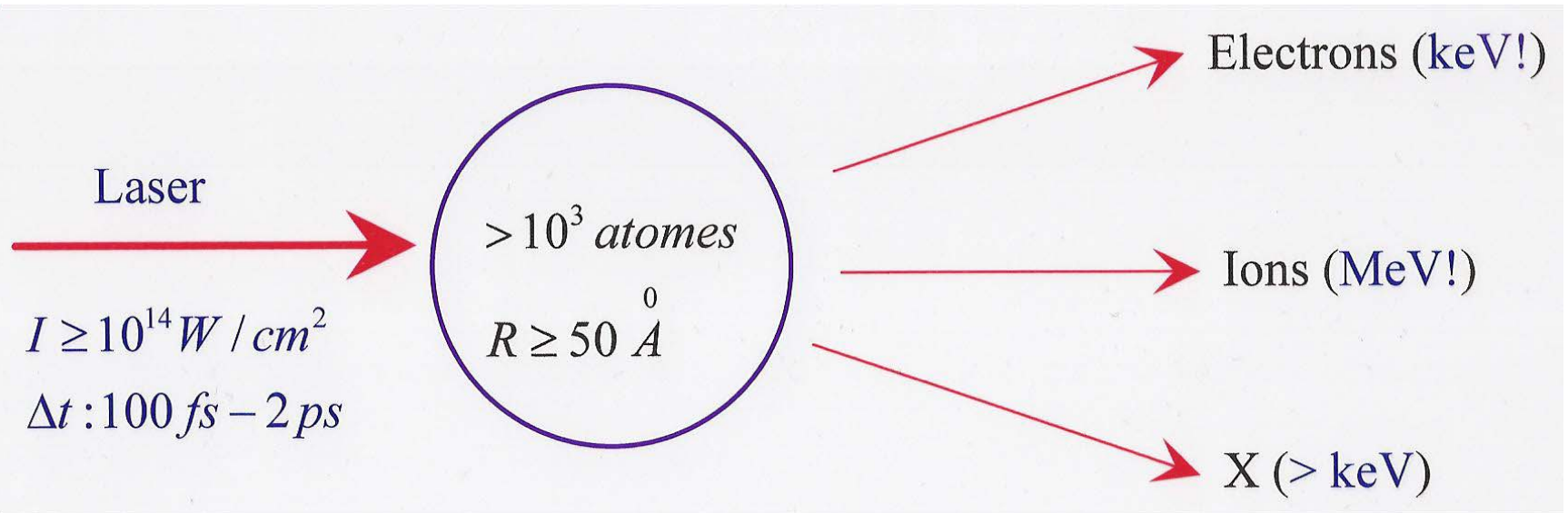
$$\text{max when } \omega t_0 \simeq 105^\circ \rightarrow \omega t_1 \simeq 351.7^\circ \rightarrow E_{\text{drift}} = 10U_p$$

PERIOD $T/2 \longrightarrow 2\omega$

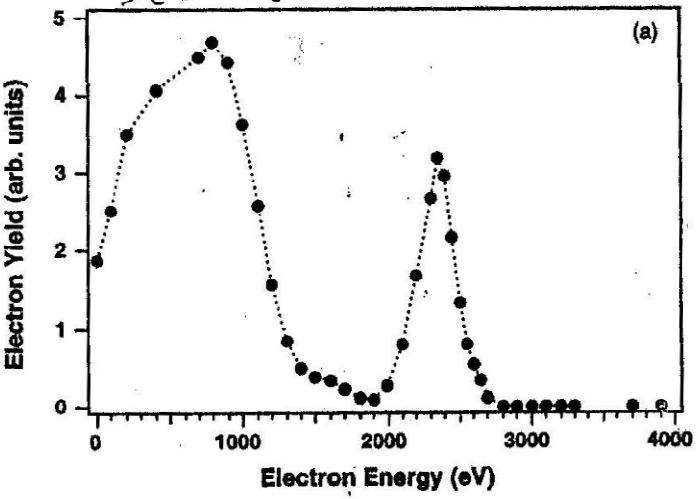
FURTHER DEVELOPMENTS

- Attosecond pulse generation (mode locking of HH)
- Coherent sources in the VUV and XUV
- Cluster explosion :
neutron sources
Highly energetic particles

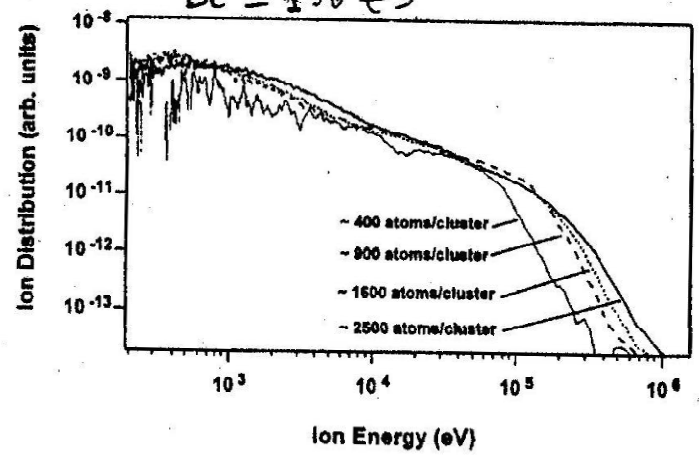
CLUSTER EXPLOSION



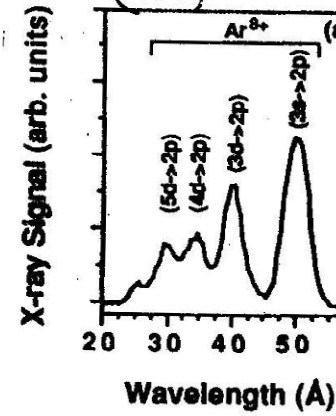
$I = 10^{16} \text{ W/cm}^2$
 $\Delta t \approx 150 \text{ fs}$



(Xe)
 $I = 2 \cdot 10^{16} \text{ W/cm}^2$
 $\Delta t \approx 150 \text{ fs}$



$I = 8 \cdot 10^{16} \text{ W/cm}^2$
 $\Delta t = 130 \text{ fs}$
 $(\text{Ar})^n$



DITMIRE et al.
 (NATURE, 8 April 99)
 FUSION!

